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ENGINEERING AND DESIGN

DESIGN OF STRUCTURES TO RESIST THE  
EFFECTS OF ATOMIC WEAPONS

SINGLE-STORY FRAME BUILDINGS

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ENGINEERING AND DESIGN

DESIGN OF STRUCTURES TO RESIST THE EFFECTS OF ATOMIC WEAPONS

SINGLE-STORY FRAME BUILDINGS

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## ENGINEERING AND DESIGN

### DESIGN OF STRUCTURES TO RESIST THE EFFECTS OF ATOMIC WEAPONS

#### SINGLE-STORY FRAME BUILDINGS

##### INTRODUCTION

7-01 PURPOSE AND SCOPE. This manual is one in a series issued for the guidance of engineers engaged in the design of permanent type military structures required to resist the effects of atomic weapons. It is applicable to all Corps of Engineers activities and installations responsible for the design of military construction.

The material is based on the results of full-scale atomic tests and analytical studies. The problem of designing structures to resist the effects of atomic weapons is new and the methods of solution are still in the development stage. Continuing studies are in progress and supplemental material will be published as it is developed.

The methods and procedures were developed through the collaboration of many consultants and specialists. Much of the basic analytical work was done by the engineering firm of Ammann and Whitney, New York City, under contract with the Chief of Engineers. The Massachusetts Institute of Technology was responsible, under another contract with the Chief of Engineers, for the compilation of material and for the further study and development of design methods and procedures.

It is requested that any errors and deficiencies noted and any suggestions for improvement be transmitted to the Office of the Chief of Engineers, Department of the Army, Attention: ENGEB.

7-02 REFERENCES. Manuals - Corps of Engineers - Engineering and Design, containing interrelated subject matter are listed as follows:

#### DESIGN OF STRUCTURES TO RESIST THE EFFECTS OF ATOMIC WEAPONS

EM 1110-345-413 Weapons Effects Data  
EM 1110-345-414 Strength of Materials and Structural Elements  
EM 1110-345-415 Principles of Dynamic Analysis and Design

EM 1110-345-416 Structural Elements Subjected to Dynamic Loads  
EM 1110-345-417 Single-Story Frame Buildings  
EM 1110-345-418 Multi-Story Frame Buildings  
EM 1110-345-419 Shear Wall Structures  
EM 1110-345-420 Arches and Domes  
EM 1110-345-421 Buried and Semi-Buried Structures

a. References to Material in Other Manuals of This Series. In the text of this manual references are made to paragraphs, figures, equations, and tables in the other manuals of this series in accordance with the number designations as they appear in these manuals. The first part of the designation which precedes either a dash, or a decimal point, identifies a particular manual in the series as shown in the table following.

<u>EM</u>	<u>paragraph</u>	<u>figure</u>	<u>equation</u>	<u>table</u>
1110-345-413	3-	3.	(3. )	3.
1110-345-414	4-	4.	(4. )	4.
1110-345-415	5-	5.	(5. )	5.
1110-345-416	6-	6.	(6. )	6.
1110-345-417	7-	7.	(7. )	7.
1110-345-418	8-	8.	(8. )	8.
1110-345-419	9-	9.	(9. )	9.
1110-345-420	10-	10.	(10. )	10.
1110-345-421	11-	11.	(11. )	11.

b. List of Symbols. Definitions of the symbols used throughout this manual series are given in a list following the table of contents in EM 1110-345-413 through EM 1110-345-416.

7-03 RESCISSIONS. (Draft) EM 1110-345-417 (Part XXIII - The Design of Structures to Resist the Effects of Atomic Weapons, Chapter 7 - Single-Story Frame Buildings).

7-04 GENERAL. This manual presents four numerical examples illustrating design procedures and principles given in EM 1110-345-413 through -416.

The examples presented are as follows:

- (1) The design of a one-story steel frame building, plastic deformation permitted.
- (2) The design of a one-story steel frame building, elastic behavior.
- (3) The design of a one-story reinforced concrete frame building, plastic deformation permitted.
- (4) The design of a one-story reinforced concrete frame building, elastic and elasto-plastic behavior.

Before illustrating the design of buildings to resist blast loads it is desirable to describe the behavior of the elements of a building frame subjected to blast loads. Accordingly, the first part of this manual is devoted to a description of the response of single-story frame buildings to vertical and lateral blast loads. In the general discussion of frames, it is assumed that the exterior walls are framed vertically between the foundation and the roof so that the columns are loaded laterally only at the top of the frame and are not subject to direct lateral loads such as to cause them to resist these loads by beam action. Columns subjected to directly applied lateral loads are discussed briefly in paragraph 7-12.

7-05 BEHAVIOR OF SINGLE-STORY FRAMES. Single-story frame buildings subjected to lateral blast loads suffer a lateral deflection which is determined by the mass, stiffness, and strength of the structure, the variation of loading with time, the distribution of load on the structure, and the dynamic behavior of the walls and roof. The lateral loads on all walls are transmitted to the roof and the foundations by vertical framing and are carried laterally by the roof slab or roof lateral bracing to the girders of the frame which in turn transmit the load to the columns. The columns carry the lateral loads to the foundation where the reactions are provided by friction and passive pressure forces. The lateral blast loads on the walls are transmitted to the frame girders by either a lateral truss system spanning between frames, or by the roof slab acting as a deep lateral beam.

The resistance of a building frame to lateral loads is a function of the stiffness of the frame columns to relative displacement of the roof and the foundation. The equivalent single-degree-of-freedom dynamic system for the single-story frame is a concentrated mass supported by a massless spring having the lateral resistance properties of the columns (fig. 7.1). The be-

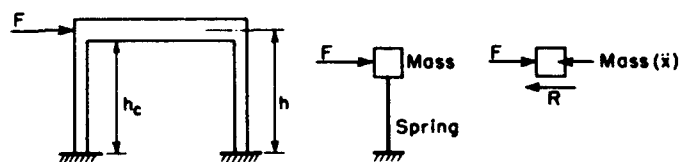


Figure 7.1. Single-story frame and equivalent dynamic system

havior of columns in a frame, and the procedure for designing the columns in single-story buildings, are discussed in paragraphs 7-06 to 7-10. The design of roof girders is covered in paragraph 7-11.

7-06 SHEAR AND MOMENT RESISTANCE OF COLUMNS. Each column resists the lateral motion of the frame through the action of shear forces and bending moments in the columns as indicated in figure 7.2. The shear resistance

in terms of the column bending moments and the axial load is

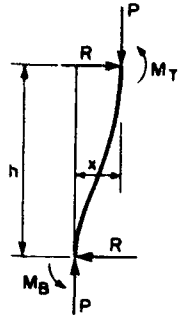


Figure 7.2. Shear resistance and bending moments in a column subjected to lateral displacement and vertical load

$$R = \frac{M_T + M_B - Px}{h} \quad (\text{elastic range}) \quad (7.1)$$

$$R = \frac{M_T + M_B - Px}{h_c} \quad (\text{plastic range}) \quad (7.1a)$$

where  $h_c$  is the clear height of the column as in figure 7.1. In the elastic range the effect of joint rotation should be included. Thus, using slope-deflection equations where  $\theta_B$  and  $\theta_T$  are joint rotations at bottom and top of the column, respectively, the moments are

$$M_T = \frac{2EI}{h} \left( 2\theta_T + \theta_B - \frac{3x}{h} \right) \quad (7.2)$$

$$M_B = \frac{2EI}{h} \left( 2\theta_B + \theta_T - \frac{3x}{h} \right)$$

In the plastic range the top and bottom moments are assumed to be equal to a maximum moment  $M_D$  so that

$$R = R_m = \frac{2M_D - Px}{h_c} \quad (7.3)$$

The value of  $M_D$  to be used in equation (7.3) is a variable dependent upon the direct stress. The effect of direct stress is discussed in paragraphs 4-07, 4-11b, and 7-07. If there is no direct stress,  $M_D$  can be replaced by  $M_P$  giving

$$R_m = \frac{2M_P}{h_c} \quad (7.3a)$$

In equations (7.1) and (7.3),  $R$  is a function of  $M$ ,  $P$ ,  $x$ , and  $h$  or  $h_c$ . In a given design two of the four factors are known;  $h$  and  $h_c$  are constants and  $P$  is of known variation with time. The remaining two terms are related; i.e., for a given column,  $M$  is a function of  $x$  and  $P$ ,



so that the variation of  $M$  and then  $R$  may be determined from the variation of  $x$  and  $P$ .

For a frame with infinitely rigid girders, in the elastic range, the relationship between  $M$  and  $x$  in any column is obtained from equation (7.2) by setting  $\theta_T = \theta_B = 0$ .

$$M = M_T = M_B = \frac{6EIx}{h^2} \quad (7.4)$$

With infinitely rigid girders in the elastic range equation (7.1) becomes

$$R = \frac{2M - Px}{h} = \frac{12EIx}{h^3} - \frac{Px}{h} \quad (7.5)$$

Equation (7.5) may be written in the form

$$R = kx - \frac{Px}{h} \quad (7.6)$$

from which the equation for  $k$  for one column is

$$k = \frac{12EI}{h^3} \quad (7.7)$$

To obtain the maximum elastic displacement  $x_e$  defined by figure 7.3, it is necessary to obtain the maximum or plastic resistance  $R_m$  and divide by the spring constant  $k$ . For a complete frame with  $n$  columns from equation (7.7)

$$k = n \frac{12EI}{h^3} \quad (7.8)$$

and from equation (7.3a), neglecting the entire effect of direct stress,

$$R_m = n \frac{2M_P}{h_c} \quad (7.9)$$

so that

$$x_e = \frac{R_m}{k} = \frac{M_P h^3}{6EI n h_c} \quad (7.10)$$

Equations (7.8) and (7.9) apply only when all the columns of the story are identical in strength and stiffness. If this condition is not true, the equations are modified as follows:

$$k = \frac{12E \sum I}{h^3} \quad (7.11)$$

$$R_m = \frac{2\Sigma M_p}{h_c} \quad (7.11a)$$

where

$\Sigma M_p$  = sum of plastic column moments in the story, and

$\Sigma I$  = sum of  $I$  values for all columns in the story.

For the case in which the direct stress is considered important, the maximum resistance equation from equation (7.3) becomes

$$R_m = \frac{2M_{Dn} - Px}{h_c} \quad (7.12)$$

and the maximum elastic displacement from equation (7.10) becomes

$$x_e = \frac{M_D h^3}{6EIh_c} - \frac{Pxh^3}{12EIh_c} \quad (7.13)$$

where  $M_D$  is a function of  $P$  (par. 7-07). Equation (7.12) should be used in numerical analyses where the effect of  $P$  and  $x$  can be introduced.

For preliminary design purposes wherein it is desirable, in order to simplify the computations, to account for the approximate effect of  $P$ , the design should be based on

$$R_m = \frac{2M_{Dn}}{h_c} \quad (7.14)$$

7-07 THE EFFECT OF DIRECT STRESS ON COLUMN RESISTANCE. The value of  $M_D$  to be used in equations (7.12) and (7.13) is variable dependent upon the direct stress (pars. 4-07 and 4-11). The relationship is different for steel and reinforced concrete. A reinforced concrete section carrying both direct stress and bending moment has a higher moment-carrying capacity for a limited but important range of axial loads than the same section carrying only bending moment. However, a structural steel section carrying both direct stress and bending moment has a lower moment-carrying capacity than the same section carrying only bending moment.

The limiting elastic deflection when significant axial loads are present is determined from equation (7.13), where  $M_D$  is determined from a curve of  $P_D$  vs  $M_D$  prepared as described in paragraphs 4-07 and 4-11 (see figs. 4.12 and 4.26). As long as the moment  $M$  as determined from equation (7.4) and the axial load  $P$  together determine a point on the  $P_D - M_D$

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graph which is inside the curve, the action of the column is elastic, and the  $M$  as calculated is used to determine  $R$  from equation (7.5). If the point determined by the values of  $P$  and  $M$  lies outside the  $P_D - M_D$  curve, the action is plastic and the limiting moment  $M_m$  is the value of  $M_D$  corresponding to the axial load  $P$ .

In the preliminary design of reinforced concrete frames, it is desirable to introduce the increased bending strength that results from axial stress in the columns. If this effect is neglected, the preliminary design is generally very conservative. By introducing the direct stress effect a more reasonable column size can be determined.

For steel columns the effect of direct stress is much less important and, in most cases, reasonable results are obtained by neglecting the effect of direct stress in the preliminary column design method of this manual. However, the effect of direct stress is usually considered in making the numerical analysis which is used to check the preliminary design. Column buckling under combined axial load and bending must be prevented in order to maintain the lateral resistance of the frame. In many designs the column section is determined by buckling considerations. For the buckling criteria refer to paragraph 4-07.

In the numerical integration method used to check the preliminary design results, the more comprehensive procedure involves consideration of the individual column direct stresses and their effect on the bending resistance of the individual column. A study has been made to determine whether this precision is necessary. In a series of typical problems, the variation of resistance was determined on two bases: (1) average direct stress equal to the sum of the column loads divided by the sum of the column areas and (2) direct stress determined separately for each column and applied to that column. It has been determined that there is very little loss in accuracy if the average direct stress is used.

7-08 THE EFFECT OF GIRDER FLEXIBILITY ON COLUMN RESISTANCE. In the preliminary design of the columns, the frame response is determined on the basis of the assumption that the joint rotations are negligible. If the girders are designed to act in the elastic range (par. 7-11), the error involved is not large. If all the columns in a story have the same section

and are of equal height, this assumption results in equal moments at the top and bottom of all columns and a linear variation of resistance with displacement up to the plastic resistance. If the assumption of infinitely stiff girders is not made, the resistance-deflection diagram for a given frame may be determined by a conventional sidesway analysis.

Neglecting the flexibility of the girders results in an overestimate of the energy absorption capacity of the frame, thus resulting in underestimates of the displacement of the structure, and the required resistance of the columns. Any procedure which reduces the required resistance to a value below that needed by the more exact procedure is unconservative. It is not desirable to incorporate the flexibility effects into the preliminary design procedure. The designs obtained by the preliminary design method should be recognized as being slightly unconservative and allowance should be made for this difference by the designer. It is desirable to include this flexibility effect when the preliminary design is checked by a numerical integration procedure.

The recommended procedure for approximating the effect of girder flexibility for use in the numerical integration analysis of single-story structures is described below. Figure 7.3 presents the form of the

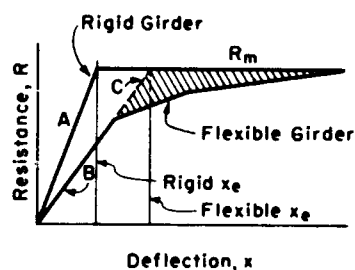


Figure 7.3. Effect of girder flexibility on resistance-deflection diagram of multibay frames

resistance-deflection diagram for a typical multi-column frame subject to lateral load only. Line A represents the resistance for infinite girder stiffness. With infinite girder stiffness, plastic hinges would develop simultaneously at both ends of all columns. If the actual girder flexibility is considered, the hinges would be found to develop successively as indicated by line B. The recommended resistance diagram is line C, an extension of the initial slope of line B to the intersection

with the line of maximum resistance. The shaded area represents the error introduced. Use of line C will result in the calculated deflections being smaller than the true deflection. However, the error involved is generally very small. To obtain the effective spring constant  $k$  for girder flexibility, it is necessary to determine the slope of line C. This can be

determined by imposing an arbitrary lateral deflection upon the frame and calculating the resistance corresponding to this deflection. The ratio of the resistance to the displacement is  $k = R/x$ . Paragraph 7-26 illustrates the simple elastic frame analysis that is needed for this determination.

7-09 EFFECT OF LATERAL DEFLECTION ON COLUMN RESISTANCE. From equation (7.1) it may be seen that the resistance is subject to reduction by the combined effect of the lateral deflection and the axial column loads. In single-story frames this effect is small and is neglected in the preliminary design procedures, but it is included in the final numerical analysis.

7-10 DESIGN OF COLUMNS. A general preliminary design procedure for plastic behavior is presented in paragraph 6-11 and for elastic behavior in paragraph 6-12. Details peculiar to application of these methods to single-story column designs are explained below and illustrated in paragraphs 7-24 and 7-34 for steel frames, and paragraphs 7-42 and 7-50 for reinforced concrete frames. The loading used in the preliminary column design is the net lateral blast load as computed from procedures in paragraph 5-09, neglecting the effect of dynamic response of wall panels and other intervening structural elements. The dynamic effects of the mass and structural properties of the walls are accounted for in the final check of the column section by using the dynamic reactions to the front and rear walls in the numerical integration. The equivalent mass concentrated at the top of the column is given by

$$m = \text{total roof mass} + 1/3 \text{ column mass} + 1/3 \text{ wall mass}$$

In the preliminary design of steel columns the frame girders are assumed to be perfectly rigid, and the axial load in the columns is neglected so that the required moments, the spring constant, and the limiting elastic deflection of the columns can be obtained from equations (7.8) and (7.10). From equation (7.9)

$$M_p = \frac{R h}{\frac{m}{2n} c} \quad (7.15)$$

The cross section required to provide the plastic bending moment resistance  $M_p$  is determined from data in paragraph 4-07 for steel columns.

The preliminary design procedure for reinforced concrete columns is different from that for steel because allowances are made for the effect of

direct stress on the bending resistance of the column. After solving for the required  $M_D$  from equation (7.14), it is necessary to determine the dimensions of the cross section. Since  $M_D$  is a function of  $P$  (par. 4-11) it is necessary to use the time average of the total axial load  $P$  for the time interval estimated to be the plastic phase.

Using equations from paragraph 4-11b for eccentrically loaded columns, a cross section is selected which will provide the necessary  $M_D$  at the average  $P$ .

Having determined the necessary cross section, the next step is to compute  $k$  and  $x_e$ . For both steel and concrete,  $k = 12EI_n/h^3$ . However, from equation (7.10)

$$\begin{aligned} x_e &= \frac{M_P h^3}{6EI_n} \text{ for steel} \\ x_e &= \frac{M_D h^3}{6EI_n} \text{ for concrete} \end{aligned} \quad (7.16)$$

These parameters are used in the remainder of the design procedure without modification by any load or mass factors because the single-story frame is considered to be directly replaceable by a single-degree system. The preliminary design is then completed in accordance with the steps of either paragraph 6-11 or paragraph 6-12.

After the girder is designed, the preliminary column design is verified by means of a step-by-step numerical integration procedure. It is required that the displacement of the top of the column determined by this computation be reasonably close to the design displacement determined from consideration of paragraph 6-26.

The more exact numerical analysis includes all the factors which have been neglected in order to simplify the preliminary procedure. These are: the effect of girder flexibility, the effect of vertical load eccentricity, the effect of direct stress, and the effect of the dynamic response of the wall and roof elements on the lateral and vertical load-time curves. As discussed in paragraph 7-07, a simplification is possible by using the average column axial loads instead of considering the individual columns separately. The effect of girder flexibility is determined as indicated in paragraph 7-08. Examples of the numerical

integration procedure applied to steel column designs are presented in paragraphs 7-26 and 7-36, and for reinforced concrete columns in paragraphs 7-44 and 7-52.

7-11 DESIGN OF ROOF GIRDERS. The design of roof girders in building frames for blast loads is a difficult problem which is complicated by the time variation of the lateral and vertical blast loads, and the difference in time required for the different girders to reach maximum stress. In general, the maximum frame moments due to the vertical loads develop before those due to the lateral loads. Conventional static loads must be considered in addition to the blast loads. In a building with openings it is possible to have internal pressures of such magnitude as to develop net upward forces on the roof girders.

To obtain the maximum lateral stiffness for the building frame, it is desirable that the roof girders in frames be designed to act elastically. To simplify design procedures, continuous-span beams can be considered as single-span elements with restraints as indicated in figure 7.4. It is recom-

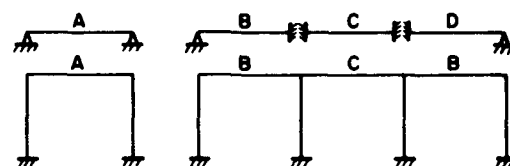


Figure 7.4. Single-span support assumptions for design of frame girders

mended that single-span frame girders such as A be designed elastically to carry vertical loads as simply supported beams. Exterior girders such as B should be designed elastically to carry vertical loads as beams fixed at the first interior support and pinned at the exterior support. Interior girders such as C should be designed elastically to carry the vertical loads as beams fixed at both ends. In order to develop the maximum lateral resistance of the frame, it is necessary to design the girders so that the plastic hinges form in the columns. To insure this behavior, the bending strength of the girders at any point must equal the moment at that point due to static and dynamic vertical loads, plus the moment due to lateral motion. The latter is computed by applying to the girder the full plastic hinge moments of all the columns simultaneously. This is conservative because it assumes that all the maximum moments develop at the same time.

Consideration of the blast loading on frame girders shows that the front girder is the critical girder in a multibay frame and the critical section is at the first interior support. At this section the frame moment

in the girder should be a fraction of the column maximum moment  $M_D$ . For a 2-bay frame use  $1/2 M_D$ . For a 3-bay frame use  $2/3 M_D$  and for a 4-bay frame use  $5/8 M_D$ .

The design procedure recommended for girders is an elastic design based upon paragraph 6-12 and consists of the following steps which are illustrated in paragraphs 7-25, 7-35, 7-43, and 7-51. In order to perform these operations, it is necessary to refer freely to other manuals for information; viz., to EM 1110-345-413 for overpressure-time variation on roofs, to EM 1110-345-414 for the equations governing the plastic moment capacity of steel and concrete girders, to EM 1110-345-415 for the elastic response characteristics of single-degree-of-freedom systems, and to EM 1110-345-416 for the factors which define the equivalent single-degree-of-freedom system.

Step 1. Obtain the vertical load-time curve for the girder from the dynamic reaction of the roof element which the girder supports. Idealize the curve to a form for which dynamic load factors are available.

Step 2. Estimate the dynamic load factor for preliminary size determination.

Step 3. Calculate  $T_n$ , the period of vibration of the equivalent single-degree-of-freedom system.  $T_n$  is a function of mass  $m$  and spring constant  $k$ . The equations for  $k$  are given in EM 1110-345-416. The mass to be used is the mass which is considered to move with the girder. In EM 1110-345-416 consideration is given to beams which have mass variations that are triangular in spanwise distribution as well as concentrated at local points.

Step 4. Using  $T_n$  and  $T$ , the time parameter of the loading, obtain a new value of dynamic load factor from figures 5.20 and 5.21.

Step 5. With the new dynamic load factor, determine the design moment required for vertical blast loads. Determine the total moment by adding the static load moments and the proper fraction of the column plastic moment. Select a size to withstand the indicated bending moments in accordance with EM 1110-345-414 requirements.

Step 6. Repeat the cycle, computing  $T_n$ ,  $T/T_n$ , and D.L.F., and check the section for the revised bending moments including the allowances



described above until satisfactory agreement is realized.

Step 7. If the loading curve cannot be approximated by the idealized shapes used in figures 5.20 and 5.21 it may be necessary to perform a step-by-step numerical integration to check the design for the loading curve.

7-12 FRAMES WITH LATERAL LOADS ON THE COLUMNS. There are certain arrangements of the structural elements in framed buildings which would require that the columns be capable of resisting directly applied transverse loads in addition to providing the resistance to lateral motion of the frame.

This condition is indicated in figures 7.5(a) and (b). Figure 7.5(a) corresponds to the case of a frame building with exterior walls which act as two-way panels. Two edges of the panel load the columns directly, and the other

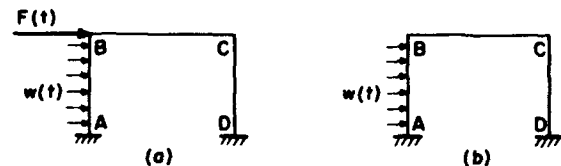


Figure 7.5. Columns subjected to directly applied lateral loads

edges transmit load to the roof and the foundation. The portion of the load transmitted to the roof is indicated by  $F(t)$ . The column loading  $w(t)$  is assumed to be uniformly distributed. Figure 7.5(b) corresponds to the case of a frame building with the exterior wall framed horizontally. In this case, there is no concentrated load  $F(t)$ .

7-13 LOADS ON FRAME STRUCTURES. The orientation of the blast wave with any element of a building should be assumed to be that which will produce the critical load on that element. The critical load for an exterior wall is produced by a blast wave acting perpendicular to the wall. Frames should be designed for the load produced by the blast moving parallel to the plane of the frame. For roof slabs the critical load is a function of two considerations, the location of the element and the direction of the blast wave. Paragraph 3-05 shows that the overpressures on the central portion of a roof of rectangular plan subjected to a blast wave moving normal to the long axis are less than the overpressures at and near the ends. For the same roof plan, all roof elements would be subjected to the same intensity of overpressure if the direction of the blast wave movement is parallel to the long axis of the building. On the central portions of square roofs the overpressures for all orientations of blast wave are less than at the edges of the roof.

The outside dimensions of the structure must be used in computing the blast loading on the structure. This means that the member sizes must be assumed initially to obtain the outside dimensions in order to compute the loads. A large difference between the assumed dimensions and final design dimensions would require a revision of loads.

7-14 FRAMING ARRANGEMENTS. In the blast resistant designs of this manual there are no radical departures from conventional framing arrangements. In reinforced concrete frame structures, the exterior columns are made independent of the exterior walls and the walls are designed to span vertically between the wall footing and the roof slab. This arrangement is desirable and most economical because it eliminates the necessity of designing the column to act as a beam spanning between the foundation and the girder in addition to providing restraint to the lateral motion of the frame.

7-15 PRELIMINARY DESIGN METHODS. The design of each element of a building consists of two steps. The first step is the preliminary design of the element using an idealized straight line load-time curve and the design charts presented in EM 1110-345-415. The second step is the numerical integration check of the preliminary design using the calculated load-time data. The following discussion deals with some of the details of the preliminary design method.

Only one mass factor and one load factor may be used in any of the preliminary design methods. Therefore, average values of these factors must be obtained for all designs in the plastic range and also for elastic designs in which there is a bilinear resistance function. In the case of a fixed-end beam designed to allow plastic deformation at midspan, the average of the elasto-plastic and plastic mass and load factors should be used to obtain the mass and load factors for use in the preliminary design. The elastic values of mass and load factors are not used in computing these average values since only a small percentage of the total deflection occurs within the elastic range.

For a simple beam designed for plastic action, the average of the elastic and plastic values of mass and load factors should be used. In the case of a fixed-end beam designed for plastic action at the support and elastic action at the centerline, the average of the elastic and

elasto-plastic mass and load factors is used in the preliminary design. In general, any reasonable method of obtaining a single set of mass and load factors will be satisfactory for use in the preliminary design.

7-16 NUMERICAL INTEGRATION ANALYSIS. The numerical integration analysis is a method of checking the preliminary design that takes into account the irregularity of the load-time curve, the variation in resistance function, and the changes in mass-load factors. In this manual each preliminary design is checked by the numerical integration method. In some cases, it may be necessary to use the numerical integration method more than once before a satisfactory design is obtained. This is particularly true where the actual load curve is of such a shape that a good approximation to it cannot be obtained by a straight line, and also in design of steel where the number of available beam sections is limited. An experienced designer may judge that a numerical integration check of some designs is unnecessary. A numerical integration analysis may be needed in some cases primarily to obtain the dynamic reactions of the element for use as the load on the supporting structure.

The load-time curves used in the numerical integration analysis are computed from either the direct-blast-pressure vs time curve or from the dynamic reactions of the supported elements. The dynamic reactions are computed by the use of the formulas of tables 6.1 to 6.6. Note that, in general, the formulas vary with the strain condition of the beam or slab. The dynamic reaction curve for an element designed to have some plastic action has a form indicated by line A in figure 7.6. Line B in figure 7.6 is the pseudostatic reaction (for a simple beam it equals one-half the applied load). The time  $t_1$  indicates the first instance after the maximum reaction develops for which the dynamic reaction is less than the pseudostatic reaction.

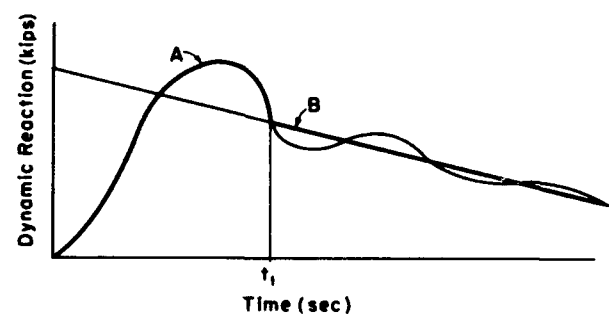


Figure 7.6. Simplification of dynamic reaction curve

If the dynamic load on the supporting structures is required for a period of time exceeding  $t_1$ , it is recommended that the load be represented

by line A before  $t_1$  and by line B after  $t_1$ . This simplifies the load shape in a reasonable manner and reduces the number of tedious steps in the numerical integration.

7-17 SUMMARY OF DESIGN EXAMPLES. In this paragraph the results of all the illustrative design examples which are contained in the remainder of this manual are summarized to provide a ready reference for preliminary designs of similar elements and buildings. Examples are for incident overpressure and duration indicated in figure 7.8.

Table 7.1. Summary of Design Examples in EM 1110-345-417

Element	Span (ft)	Size	$\alpha\beta = \frac{x_m}{x_e}$	Maximum Deflection Range	$C_R$	$T/T_n$	D.L.F.	$C_W$	$W_n$ (ft-kips)	$E$ (ft-kips)
R/C wall slab	17.5	11 in.	4.6	Plastic	0.70	1.07	----	0.365	7.42	7.62
R/C roof slab	6.67	3-3/4 in.	1.8	Plastic	1.3	18.3	----	0.0015	0.177	0.516
Steel purlin	18.0	16 W 36	6.9	Plastic	1.07	10.3	----	0.008	15.4	22.0
Steel girder	20.0	36 W 160	----	Elastic	----	0.69	1.38	----	----	----
Steel column	14.5	10 W 77	10.1	Plastic	0.22	0.27	----	0.88	157.0	236.0
R/C wall slab	11.67	14 in.	----	Elastic	----	1.57	1.7	----	----	----
R/C roof slab	5.33	4-1/4 in.	----	Elastic	----	37.0	2.0	----	----	----
Steel purlin	18.0	16 W 40	----	Elasto-plastic	----	10.8	1.95	----	----	----
Steel girder	20.0	36 W 150	----	Elastic	----	0.69	1.38	----	----	----
Steel column	8.67	12 W 120	----	Elastic	----	0.593	1.3	----	----	----
R/C wall slab	15.83	10.5 in.	3.5	Plastic	0.77	1.24	----	0.31	5.95	6.48
R/C roof slab	18.0	7-3/4 in.	5.5	Plastic	1.0	5.7	----	0.028	4.55	4.57
R/C girder	20.0	20 in. x 45 in. T	----	Elastic	----	1.25	1.18	----	----	----
R/C column	13.0	12 in. x 20 in.	6.0	Plastic	0.324	0.338	----	0.83	92.1	114.0
R/C wall slab	14.75	12-1/2 in.	----	Elasto-plastic	1.26	1.67	----	0.118	1.76	1.71
R/C roof slab	18.0	9-1/2 in.	----	Elasto-plastic	----	6.6	1.92	----	----	----
R/C girder	16.0	20 in. x 42 in. T	----	Elastic	----	1.59	1.18	----	----	----
R/C column	11.83	18 in. x 28 in.	----	Elastic	----	0.48	1.15	----	----	----

# NUMERICAL EXAMPLE, DESIGN OF A ONE-STORY STEEL FRAME BUILDING, PLASTIC DEFORMATION PERMITTED

7-18 GENERAL. This numerical example presents the design of one bay of a windowless one-story, steel, rigid-frame building with plastic deformation permitted (fig. 7.7). The design overpressure (10 psi from an 18-KT weapon) is arbitrarily selected for illustrative purposes. In an actual case the design overpressure would be determined by evaluating a group of considerations including many nonstructural design considerations. The example includes only the designs of the major elements of the structure, including the roof slab, purlins, wall slab, columns and girders of the frame, and the foundation.

One-way reinforced concrete slabs are used for the roof and walls.

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The concrete compressive strength is specified at 3000 psi and intermediate grade reinforcing steel is used. In accordance with EM 1110-345-414, a uniform dynamic increase factor of 1.3 is used, giving the following strength properties for use in the design concrete

$$\begin{aligned} f'_c &= 3000 \text{ psi} \\ f'_{dc} &= 3900 \text{ psi} \\ E_c &= 3(10)^6 \text{ psi} \\ n &= 10 \end{aligned}$$

reinforcing steel

$$\begin{aligned} f_y &= 40,000 \text{ psi} \\ f_{dy} &= 52,000 \text{ psi} \end{aligned}$$

The purlins, columns, and girders are wide flange structural shapes with welded connections. The strength properties are specified in EM 1110-345-414.

The structure is to be located upon a compact sand-gravel mixture exhibiting the following properties (par. 4-15).

Normal load-bearing capacity = 10 kips/sq ft

Ultimate load-bearing capacity = 30 kips/sq ft

Coefficient of friction (soil on soil) = 0.50

Coefficient of friction (concrete on soil) = 0.75

Unit weight of soil = 100 lb/ft<sup>3</sup>

Normal component of passive pressure coefficient,  $K_{p\phi} = 10$

Modulus of elasticity,  $E = 40,000 \text{ psi}$

It must be emphasized that the primary purposes of this example and those that follow are to illustrate the design techniques and philosophy presented in the previous manuals. Presentation on this example should not be considered a recommendation of the structural system for use in blast-resistant buildings.

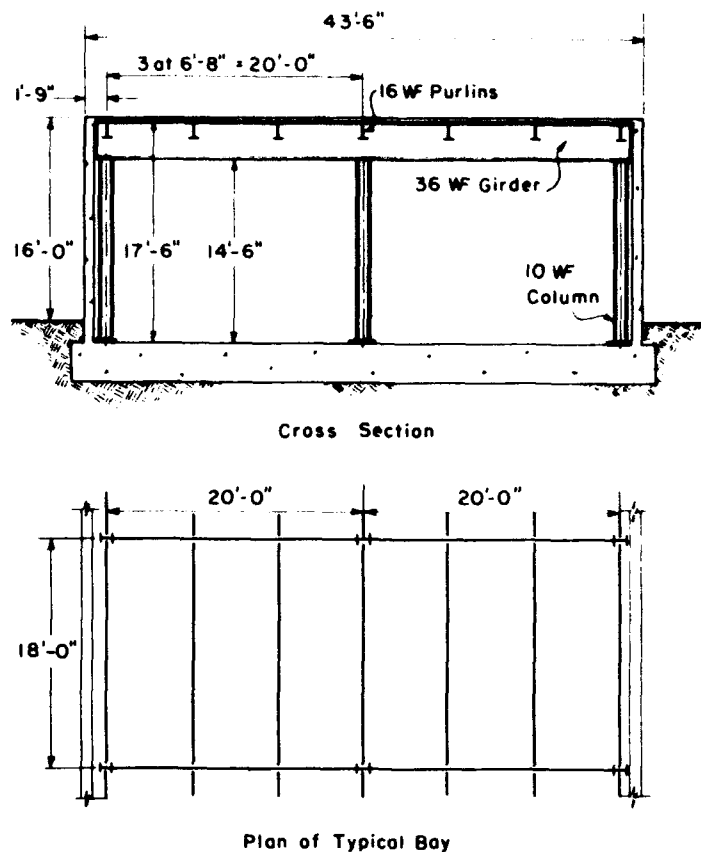


Figure 7.7. Plan and section of steel frame building

7-19 DESIGN PROCEDURE. In general blast-resistant design proceeds from the outside to the inside because the dynamic reactions of the outside elements are used for the loading on the supporting members. The steps in the design procedure are as follows:

Step 1. Compute the pressure variations from which the design loads can be obtained. The following curves are needed in addition to some local roof overpressure curves which are introduced in paragraph 7-22:

- (1) Incident overpressure vs time curve (fig. 7.8)
- (2) Front face overpressure vs time curve (fig. 7.9)
- (3) Rear face overpressure vs time curve (fig. 7.10)
- (4) Net lateral overpressure vs time curve (fig. 7.11)
- (5) Average roof overpressure vs time curve (fig. 7.12)

Step 2. Using the procedure of paragraph 6-11 for design with plastic deformation and a triangular load-time curve idealized from the front face overpressure-time curve make a preliminary design of a wall slab. Check the design using the numerical integration procedure of paragraph 5-08 and the computed front face overpressure-time curve. In this analysis the dynamic reaction of the wall slab at the roof and foundation are obtained for later use.

Step 3. Design the roof slab by the same procedure used for design of the wall slab with a triangular load-time curve idealized from the incident overpressure-time curve for the preliminary step, and use the actual incident overpressure-time curve in the numerical integration to check the slab deflection. The dynamic reactions are obtained for use in the purlin design.

Step 4. Base the purlin preliminary design on the same idealized load-time curve used in the roof slab design. However, for the numerical integration analysis use the load obtained from the roof slab dynamic reaction. From the numerical analysis of the purlin obtain the design load for the girder.

Step 5. Make a preliminary design of the columns assuming the girders to be infinitely rigid and neglecting the effect of axial load on the column. A triangular load-time curve idealized from the net lateral pressure-time curve is used in the preliminary design.

Step 6. Design the frame girders using the procedure of paragraph 6-12 for elastic design. In this preliminary design the load-time curve is idealized from the variation of purlin dynamic reactions.

Step 7. Check the preliminary column design by determining the maximum lateral deflection of the frame considering the relative flexibility of the columns and girders and the effect of axial load on column resistance. Use the wall slab dynamic reactions at the roof line for the design lateral load on the frame.

Step 8. Design the foundation.

7-20 LOAD DETERMINATION. The computation of the various pressure-time curves is explained in detail in EM 1110-345-413. In this example the methods are illustrated by presenting the computations for one point on each of the curves. The dimensions of the structure used in the load computations are the outside dimensions of the building which are determined at this stage of the design by estimating the sizes of the slabs and girders (fig. 7.7).

a. Incident Overpressure vs Time Curve. Assumptions of the incident overpressure and time duration used in the illustrative examples in this manual are given in figure 7.8.

$$t_o = W^{1/3} (0.262) = 18^{1/3} (0.262) = 0.685 \text{ sec}$$

The incident overpressure-time curve (fig. 7.8) is obtained from figure 3.4b.

b. Front Face Overpressure vs Time Curve (Fig. 7.9).

$$c_{\text{refl}} = 1290 \text{ fps (fig. 3.21)}$$

$$t_c = \frac{3h'}{c_{\text{refl}}} = \frac{3(16)}{1290} = 0.0372 \text{ sec}$$

$$P_{\text{refl}} = 25.3 \text{ psi (fig. 3.20)}$$

$$q_o = 2.23 \text{ psi (fig. 3.23)}$$

$$\text{Overpressure} = P_s + 0.85q$$

q is obtained using table 3.2.  $P_s$  is obtained using table 3.1.

For example, for  $t = 0.100 \text{ sec}$ ,  $t/t_o = 0.146$

$$q = 2.23(0.513) = 1.14 \text{ psi (table 3.2)}$$

$$P_s = 10.0(0.738) = 7.38 \text{ psi (table 3.1)}$$

$$\bar{P}_{\text{front}} = 7.38 + 0.85(1.14) = 8.35 \text{ psi}$$

Following the procedure of figure 3.25, the front face overpressure vs time curve may be drawn.

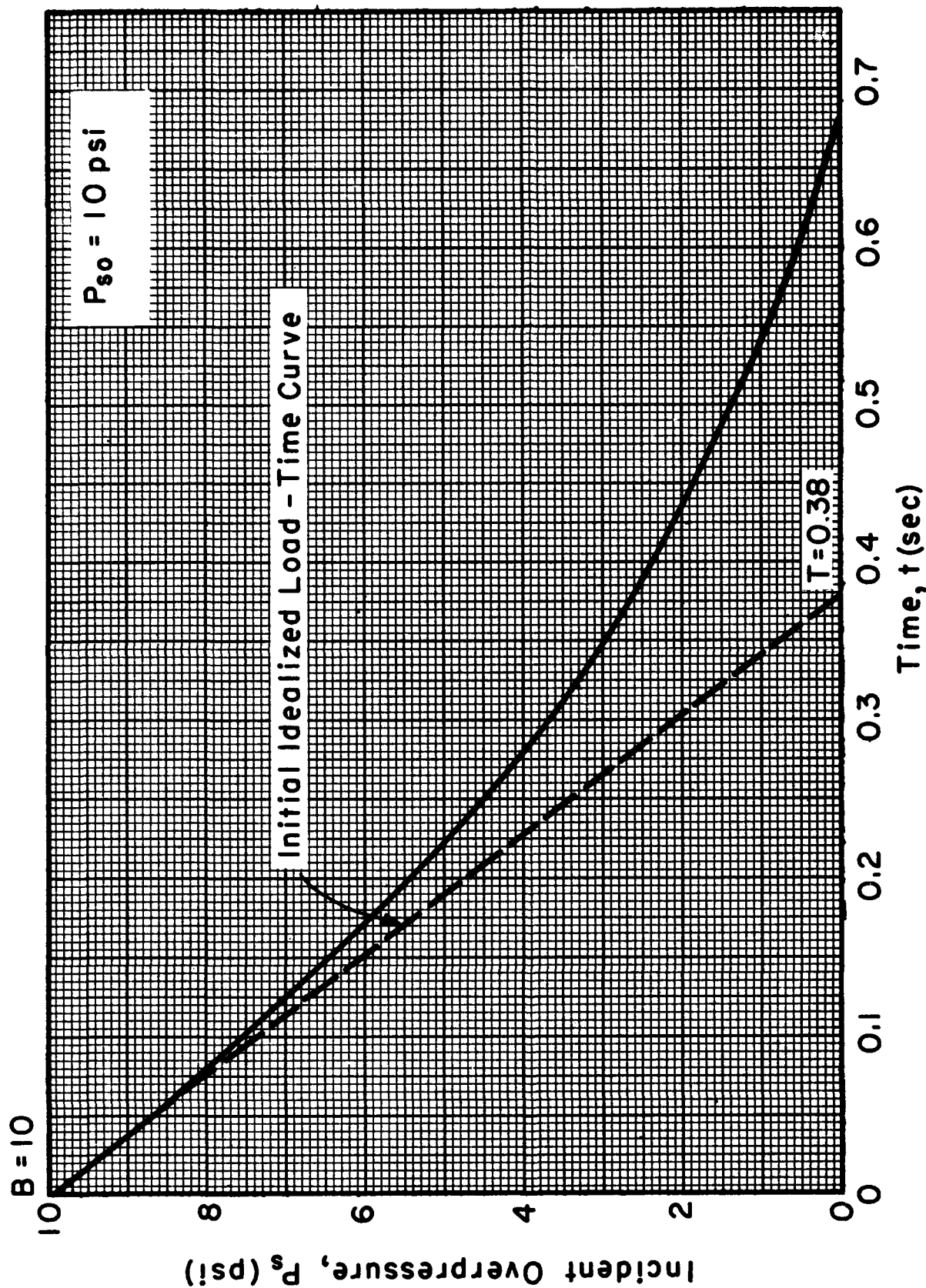


Figure 7.8. Incident overpressure vs time curve



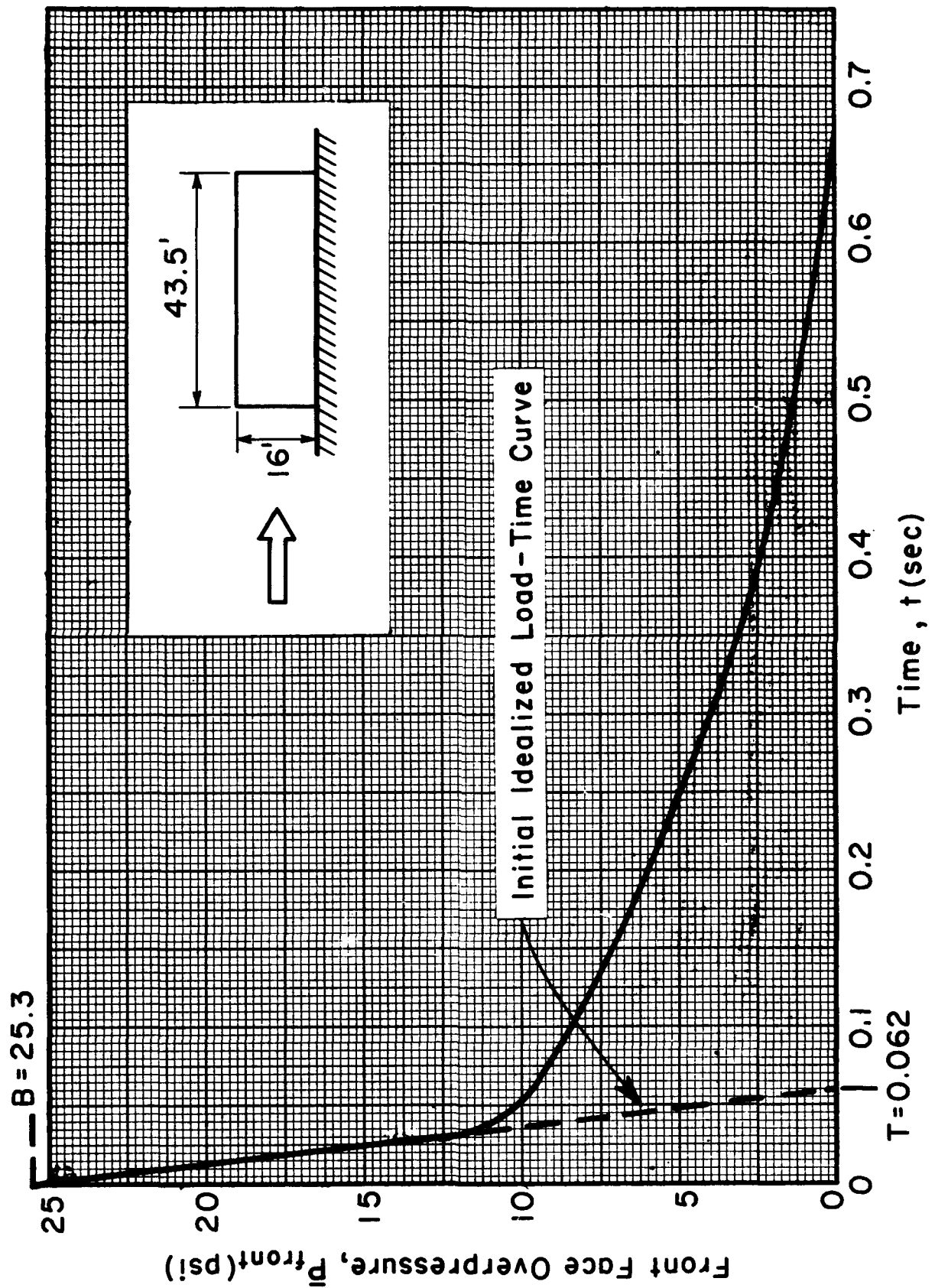


Figure 7.9. Front face overpressure vs time curve

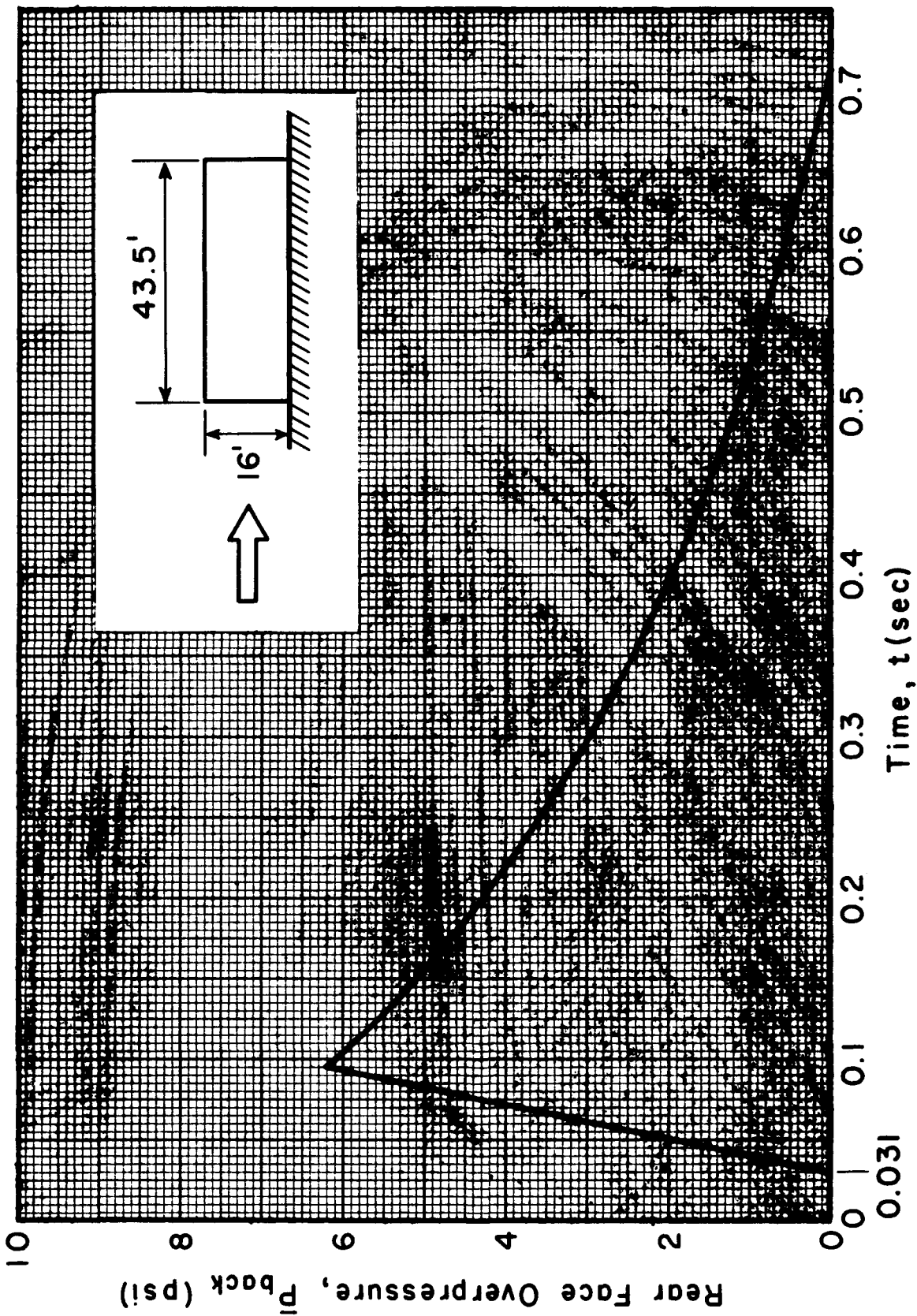


Figure 7.10. Rear face overpressure vs time curve

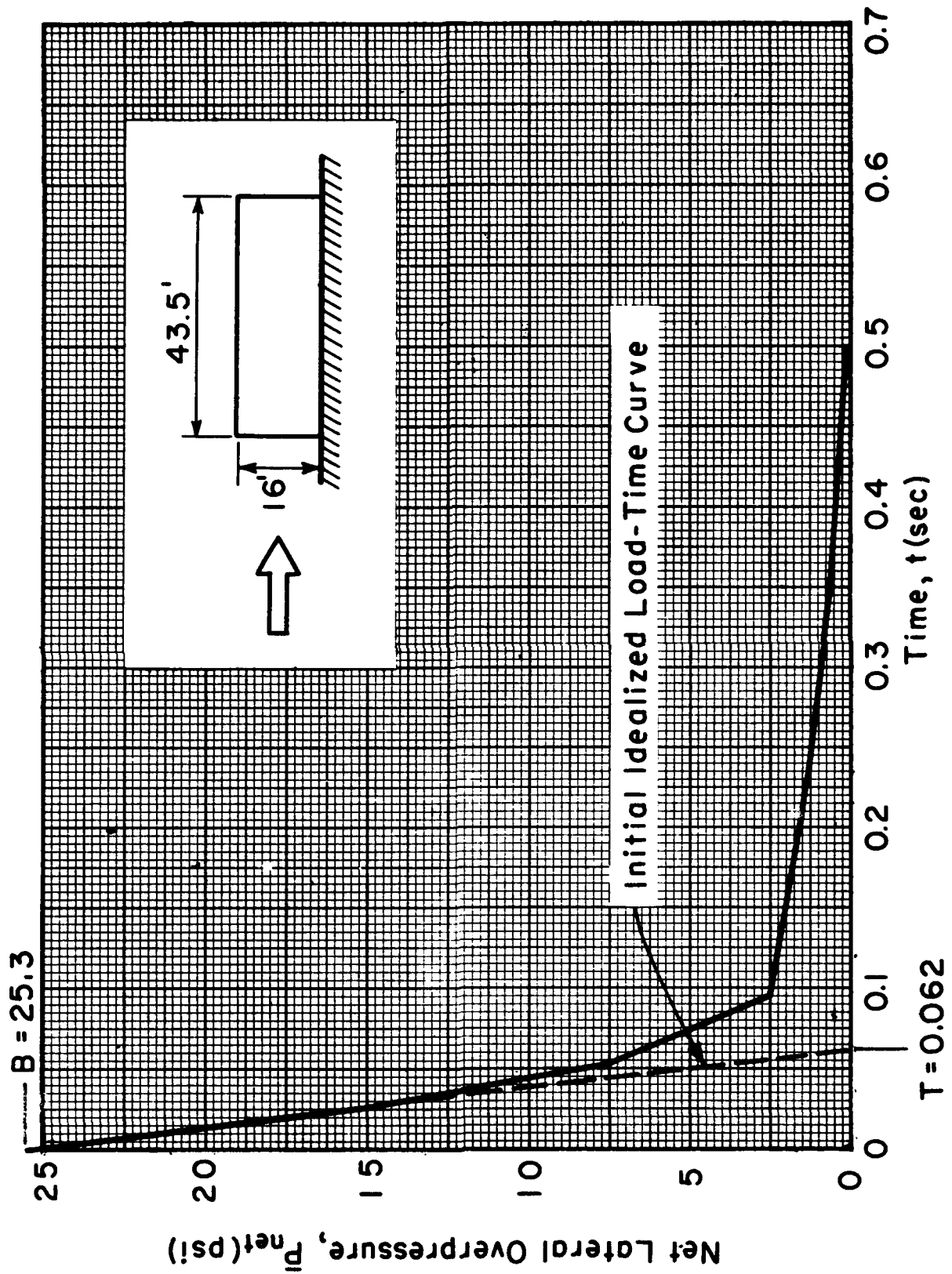


Figure 7.11. Net lateral overpressure vs time curve

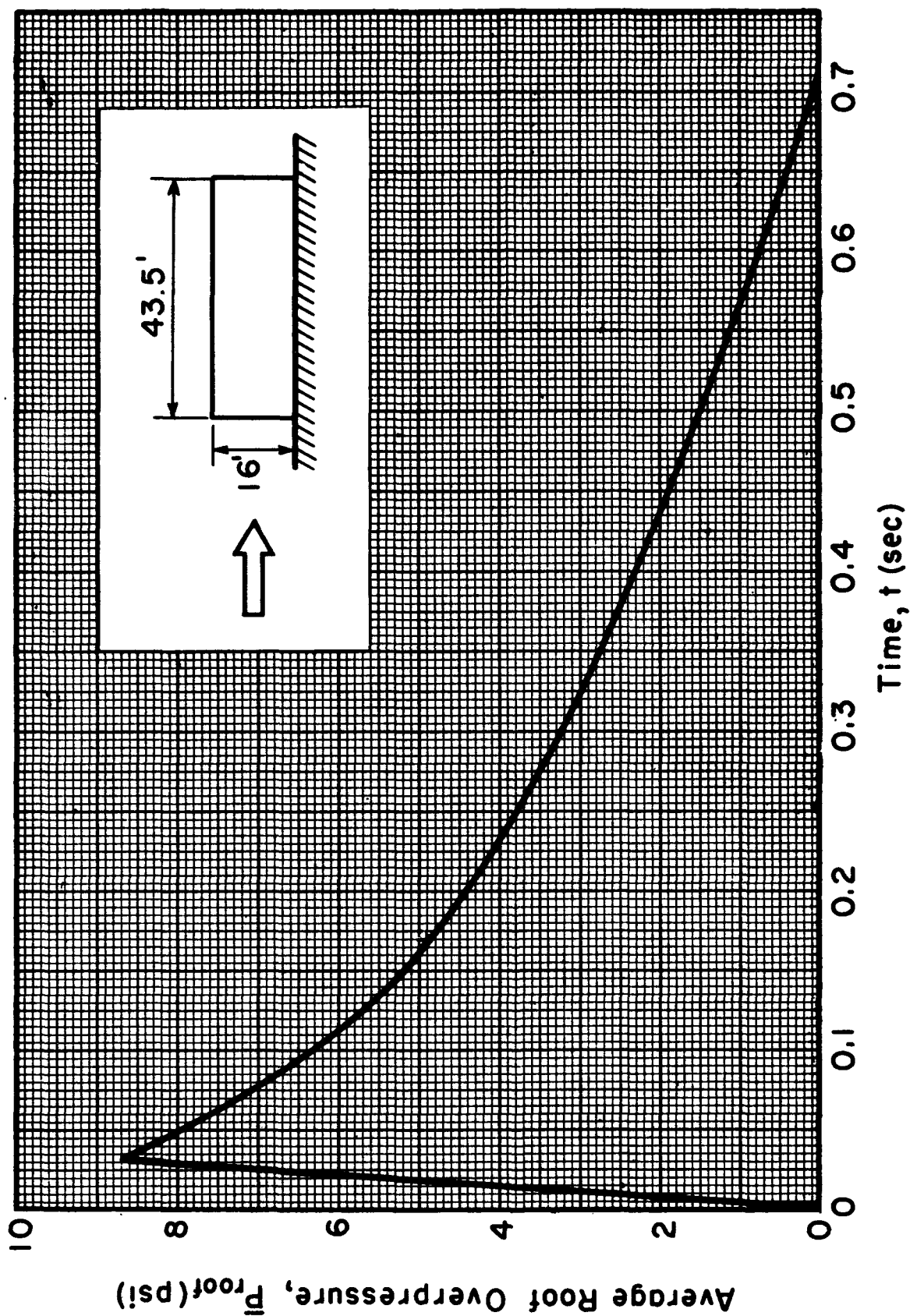


Figure 7.12. Average roof overpressure vs time curve

c. Rear Face Overpressure vs Time Curve (Fig. 7.10).Length of structure,  $L = 43.5$  ft $U_o = 1403$  fps (fig. 3.9),  $c_o = 1115$  fps (par. 3-08a)

$$t_d = \frac{L}{U_o} = \frac{43.5}{1403} = 0.0310 \text{ sec}$$

$$t_b = \frac{4h'}{c_o} = \frac{4(16)}{1115} = 0.0573 \text{ sec}$$

At time  $t_d + t_b = 0.088$  sec, using table 3.1,  $P_{sb} = 10.0(0.842)$   
 $= 8.42$  psi

At time  $t_d + t_b$ , from figure 3.27b,  $\bar{P}_{back}/P_{sb} = 0.735$

Therefore,  $\bar{P}_{back} = 0.735(8.42) = 6.20$  psi

For times in excess of  $t = t_d + t_b$ , the ratio of  $\bar{P}_{back}/P_s$  is as given in figure 3.27b.

d. Net Lateral Overpressure vs Time Curve (Fig. 7.11). At any time  $t$ ,  $\bar{P}_{net} = \bar{P}_{front} - \bar{P}_{back}$

e. Average Roof Overpressure (Fig. 7.12). For the blast propagation direction normal to the long side of the building, at time  $t = L/U_o = 0.0310$  sec

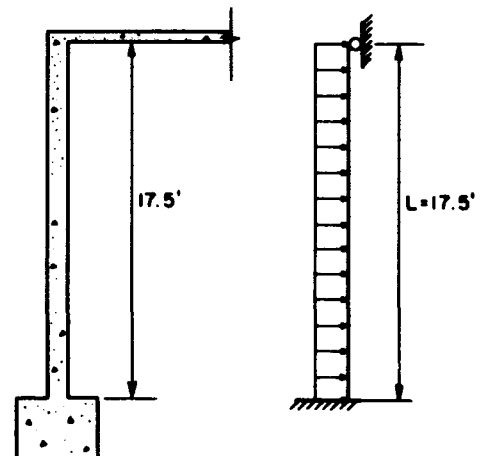
$$\frac{\bar{P}_{roof}}{P_s} = 0.9 + 0.1 \left[ \frac{1 - P_{so}}{14.7} \right]^2 = 0.91 \text{ (fig. 3.34)}$$

$P_s = 10.0(0.957) = 9.57$  psi (table 3.1)

Therefore,  $\bar{P}_{roof} = 0.91(9.57) = 8.71$  psi at  $t = 0.0310$  sec

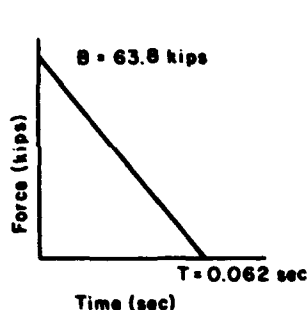
For times in excess of  $t = L/U_o$ , the ratio of  $\bar{P}_{roof}/P_s$  is as given in figure 3.34.

7-21 DESIGN OF WALL SLAB. The wall is designed as a one-way reinforced concrete slab spanning from a fixed support at the foundation to a pinned support at the roof slab. The slab is permitted to deform into the plastic region by developing plastic hinges at the foundation and near midheight. The span length of the slab is equal to the clear height of the wall.



The preliminary plastic design procedure is described and illustrated by an example in paragraph 6-11. Dead loads are not considered in designing vertical wall slabs. The design calculations are made for a one-foot width of slab.

a. Design Loading. The design load as idealized from the computed loading shown by figure 7.9 is defined by:



$$B = \frac{25.3(144)17.5}{1000} = 63.8 \text{ kips}$$

$$T = 0.062 \text{ sec}$$

$$H = \frac{BT}{2} = \frac{(63.8)(0.062)}{2} = 1.98 \text{ kip-sec (par. 6-11)}$$

b. Dynamic Design Factors. (Refer to table 6.1.)

Elastic range:

$$K_L = 0.58,$$

$$K_M = 0.45,$$

$$K_{IM} = 0.78$$

$$R_{lm} = \frac{8M_{Ps}}{L},$$

$$k_1 = \frac{185EI}{L^3}$$

$$V_1 = 0.26R + 0.12P,$$

$$V_2 = 0.43R + 0.19P$$

Elasto-plastic range:

$$K_L = 0.64,$$

$$K_M = 0.50,$$

$$K_{IM} = 0.78$$

$$R_m = \frac{4}{L} (M_{Ps} + 2M_{Pm}),$$

$$k_{ep} = \frac{384EI}{5L^3}$$

$$V = 0.39R + 0.11P$$

Plastic range:

$$K_L = 0.50,$$

$$K_M = 0.33,$$

$$K_{IM} = 0.66$$

$$R_m = \frac{4}{L} (M_{Ps} + 2M_{Pm})$$

$$V = 0.38R + 0.12P$$

Average values:

$$K_L = \frac{0.64 + 0.50}{2} = 0.57$$

$$K_M = \frac{0.50 + 0.33}{2} = 0.42$$

$$R_m = \frac{4}{L} (M_{Ps} + 2M_{Pm})$$

$$k_E = \frac{160EI}{L^3}$$

c. First Trial - Actual Properties.

$$\text{Let } M_{Ps} = M_{Pm} = M_P$$

$$\text{Assume } p = 0.015 \text{ (par. 4-10)}$$

$$\text{Let } \alpha\beta = 5 \text{ (par. 6-26)}$$

$$\text{Assume } C_R = 0.7 \text{ (experience)}$$

$$R_m = C_R B = 0.7(63.8) = 44.7 \text{ kips}$$

$$M_P = p f_{dy} b d^2 \left( 1 - \frac{p f_{dy}}{1.7 f'_{dc}} \right) \text{ (eq 4.16)}$$

$$= 0.015(52) (1) d^2 \left[ 1 - \frac{0.015(52)}{1.7(3.9)} \right] = 0.688 d^2 \text{ kip-ft (d in inches)}$$

$$R_m = \frac{12 M_P}{L} = \frac{(12) 0.688 d^2}{17.5} = 44.7, \therefore d = 9.73 \text{ in.}$$

$$\text{Try } h = 11 \text{ in., } d = 9.75 \text{ in., } p = 0.015$$

$$M_P = 0.688(9.75)^2 = 65.4 \text{ kip-ft}$$

$$R_m = \frac{12 M_P}{L} = \frac{12(65.4)}{17.5} = 44.8 \text{ kips}$$

$$I_g = b h^3 / 12 = (11)^3 = 1331 \text{ in.}^4$$

$$I_t = b d^3 \left[ \frac{k^3}{3} + p(1 - k)^2 \right]$$

$$= 12(d)^3 \left[ \frac{(0.42)^3}{3} + 0.015(1 - 0.42)^2 \right]$$

$$= 0.905 d^3 = 0.905(9.75)^3 = 839 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 0.5(1331 + 839) = 1085 \text{ in.}^4$$

$$k_E = \frac{160 E I}{L^3} = \frac{(160) 3(10)^3 1085}{144(17.5)^3} = 675 \text{ kips/ft}$$

$$y_E = \frac{R_m}{k_E} = \frac{44.8}{675} = 0.0664 \text{ ft}$$

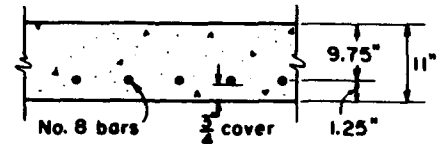
$$y_m = \alpha\beta y_E = 5(0.0664) = 0.3320 \text{ ft (par. 6-26)}$$

$$\text{Weight} = \frac{11(150)17.5}{(12)1000} = 2.406 \text{ kips}$$

$$\text{Mass } m = \frac{2.406}{32.2} = 0.0747 \frac{\text{kip-sec}^2}{\text{ft}}$$

d. First Trial - Equivalent System Properties.

$$R_{me} = K_L R_m = 0.57(44.8) = 25.5 \text{ kips (eq 6.12)}$$



$$H_e = K_L H = 0.57(1.98) = 1.13 \text{ kip-sec (eq 6.2)}$$

$$m_e = K_M m = 0.42(0.0747) = 0.0314 \text{ kip-sec}^2/\text{ft (eq 6.2)}$$

$$W_P = \frac{(H_e)^2}{2m_e} = \frac{(1.13)^2}{2(0.0314)} = 20.33 \text{ ft-kips (eq 6.10)}$$

$$T_n = 2\pi \sqrt{\frac{K_{LM} m}{k_E}} = 6.28 \sqrt{\frac{0.78(0.0747)}{675}} = 0.058 \text{ sec (eq 6.14)}$$

e. Work Done vs Energy Absorption Capacity.

$$C_T = T/T_n = 0.062/0.058 = 1.07$$

$$C_R = R_m/B = 44.8/63.8 = 0.70 \text{ (eqs 6.15, 6.16)}$$

$$t_m/T = 0.8 \text{ (fig. 5.29)}$$

$$t_m = (0.8)0.062 = 0.049 \text{ sec}$$

The idealized load-time curve is satisfactory since it agrees reasonably with the actual load-time curve (fig. 7.9) (see par. 5-13).

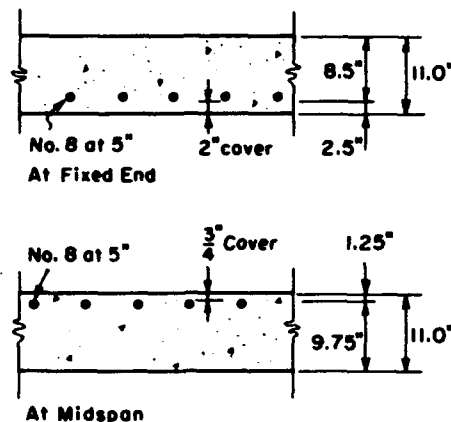
$$C_W = 0.365 \text{ (fig. 5.27)}$$

$$W_m = C_W W_P = 0.365(20.33) = 7.42 \text{ ft-kips (eq 6.17)}$$

$$E = R_{me}(y_m - 0.5y_E) = 25.5 [0.3320 - 0.5(0.0664)] \\ = 7.62 \text{ ft-kips (eq 6.18)}$$

$E > W$ ,  $\therefore$  the selected proportions are satisfactory as a preliminary design.

f. Preliminary Design for Bond Stress. It is now necessary to



select the reinforcing steel for the critical cross sections. At the fixed end of the wall the cover requirement results in a smaller value of  $d = 8.5$  in. than at midspan,  $d = 9.75$  in. To achieve approximately the same  $p$  at both critical sections several values of  $p$  are investigated to obtain the value of  $p$

$$\text{for which } \frac{8M_{Pm} + 4M_{Ps}}{12} = M_P = 65.4 \text{ kip-ft. A}$$

plot of equation (4.16) simplifies this

computation. From such a plot  $p = 0.016$ .



At the fixed end

$$\text{Estimated } V_{\max} = 0.5R_m = 0.5(44.8) = 22.4 \text{ kips}$$

$$\text{Allowable } u = 0.15f'_c = 0.15(3000) = 450 \text{ psi}$$

$$\Sigma o = \frac{V}{u_j d} = \frac{8(22,400)}{7(450)8.5} = 6.7 \text{ in.}$$

$$A_s = pbd = 0.016(12)(8.5) = 1.63 \text{ in.}^2$$

$$\text{Try \#8 at 5 in., } A_s = 1.9 \text{ in.}^2, \Sigma o = 7.5 \text{ in.}$$

$$p = \frac{1.9}{12(8.5)} = 0.0186$$

At the pinned end

$$\text{Estimated } V_{\max} = (1/3)R_m = 1/3(44.8) = 15.0 \text{ kips}$$

$$\Sigma o = \frac{V}{u_j d} = \frac{8(15,000)}{7(450)9.75} = 3.9 \text{ in.}$$

$$A_s = pbd = 0.016(12)(9.75) = 1.87 \text{ in.}^2$$

$$\text{Try \#8 at 5 in., } A_s = 1.9 \text{ in.}^2, \Sigma o = 7.5 \text{ in.}$$

$$p = \frac{1.9}{12(9.75)} = 0.0162$$

g. Determination of Maximum Deflection and Dynamic Reactions by Numerical Integration.

$$\begin{aligned} M_{Pm} &= pf_{dy} bd^2 \left[ 1 - \frac{pf_{dy}}{1.7f'_{dc}} \right] = 0.0162(52)(1)(9.75)^2 \left[ 1 - \frac{(0.0162)52}{1.7(3.9)} \right] \\ &= 70.0 \text{ kip-ft (eq 4.16)} \end{aligned}$$

$$M_{Ps} = 0.0186(52)(1)(8.5)^2 \left[ 1 - \frac{0.0186(52)}{1.7(3.9)} \right] = 59.5 \text{ kip-ft}$$

$$I_g = bh^3/12 = (11)^3 = 1331 \text{ in.}^4$$

$$\begin{aligned} I_t &= bd^3 \left[ k^3/3 + np(1 - k)^2 \right] = 12(9.75)^3 \left[ \frac{0.43^3}{3} + 0.162(1 - 0.43)^2 \right] \\ &= 880 \text{ in.}^4 \end{aligned}$$

$$I_a = 0.5(I_g + I_t) = 0.5(1331 + 880) = 1105 \text{ in.}^4$$

$$\text{Weight} = \frac{11(150)17.5}{12(1000)} = 2.406 \text{ kips}$$

$$\text{Mass } m = \frac{2.406}{32.2} = 0.0747 \text{ kip-sec}^2/\text{ft}$$

Elastic range:

$$R_{lm} = \frac{8M_{Ps}}{L} = \frac{8(59.5)}{17.5} = 27.2 \text{ kips}$$

$$k_1 = \frac{185EI}{L^3} = \frac{(185)(3)(10)^3 1105}{144(17.5)^3} = 793 \text{ kips/ft}$$

$$y_e = \frac{R_{lm}}{k_1} = \frac{27.2}{793} = 0.0344 \text{ ft}$$

Elasto-plastic range:

$$R_m = 4 \left( \frac{M_{Ps} + 2M_{Pm}}{L} \right) = \frac{4[59.5 + 2(70)]}{17.5} = 45.6 \text{ kips}$$

$$k_{ep} = \frac{384EI}{5L^3} = \frac{384}{5(185)} k_1 = 329 \text{ kips/ft}$$

$$y_{ep} = y_e + \frac{R_m - R_{lm}}{k_{ep}} = 0.0344 + \frac{45.6 - 27.2}{329} = 0.090 \text{ ft}$$

Plastic range:

$$R_m = 45.6 \text{ kips}$$

Since  $M_{Pm} \neq M_{Ps}$  the formula for  $k_E$  in table 6.1 is not usable. To obtain a value for  $y_E$  and  $k_E$ , an "effective resistance" line is selected on

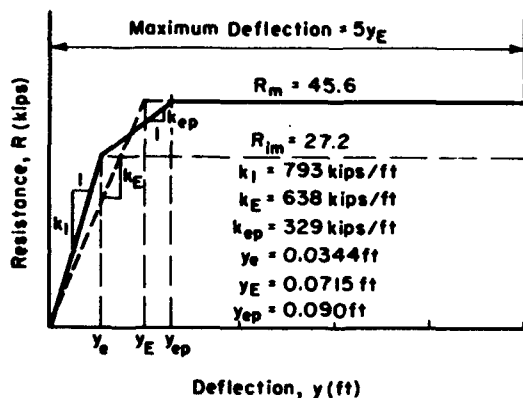


figure 7.13 so that the area under it up to  $y_E$  is equal to the area under the calculated elasto-plastic resistance line.

The required value of  $y_E = 0.0715 \text{ ft}$ .

$$k_E = \frac{R_m}{y_E} = \frac{45.6}{0.0715} = 638 \text{ kips/ft}$$

$$y_m = \alpha \beta y_E = 5(0.0715) = 0.3575 \text{ ft}$$

$$T_n = 2\pi \sqrt{\frac{K_{LM}^m}{k_E}} = 6.28 \sqrt{\frac{0.78(0.0747)}{638}} = 0.06 \text{ sec}$$

The basic equation for the numerical integration in table 7.2 is  $y_{n+1} = \ddot{y}_n(\Delta t)^2 + 2y_n - y_{n-1}$  (table 5.3) where

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(\Delta t)^2}{K_{LM}^m}$$

Table 7.2. Determination of Maximum Deflection and Dynamic Reactions for Front Wall Slab

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> - R <sub>n</sub> (kips)	$\ddot{y}_n(\Delta t)^2$ (ft)	y <sub>n</sub> (ft)	V <sub>1n</sub> (kips)	V <sub>2n</sub> (kips)
0	63.8	0	31.9	0.01369	0	7.7	12.1
0.005	58.7	10.8	47.9	0.02055	0.01369	9.8	15.8
0.010	53.5	31.6	21.9	0.00939	0.04793	18.2	18.2
0.015	48.4	45.6	2.8	0.00142	0.09156	23.1	23.1
0.020	43.2	45.6	-2.4	-0.00122	0.13661	22.5	22.5
0.025	38.1	45.6	-7.5	-0.00380	0.18044	21.9	21.9
0.030	32.9	45.6	-12.7	-0.00644	0.22047	21.3	21.3
0.035	29.0	45.6	-16.6	-0.00842	0.25406	20.8	20.8
0.040	27.3	45.6	-18.3	-0.00928	0.27923	20.6	20.6
0.045	26.3	45.6	-19.3	-0.00979	0.29512	20.4	20.4
0.050	25.6	45.6	-20.0	-0.01014	0.30122*	20.4	20.4
0.055	24.9	42.4	-17.5	-0.00751	0.29718	14.0	22.9
0.060	24.4	33.2	-8.8	-0.00378	0.28563	11.5	18.9
0.065	23.9	21.1			0.27030	8.4	13.6

\* (y<sub>n</sub>)<sub>max</sub> = 0.30 ft.

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)25(10^{-6})}{0.78(0.0747)} = 4.29(10^{-4})(P_n - R_n) \text{ ft, elastic range}$$

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)25(10^{-6})}{0.78(0.0747)} = 4.29(10^{-4})(P_n - R_n) \text{ ft, elasto-plastic range}$$

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)25(10^{-6})}{0.66(0.0747)} = 5.07(10^{-4})(P_n - R_n) \text{ ft, plastic range}$$

The time interval  $\Delta t = 0.005$  sec is approximately  $T_n/10 = 0.006$  (par. 5-08).

The dynamic reaction equations are listed in paragraph 7-21b. The P<sub>n</sub> values for the second column are obtained from figure 7.9, multiplying by  $144(17.5)/1000 = 2.52$ .

The maximum deflection, (y<sub>n</sub>)<sub>max</sub>, computed in table 7.2, is 0.30 ft which is less than the allowable y<sub>m</sub> of 0.35 ft.

$$y_E = 0.068 \text{ ft}$$

$$\alpha\beta = \frac{(y_n)_{\max}}{y_E} = \frac{0.30}{0.0715} = 4.2 < 5.0; \text{ OK}$$

h. Shear and Bond Strength. For bottom of wall (fixed end of idealized slab):

$$V_{\max} = 23.1 \text{ kips (table 7.2)}$$

For no shear reinforcement (eq 4.24a)

$$v_y = 0.04f'_c + 5000p = 0.04(3000) + 5000(0.0162) = 120 + 81 = 201 \text{ psi}$$

$$v = \frac{8V}{7bd} = \frac{8(23,100)}{7(12)8.5} = 258 \text{ psi}$$

Shear reinforcement required for  $258 - 201 = 57 \text{ psi}$ . Contribution of shear reinforcement to allowable shear stress  $\approx rf_y$ .

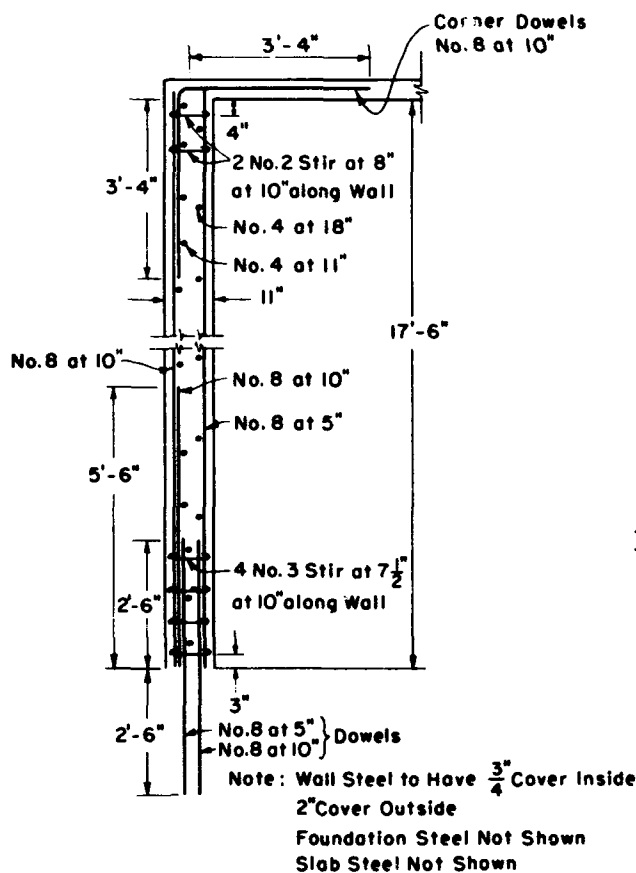
$$r = \frac{57}{40,000} = 0.0014$$

$$\text{Try 1 \#3, } A_s = 0.11 \text{ in.}^2$$

$$r = \frac{A_s}{bs} = \frac{0.11}{10s} = 0.0014, \therefore s = 7.8 \text{ in., use } s = 7.5 \text{ in.}$$

For top of wall (pinned end of idealized slab):

$$V_{\max} = 23.1 \text{ kips (table 7.2)}$$



$$v_y = 201 \text{ psi}$$

$$v = \frac{8V}{7bd} = \frac{8(23,100)}{7(12)9.75} = 226 \text{ psi}$$

Shear reinforcement required for

$$226 - 201 = 25 \text{ psi}$$

$$r = 25/40,000 = 0.0006$$

$$\text{Try 1 \#2, } A_s = 0.05 \text{ in.}^2$$

$$r = \frac{A_s}{bs} = \frac{0.05}{10s} = 0.0006,$$

$$\therefore s = 8.3 \text{ in.,}$$

$$\text{use } s = 8 \text{ in.}$$

Bond:

$$u = \frac{8V}{7\sigma_{od}} = \frac{8(23,100)}{7(7.5)(8.5)} = 415 \text{ psi}$$

$$\begin{aligned} \text{Allowable } u &= 0.15 f'_c \\ &= 0.15(3000) = 450 \text{ psi} > 415 \text{ psi; OK} \end{aligned}$$

i. Summary. 11-in. slab,

$$p = 0.0162$$

**Shear reinforcement:**

Bottom of wall #3 at 7-1/2 in.

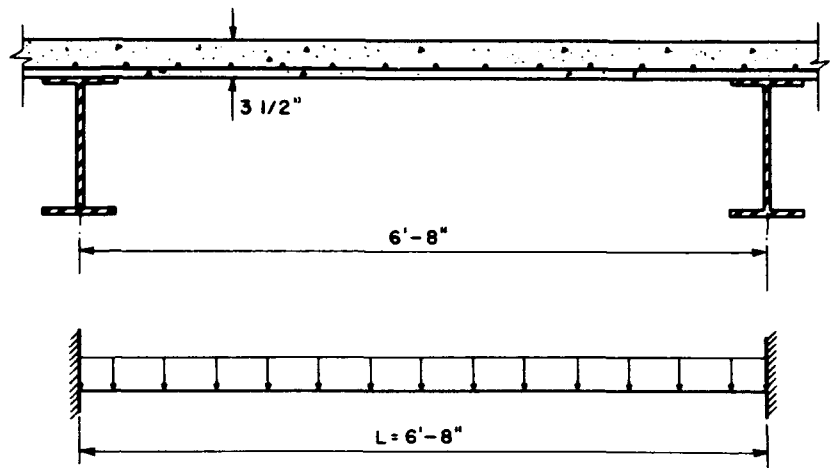
Top of wall #3 at 8 in.

7-22 DESIGN OF ROOF SLAB. The roof slab is designed as a one-way reinforced concrete slab spanning continuously over purlins located at the third-points of the supporting girder.

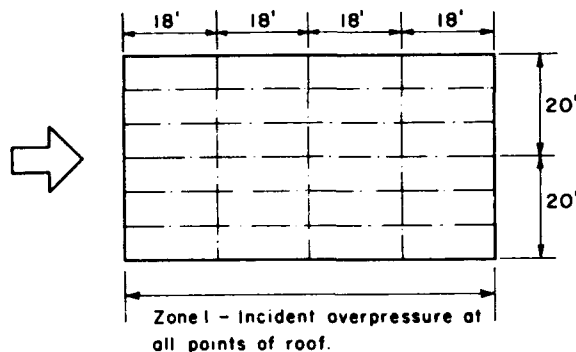
As stated previously, the general arrangement of the members in this example is not selected as the result of economic studies. This example is intended primarily to illustrate design technique.

The slab is permitted to deform into the plastic region by developing plastic hinges at both supports and midspan. In the design procedures of

this manual only single-span elements can be handled, therefore in using the preliminary plastic design procedure of paragraph 6-11 a one-foot width of slab is considered to be a fixed-end beam spanning 6 ft 8 in., the purlin spacing.



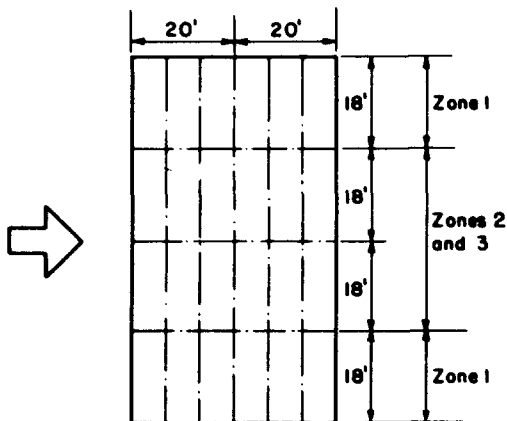
a. Loading. The critical slab loading is the incident overpressure vs time curve (fig. 7.8). This loading results from the blast wave moving



parallel to the long axis of the building. Since the slab is framed perpendicular to the direction of the blast wave the load may be considered to be uniformly distributed along each slab span. The individual one-foot slab elements along the purlin reach their maximum deflections at different times;

however, they provide little restraint to adjacent elements. This effect is neglected in this example.

For the blast wave moving perpendicular to the long axis of this



building the design loads on the slab are reduced from the incident overpressure loads for two reasons: (1) the time required to build up load on the slab is finite and (2) the Zone 3 overpressures are less than the incident overpressure (EM 1110-345-413).

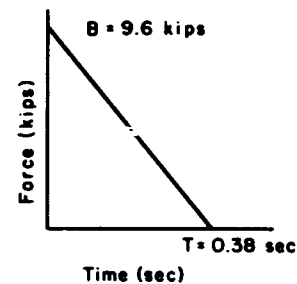
The design load as idealized from the computed loading shown by figure 7.8 is defined by

$$B = 10 \text{ psi} = \frac{10(144)(6.67)}{1000} = 9.6 \text{ kips}$$

$$T = 0.38 \text{ sec}$$

$$H = \frac{BT}{2} = \frac{(9.6)(0.38)}{2} = 1.83 \text{ kip-sec (par. 6-11)}$$

b. Dynamic Design Factors. (Refer to table 6.1.)



Elastic range:

$$K_L = 0.53,$$

$$K_M = 0.41,$$

$$K_{IM} = 0.77$$

$$R_{lm} = 12 \frac{M_{Ps}}{L},$$

$$k_l = \frac{384EI}{L^3}$$

$$V = 0.36R + 0.14P$$

Elasto-plastic range:

$$K_L = 0.64,$$

$$K_M = 0.50,$$

$$K_{IM} = 0.78$$

$$R_m = \frac{8}{L} (M_{Ps} + M_{Pm}),$$

$$k_{ep} = \frac{384EI}{5L^3}$$

$$V = 0.39R + 0.11P$$

Plastic range:

$$K_L = 0.50,$$

$$K_M = 0.33,$$

$$K_{IM} = 0.66$$

$$R_m = \frac{8}{L} (M_{Ps} + M_{Pm})$$

$$V = 0.38R_m + 0.12P$$

Average values:

$$K_L = 0.5(0.64 + 0.50) = 0.57, \quad K_{IM} = 0.5(0.78 + 0.77) = 0.77$$

$$K_M = 0.5(0.50 + 0.33) = 0.42$$

$$R_m = \frac{8}{L} (M_{Ps} + M_{Pm})$$

$$k_E = \frac{307EI}{L^3}$$

c. First Trial - Actual Properties.

$$\text{Let } M_{Ps} = M_{Pm} = M_P$$

$$\text{Assume } p = 0.015 \text{ (par. 4-10)}$$

$$\text{Let } \alpha\beta = 5 \text{ (par. 6-26)}$$

$$\text{Assume } C_R = 1.0 \text{ (experience)}$$

$$R_m = C_R B = 1.0(9.6) = 9.6 \text{ kips}$$

$$M_P = p f_{dy} b d^2 \left( 1 - \frac{p f_{dy}}{1.7 f'_{dc}} \right) \text{ (eq 4.16)}$$

$$= 0.015(52)(1)d^2 \left[ 1 - \frac{0.015(52)}{1.7(3.9)} \right] = 0.688d^2 \text{ kip-ft (d in inches)}$$

$$R_m = \frac{16M_P}{L} = \frac{(16)0.688d^2}{6.67} = 9.6, \therefore d = 2.4 \text{ in.}$$

$$\text{Try } h = 3.5 \text{ in., } d = 2.5 \text{ in., } p = 0.015, \quad np = 0.15$$

$$M_P = 0.688(2.5)^2 = 4.3 \text{ kip-ft}$$

$$R_m = \frac{16M_P}{L} = \frac{16(4.3)}{6.67} = 10.3 \text{ kips}$$

$$I_g = bh^3/12 = (3.5)^3 = 42.9 \text{ in.}^4$$

$$I_t = bd^3 \left[ \frac{k^3}{3} + np(1 - k)^2 \right]$$

$$= 12(d)^3 \left[ \frac{(0.42)^3}{3} + 0.15(1 - 0.42)^2 \right] = 0.905d^3 = 0.905(2.5)^3$$

$$= 14.1 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 0.5(14.1 + 42.9) = 28.5 \text{ in.}^4$$

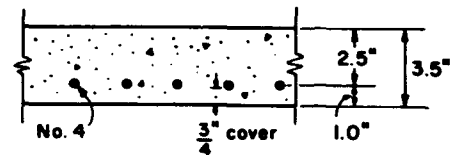
$$k_E = \frac{307EI}{L^3} = \frac{(307)3(10)^3 28.5}{(6.67)^3 144} = 615 \text{ kips/ft}$$

$$y_E = \frac{R_m}{k_E} = \frac{10.3}{615} = 0.0168 \text{ ft}$$

$$y_m = \alpha\beta y_e = 5(0.0168) = 0.084 \text{ ft (par. 6-26)}$$

The roofing weight is 6 psf.

$$\text{Weight} = \left[ \frac{3.5(150)}{(12)} + 6.0 \right] \frac{6.67}{1000} = 0.332 \text{ kips}$$



$$\text{Mass } m = \frac{0.332}{32.2} = 0.0103 \text{ kip-sec}^2/\text{ft}$$

d. First Trial - Equivalent System Properties.

$$R_{me} = K_L R_m = 0.57(10.3) = 5.87 \text{ kips (eq 6.12)}$$

$$H_e = K_L H = 0.57(1.83) = 1.04 \text{ kip-sec (eq 6.2)}$$

$$m_e = K_M m = 0.42(0.0103) = 0.00432 \text{ kip-sec}^2/\text{ft (eq 6.2)}$$

$$W_P = \frac{(H_e)^2}{2m_e} = \frac{(1.04)^2}{2(0.00432)} = 125 \text{ ft-kips (eq 6.10)}$$

$$T_n = 2\pi \sqrt{K_{IM} \frac{m}{k_E}} = 6.28 \sqrt{0.77(0.0103)/615} = 0.0226 \text{ sec (eq 6.14)}$$

e. Work Done vs Energy Absorption Capacity.

$$C_T = T/T_n = 0.38/0.0226 = 16.9$$

$$C_R = R_m/B = 10.3/9.6 = 1.075 \text{ (eqs 6.15, 6.16)}$$

$$t_m/T = 0.09 \text{ (fig. 5.29)}$$

$$t_m = (0.09)0.38 = 0.034 \text{ sec}$$

Idealized load-time curve is satisfactory at  $t = 0.034$

$$C_W = 0.005 \text{ (fig. 5.27)}$$

$$W_m = C_W W_P = 0.005 (1.25) = 0.625 \text{ ft-kips (eq 6.17)}$$

$$E = R_{me} (y_m - 0.5y_E) = 5.87 [0.084 - 0.5(0.0168)] \\ = 0.445 \text{ ft-kips (eq 6.18)}$$

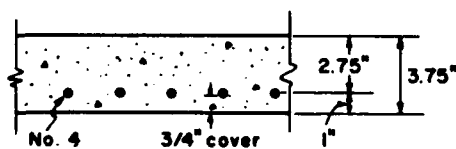
$E < W$ ,  $\therefore$  the selected proportions are unsatisfactory as a preliminary design.

f. Second Trial - Actual Properties.

$$R_m = K_L \frac{0.5(W_m + E)}{(y_m - 0.5y_E)} = \frac{0.5(0.625 + 0.445)}{(0.57) [0.084 - 0.5(0.0168)]} \\ = 12.3 \text{ kips (eq 6.19)}$$

$$R_m = \frac{16M_P}{L} = \frac{(16)0.688d^2}{6.67} = 12.3, \therefore d = 2.7 \text{ in.}$$

$$\text{Try } h = 3-3/4 \text{ in., } d = 2-3/4 \text{ in., } p = 0.015$$



$$M_P = 0.688d^2 = 0.688(2.75)^2 = 5.2 \text{ kip-ft}$$

$$R_m = \frac{16M_P}{L} = \frac{16(5.2)}{6.67} = 12.5 \text{ kips}$$



$$I_g = \frac{bh^3}{12} = (3.75)^3 = 53.0 \text{ in.}^4$$

$$I_t = 0.905d^3 = 0.905(2.75)^3 = 18.8 \text{ in.}^4 \quad (k = 0.42)$$

$$I_a = 0.5(I_g + I_t) = 0.5(53.0 + 18.8) = 35.9 \text{ in.}^4$$

$$k_E = \frac{307EI}{L^3} = \frac{(307)(3)(10)^3 35.9}{(6.67)^3 (144)} = 775 \text{ kips/ft}$$

$$y_E = \frac{R_m}{k_E} = \frac{12.5}{775} = 0.0161 \text{ ft}$$

$$y_m = \alpha \beta y_E = 5(0.0161) = 0.0805 \text{ ft (par. 6-26)}$$

$$\text{Weight} = \left[ \frac{3.75(150) + 6.0}{(12)} \right] \frac{6.67}{1000} = 0.352 \text{ kips}$$

$$\text{Mass } m = \frac{0.352}{32.2} = 0.0109 \text{ kip-sec}^2/\text{ft}$$

g. Second Trial - Equivalent System Properties.

$$R_{me} = K_L R_m = 0.57(12.5) = 7.13 \text{ kips (eq 6.12)}$$

$$H_e = K_L H = 0.57(1.83) = 1.04 \text{ kip-sec (eq 6.2)}$$

$$m_e = K_M m = 0.42(0.0109) = 0.00459 \text{ kip-sec}^2/\text{ft (eq 6.2)}$$

$$W_P = \frac{(H_e)^2}{2m_e} = \frac{(1.04)^2}{2(0.00459)} = 118 \text{ ft-kips (eq 6.10)}$$

$$T_n = 2\pi \sqrt{K_{LM} m / k_E} = 6.28 \sqrt{0.77(0.0109) / 775} = 0.0207 \text{ sec}$$

h. Work Done vs Energy Absorption Capacity.

$$C_T = T / T_n = 0.38 / 0.0207 = 18.3$$

$$C_R = R_m / B = 12.5 / 9.6 = 1.3 \text{ (eqs 6.15, 6.16)}$$

$$t_m / T = 0.04 \text{ (fig. 5.29)}$$

$$t_m = (0.04)0.38 = 0.015 \text{ sec}$$

$$C_W = 0.0015 \text{ (fig. 5.27)}$$

$$W_m = C_W W_P = 0.0015(118) = 0.177 \text{ ft-kips (eq 6.17)}$$

$$E = R_{me} (y_m - 0.5y_E) = 7.13 [0.0805 - 0.5(0.0161)] \\ = 0.516 \text{ ft-kips (eq 6.18)}$$

$$E \gg W$$

Although the difference between E and W is great no other trial

is justified since slab thickness was increased by only 1/4 in. Therefore the selected proportions are satisfactory as a preliminary design.

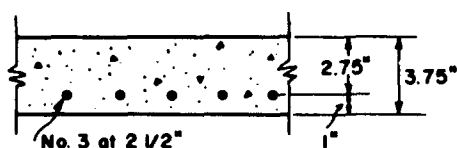
1. Preliminary Design for Bond Stress.

$$\text{Estimated } V_{\max} = 0.5R_m = 0.5(12.5) = 6.25 \text{ kips}$$

$$\text{Allowable } u = 0.15f'_c = 0.15(3000) = 450 \text{ psi (par. 4-09)}$$

$$\Sigma o = \frac{V}{u_j d} = \frac{8(6250)}{450(7)2.75} = 5.77 \text{ in.}$$

$$\text{Try \#3 at 2-1/2 in., } A_s = 0.53 \text{ in.}^2, \Sigma o = 5.7 \text{ in.}$$



$$p = A_s / bd = \frac{0.53}{12(2.75)} = 0.16,$$

$$np = 10(0.016) = 0.16$$

j. Determination of Maximum Deflection and Dynamic Reactions by Numerical Integration.

$$M_p = pf_{dy} bd^2 \left[ 1 - \frac{pf_{dy}}{1.7f'_{dc}} \right]$$

$$= 0.016(52)(1)(2.75)^2 \left[ 1 - \frac{(0.016)52}{1.7(3.9)} \right] = 5.5 \text{ kip-ft (eq 4.16)}$$

$$I_g = bh^3/12 = (3.75)^3 = 53.0 \text{ in.}^4$$

$$k = \sqrt{n^2 p^2 + 2np} - np = 0.428$$

$$I_t = bd^3 \left[ \frac{k^3}{3} + np(1 - k)^2 \right]$$

$$= 12(2.75)^3 \left[ \frac{0.428^3}{3} + 0.16(1 - 0.428)^2 \right] = 0.945(2.75)^3 = 19.6 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 0.5(53.0 + 19.6) = 36.3 \text{ in.}^4$$

$$\text{Weight} = \left[ \frac{3.75(150)}{12} + 6.0 \right] \frac{6.67}{1000} = 0.352 \text{ kips}$$

$$\text{Mass } m = \frac{0.352}{32.2} = 0.0109 \text{ kip-sec}^2/\text{ft}$$

Elastic range:

$$R_{lm} = \frac{12M_p}{L} - \text{weight} = \frac{12(5.5)}{6.67} - 0.352 = 9.5 \text{ kips}$$

$$k_1 = \frac{384EI}{L^3} = \frac{(384)(10)^3 36.3}{(6.67)^3 144} = 980 \text{ kips/ft}$$

$$y_e = \frac{R_{lm}}{k_1} = \frac{9.5}{980} = 0.0097 \text{ ft}$$

Elasto-plastic range:

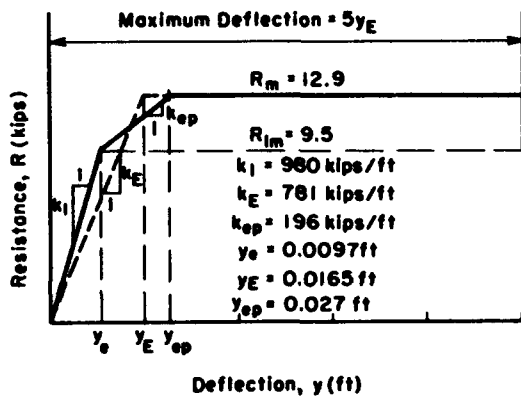
$$R_m = \frac{16M_p}{L} - \text{weight} = \frac{16(5.5)}{6.67} - 0.352 = 12.9 \text{ kips}$$

$$k_{ep} = \frac{384EI}{5L^3} = \frac{1}{5} k_1 = 196 \text{ kips/ft}$$

$$y_{ep} = y_e + \frac{R_m - R_{lm}}{k_{ep}} = 0.0097 + \frac{12.9 - 9.5}{196} = 0.027 \text{ ft}$$

Plastic range:

$$R_m = \frac{16M_p}{L} - \text{weight} = 12.9 \text{ kips}$$



General:

$$k_E = \frac{307}{384} k_1 = \frac{307}{384} (980) = 781 \text{ kips/ft}$$

$$y_E = \frac{12.9}{781} = 0.0165 \text{ ft}$$

$$y_m = \alpha y_E = 5(0.0165) = 0.0825 \text{ ft}$$

$$T_n = 2\pi \sqrt{\frac{K_{IM}^m}{k_E}} = 6.28 \sqrt{\frac{0.77(0.0109)}{781}} = 0.0206 \text{ sec}$$

Figure 7.14. Resistance function for 3-3/4-in. slab spanning 6.67 ft

The basic equation for the numerical integration in table 7.3 is:

$$y_{n+1} = \ddot{y}_n (\Delta t)^2 + 2y_n - y_{n-1} \text{ (table 5.3)}$$

where

$$\ddot{y}_n (\Delta t)^2 = \frac{(P_n - R_n)(\Delta t)^2}{K_{IM}^{(m)}} = \frac{(P_n - R_n)(0.00171)^2}{K_{IM}^{(m)}}$$

$$\ddot{y}_n (\Delta t)^2 = \frac{(P_n - R_n)(0.00171)^2}{0.77(0.0109)} = 3.48(10^{-4})(P_n - R_n) \text{ ft, elastic range}$$

$$\ddot{y}_n (\Delta t)^2 = \frac{(P_n - R_n)(0.00171)^2}{0.78(0.0109)} = 3.439(10^{-4})(P_n - R_n) \text{ ft, elasto-plastic range}$$

$$\ddot{y}_n (\Delta t)^2 = \frac{(P_n - R_n)(0.00171)^2}{0.66(0.0109)} = 4.064(10^{-4})(P_n - R_n) \text{ ft, plastic range}$$

Table 7.3. Determination of Maximum Deflection and Dynamic Reactions for Roof Slab

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> - R <sub>n</sub> (kips)	y <sub>n</sub> (Δt) <sup>2</sup> (ft)	y <sub>n</sub> (ft)	V <sub>n</sub> (kips)
0	9.6	0	4.80	0.00167	0	1.34
0.00171	9.55	1.64	7.91	0.00275	0.00167	1.93
0.00342	9.50	5.97	3.53	0.00123	0.00609	3.48
0.00513	9.45	9.90	-0.45	-0.00015	0.01174	4.90
0.00685	9.40	10.98	-1.58	-0.00054	0.01724	5.31
0.00856	9.35	11.95	-2.60	-0.00089	0.02220	5.69
0.01027	9.30	12.90	-3.60	-0.00146	0.02627	6.02
0.01198	9.25	12.90	-3.65	-0.00148	0.02888	6.01
0.01370	9.20	12.90	-3.70	-0.00150	0.03001*	6.00
0.01541	9.16	12.54	-3.38	-0.00118	0.02964	5.79
0.01712	9.11	11.02	-1.91	-0.00066	0.02809	5.24
* (y <sub>n</sub> ) <sub>max</sub> = 0.03 ft.						

The time interval  $\Delta t = 0.00171$  sec is approximately  $T_n/10 = 0.00206$  (par. 5-08). The value used is  $t_o/40$  because the incident overpressure data is presented in terms of  $t_o$  (EM 1110-345-413).

The dynamic reaction equations are listed in paragraph 7-22b. The  $P_n$  values for the second column are obtained from figure 7.8, multiplying by  $144(6.67)/1000 = 0.96$ .

The maximum deflection  $(y_n)_{max}$  computed in table 7.3 is 0.03 ft which is less than the allowable  $y_m$  of 0.08 ft. This is satisfactory because thin slabs are very sensitive. Note the variation in  $E$  and  $W$  in the two trials (pars. 7-22e and h).

$$\alpha\beta = \frac{(y_n)_{max}}{y_E} = \frac{0.03}{0.0165} = 1.8; \text{ OK}$$

k. Shear Strength and Bond Stress.

$V_{max} = 6.02$  kips (table 7.3)

For no shear reinforcement

Allowable  $v_y = 0.04f'_c + 5000p$  (eq 4.24a)

$$v_y = 0.04(3000) + 5000(0.016) = 120 + 80 = 200 \text{ psi}$$

$$v = \frac{8V}{7bd} = \frac{8(6020)}{7(12)(2.75)} = 209 \text{ psi}$$

Slightly overstressed, OK to use

$$u = \frac{8V}{7\sigma_b d} = \frac{8(6020)}{7(5.7)(2.75)} = 440 \text{ psi}$$

$$\text{Allowable } u = 0.15f'_c = 0.15(3000) = 450 \text{ psi; OK}$$

1. Summary.

3-3/4-in. slab, #3 at 2-1/2 in.

$$p = 0.016$$

No shear reinforcement

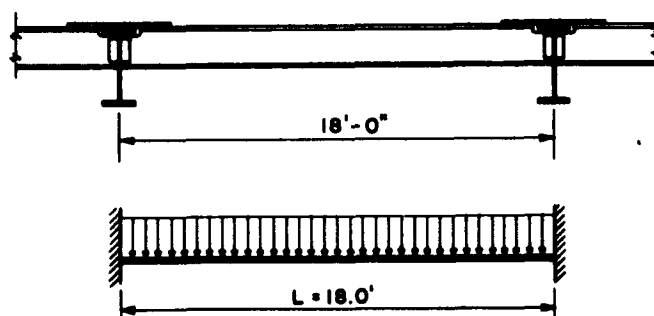
7-23 DESIGN OF ROOF PURLINS. The purlins are framed flush with the tops of the girders and are provided with moment-resisting connections. Connections attached to the top flanges of the purlins are embedded in the concrete slab to provide lateral support to the top compression flange and to prevent separation of slab from purlin in reversals.

Although composite behavior of the slab and purlin can be expected to develop to a limited extent, preliminary computations showed that design for independent behavior of slab and purlin is more desirable for this arrangement of members (pars. 4-12 and 6-23).

The purlins are designed for plastic behavior so that hinges are considered to develop at midspan and at the supports. In the design procedures of this manual only single-span elements can be handled.

Therefore, the continuity of the purlins is accounted for approximately by designing interior purlins as fixed-end beams spanning 18 ft between girder centerlines.

Depending on the exterior support condition the exterior purlins are designed as fixed-pinned beams or as fixed-fixed beams. In this example a typical interior purlin is designed.

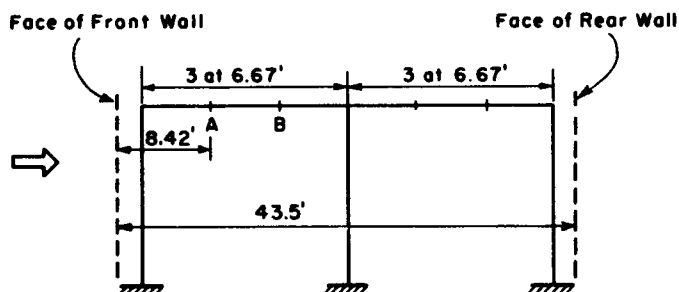


a. Loading. To present a complete picture of the loads that need be considered acting on purlins, two directions for the blast wave are considered (par. 3-09).

For the blast wave moving normal to the long axis of the building and thus normal to the axis of the purlin, the loading may be considered to be uniformly distributed along the length of the purlin. For this condition the pressure vs time variation at each point along the roof is a function of its position (par. 3-09). In addition the load on a purlin is a function of the length of the slab spans because the load on the purlin builds up to a maximum value in the time required for the blast wave to traverse the two adjoining slabs. In the preliminary design of the purlins the design load is the simplest form of the roof load obtained from the incident overpressure vs time curve. The rise time, slab dynamic reactions, and local variation are all neglected in this preliminary step.

For the blast wave moving parallel to the long axis of the building and thus parallel to the axis of the purlin, the load varies along the span as a result of the time required for the blast wave to traverse the purlin span. At any point along the purlin the time variation of the load is the same and defined by the incident overpressure vs time curve.

In the calculations that follow the load vs time curves for the purlin are obtained first for the blast wave moving parallel to the long axis of the building and then for the blast wave moving parallel to the short axis of the building.



The purlin obtained by the preliminary design procedure is analyzed for both loads in tables 7.5, 7.6, and 7.7.

Blast wave moving perpendicular to the long axis of the building:

From the procedure of para-

graph 3-09d the data for point (A) at purlin (A) are:

$$t_d = \frac{L'}{U_0} = \frac{8.42}{1403} = 0.006 \text{ sec}$$

$$v = \left[ 0.042 + (0.108) \frac{6.67}{43.5} \right] 1403 = 88.1 \text{ fps}$$

$$t_m = \frac{L'}{v} = \frac{8.42}{88.1} = 0.096 \text{ sec}$$

$$0.5(t_d + t_m) = 0.5(0.006 + 0.096) = 0.051 \text{ sec}$$

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$$t_m + 15 \frac{h}{U_o} = 0.096 + 15 \frac{16}{1403} = 0.266 \text{ sec}$$

$$4 \left( \frac{P_{so}}{14.7} \right) \left( \frac{L'}{L} - 1 \right) + 1 = 4 \left( \frac{10}{14.7} \right) \left( \frac{8.42}{43.5} - 1 \right) + 1 = -1.2$$

This value must be  $\geq 0$ ; it is therefore taken as zero.

The resulting variations in the ratio of local roof overpressure to incident overpressure for both points (A) and (B) are plotted in figure 7.15. The calculations for point (B) are not shown but are similar to those for point (A).

By combining figure 7.15 and the incident overpressure curve (fig. 7.8) the variation of local roof overpressure with time is determined (fig. 7.16) for points (A) and (B). The calculations of local roof overpressure for point (A) are contained in table 7.4. The calculations of data for point (B) are not shown.

Table 7.4. Computation of Local Roof Overpressure at Point (A)

t (sec)	t - t <sub>d</sub> (sec)	$\frac{t - t_d}{t_o}$	$\frac{P_s}{P_o}$ (fig. 7.15)	P <sub>s</sub> (fig. 7.8)	$\frac{P_{\text{roof}}}{P_s}$	P <sub>roof</sub> (psi)
0.00475	0	0	1.0	10.0	1.0	10.0
0.0429	0.03815	0.0558	0.893	8.93	1.0	8.93
0.07325	0.0685	0.1	0.814	8.14	0.205	1.67
0.0810	0.07625	0.1115	0.795	7.95	0.0	0.0
0.14175	0.1370	0.20	0.655	6.55	0.380	2.49
0.21025	0.2055	0.30	0.519	5.19	0.810	4.20
0.24100	0.23625	0.346	0.463	4.63	1.0	4.63
0.27875	0.274	0.4	0.402	4.02	1.0	4.02
0.34725	0.3425	0.5	0.303	3.03	1.0	3.03
0.41575	0.411	0.6	0.220	2.20	1.0	2.20
0.48375	0.479	0.7	0.149	1.49	1.0	1.49
0.55275	0.548	0.8	0.090	0.90	1.0	0.90
0.62075	0.616	0.9	0.041	0.41	1.0	0.41
0.68975	0.685	1.0	0.0	0.0	1.0	0.0

The data in the last column are plotted in figure 7.16.

The roof slab is analyzed in table 7.5 for both local overpressure vs time curves presented in figure 7.16. The analysis in table 7.5 is based

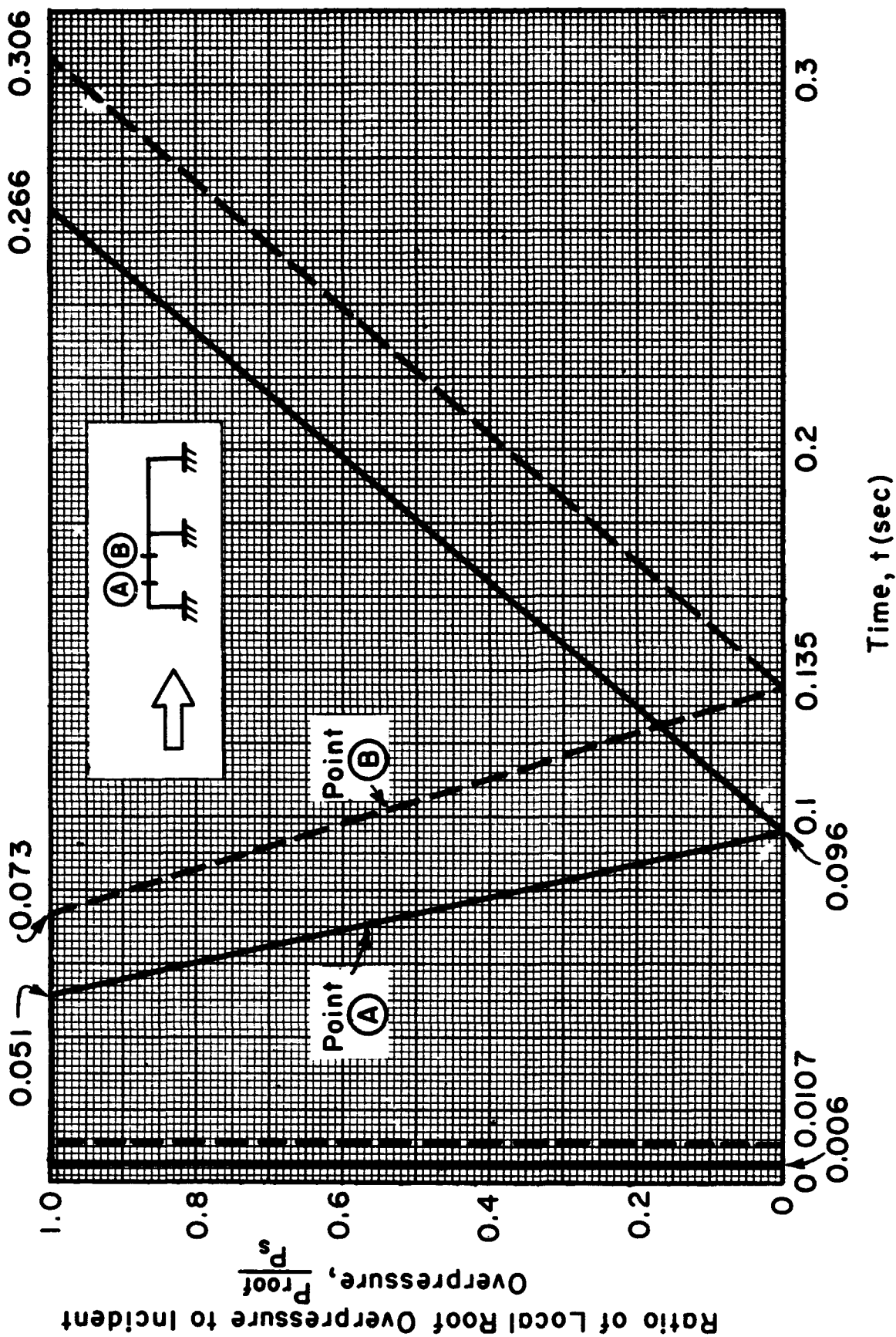


Figure 7.15. Ratio of local roof overpressure to incident overpressure for points (A) and (B)



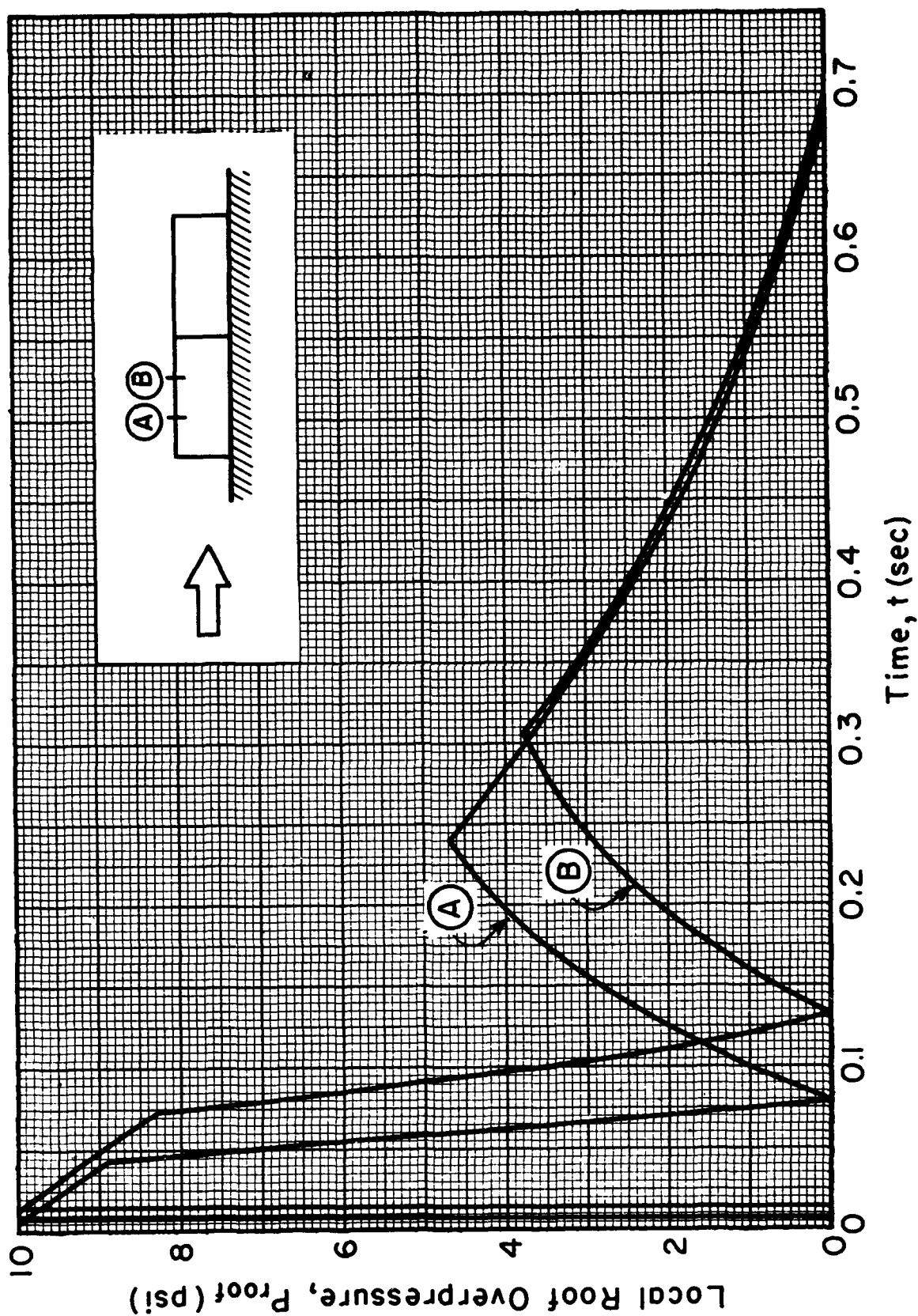


Figure 7.16. Local roof overpressure vs time for points (A) and (B)

Table 7.5. Determination of Dynamic Reactions for Roof Slab,  
Local Roof Overpressure at Purlins (A) and (B)

t (sec)	Purlin (A)						Purlin (B)	
	$P_n$ (kips)	$R_n$ (kips)	$P_n - R_n$ (kips)	$y_n(\Delta t)^2$ (ft)	$y_n$ (ft)	$V_n^*$ (kips)	$P_n$ (kips)	$V_n^*$ (kips)
0	0	0	0.62	+0.00030	0	0	0	0
0.002	3.97	0.29	3.68	0.00175	0.00030	0.66	3.97	0.66
0.004	7.94	2.30	5.64	0.00268	0.00235	1.94	7.94	1.94
0.006	9.47	6.94	2.53	0.00120	0.00708	3.83	9.47	3.83
0.008	9.42	10.15	-0.73	-0.00034	0.01301	5.00	9.42	5.00
0.010	9.36	11.24	-1.88	-0.00088	0.01860	5.41	9.36	5.41
0.012	9.30	12.17	-2.87	-0.00134	0.02331	5.77	9.30	5.77
0.014	9.25	12.83	-3.58	-0.00168	0.02668	6.02	9.25	6.02
0.016	9.20	12.90	-3.70	-0.00206	0.02837	6.00	9.20	6.00
0.018	9.14	12.54	-3.40	-0.00162	0.02800	5.79	9.14	5.79
0.020	9.09	10.59	-1.50	-0.00071	0.02601	5.08	9.09	5.08
0.022	9.03	7.94			0.02331	4.51	9.03	4.51
0.024	8.98					4.49	8.98	4.49
0.026	8.93					4.46	8.93	4.46
0.028	8.88					4.44	8.88	4.44
0.030	8.82					4.41	8.82	4.41
0.032	8.76					4.38	8.76	4.38
0.034	8.70					4.35	8.70	4.35
0.036	8.64					4.32	8.64	4.32
0.038	8.60					4.30	8.60	4.30
0.040	8.56					4.28	8.54	4.27
0.042	8.10					4.05	8.50	4.25
0.044	7.64					3.82	8.44	4.22
0.046	7.18					3.59	8.38	4.19
0.048	6.70					3.35	8.34	4.17
0.050	6.24					3.12	8.28	4.14
0.052	5.78					2.89	8.22	4.11
0.054	5.32					2.66	8.18	4.09
0.056	4.86					2.43	8.12	4.06
0.058	4.38					2.19	8.08	4.04
0.060	3.88					1.94	8.02	4.01
0.062	3.46					1.73	7.96	3.98
0.064	3.00					1.50		
0.066	2.54					1.27		
0.068	2.06					1.03		

\* From  $t = 0.020$  the values of  $V_n$  are equal to  $0.50P_n$ .

on the following data developed in paragraph 7-22j and the resistance diagram for the roof slab (fig. 7.14):

$$\text{Elastic range: } y_n(\Delta t)^2 = 3.48(10^{-4})(P_n - R_n) \text{ ft}$$

$$\text{Elasto-plastic range: } y_n(\Delta t)^2 = 3.439(10^{-4})(P_n - R_n) \text{ ft}$$

$$\text{Plastic range: } y_n(\Delta t)^2 = 4.064(10^{-4})(P_n - R_n) \text{ ft}$$

The object of this computation is to determine the slab dynamic reactions on the purlins.

The maximum response of the slab to these loads occurs before there is any difference between the loads at (A) and (B), thus the dynamic reactions of the slabs at purlins (A) and (B) are the same until the dynamic reactions are based on the applied load  $P$  above. The last two columns of table 7.5 show the applied load and the dynamic reactions at purlin (B).

To obtain the design load for purlins (A) and (B) the dynamic reaction data from table 7.5 are plotted in figure 7.17. The total purlin load is equal to the sum of the reactions of the slabs forward and aft of the purlin. In figure 7.17, it may be seen that the same dynamic reactions are plotted with a time lag

$$t_{\text{lag}} = \frac{6.67}{1403} = 0.0048 \text{ sec}$$

The loads from figure 7.17 are used in tables 7.6 and 7.7 (par. 7-23j) to check the preliminary purlin design.

Blast wave moving parallel to the long axis of the building:

The design load on the purlin is determined in figure 7.17a using the slab dynamic reactions obtained in table 7.3 for incident overpressure. The variation of the average load on the purlin with time is found by plotting the same dynamic reaction curve with a time lag

$$t_d = \frac{18}{1403} = 0.0128 \text{ sec}$$

The variation in slab dynamic reaction with time is the same at each point along the purlin. The load curve from figure 7.17a is used in table 7.8 to check the preliminary purlin design.

Preliminary design:

For preliminary design it is desirable to use a simple load-time

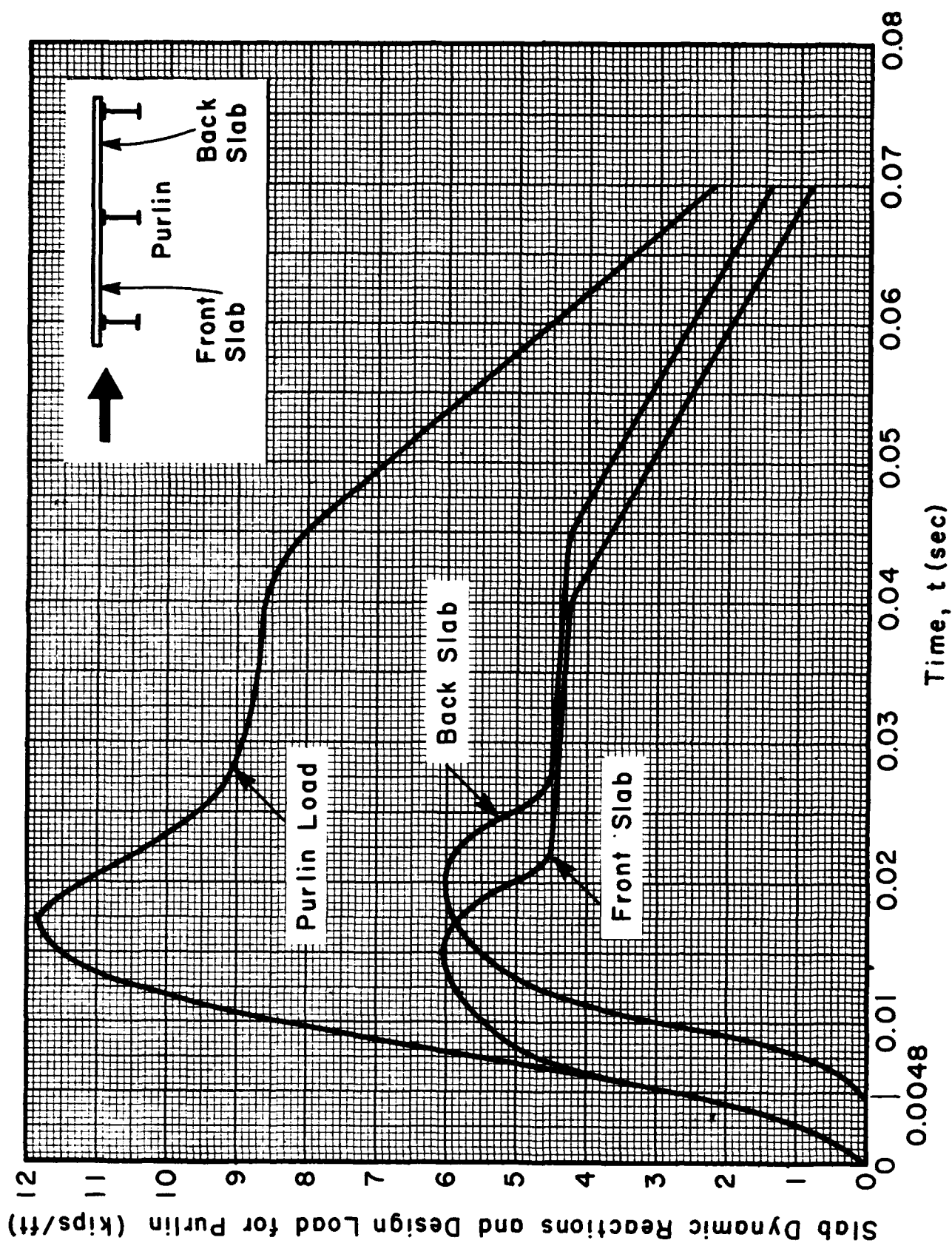


Figure 7.17. Design loads for purlins (A) and (B) for local roof overpressure

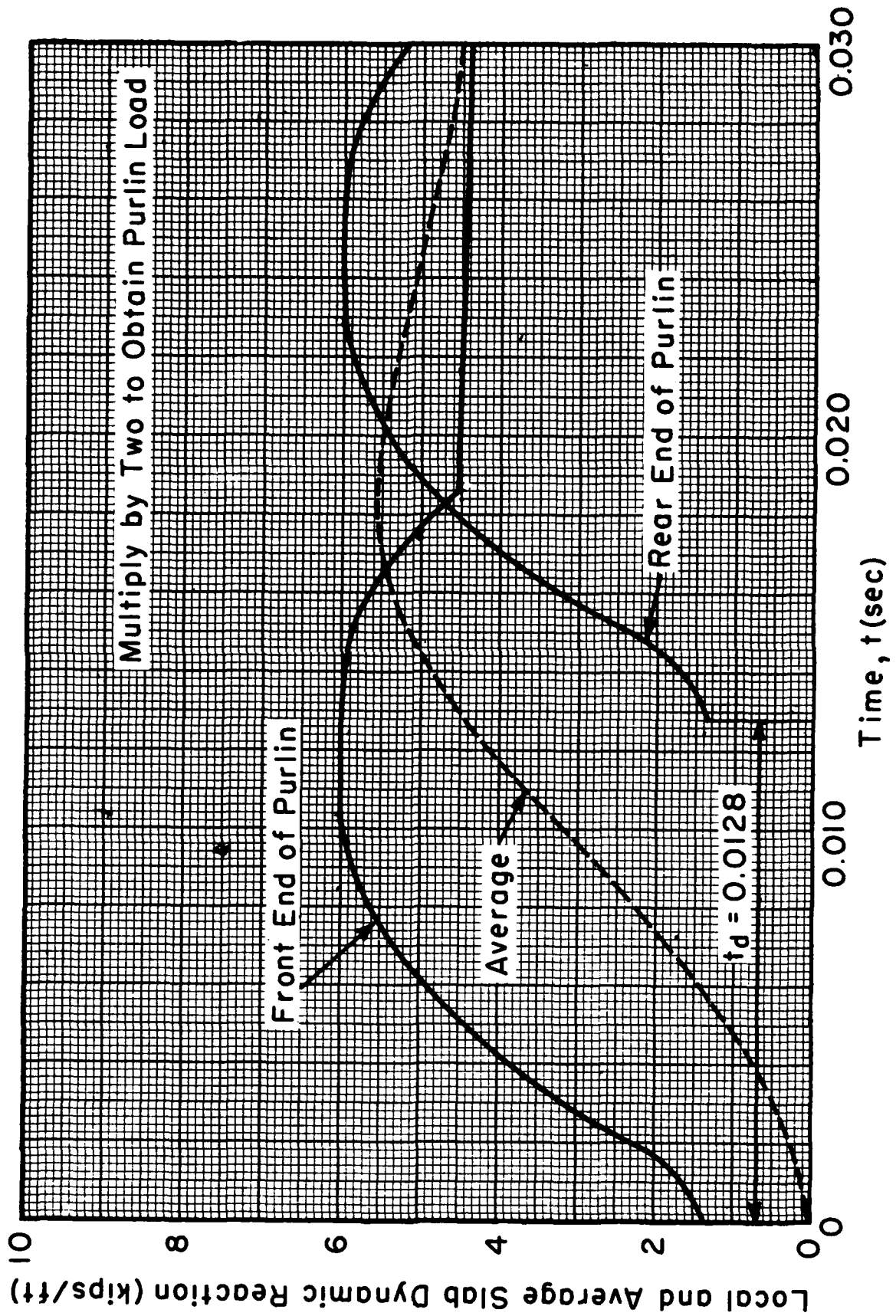
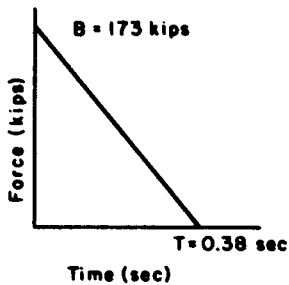


Figure 7.17a. Design loads for any purlin for incident overpressure

curve. The design load as idealized from the computed loading shown by figure 7.8 is defined by



$$B = 10 \text{ psi} = \frac{10(144)(6.67)(18)}{1000} = 173 \text{ kips}$$

$$T = 0.38 \text{ sec}$$

$$H = \frac{BT}{2} = \frac{(173)(0.38)}{2} = 32.9 \text{ kip-sec (par. 6-11)}$$

b. Dynamic Design Factors. (Refer to table 6.1.)

Elastic range:

$$K_L = 0.53,$$

$$K_M = 0.41,$$

$$K_{LM} = 0.77$$

$$R_{lm} = \frac{12M_{Ps}}{L}$$

$$V_1 = 0.36R + 0.14P, \quad k_1 = \frac{384EI}{L^3}$$

Elasto-plastic range:

$$K_L = 0.64,$$

$$K_M = 0.50,$$

$$K_{LM} = 0.78$$

$$R_m = \frac{8}{L}(M_{Ps} + M_{Pm}), \quad k_{ep} = \frac{384EI}{5L^3}$$

$$V = 0.39R + 0.11P$$

Plastic range:

$$K_L = 0.50,$$

$$K_M = 0.33,$$

$$K_{LM} = 0.66$$

$$R_m = \frac{8}{L}(M_{Ps} + M_{Pm})$$

$$V = 0.38R_m + 0.12P$$

Average values:

$$K_L = 0.5(0.64 + 0.50) = 0.57$$

$$K_M = 0.5(0.50 + 0.33) = 0.42$$

$$K_{LM} = 0.5(0.78 + 0.77) = 0.77$$

$$R_m = \frac{8}{L}(M_{Ps} + M_{Pm})$$

$$k_E = \frac{307EI}{L^3}$$

c. First Trial - Actual Properties.

$$M_{Ps} \equiv M_{Pm} = M_P$$

Let  $\alpha\beta = 6$  (par. 6-26)

Assume  $C_R = 1.0$  (experience)

$$R_m = C_R B = 1.0(173) = 173 \text{ kips}$$

$$M_P = 1.05 S f_{dy} = \frac{1.05(41.6)S}{12} = 3.64S \text{ kip-ft (S in in.}^3 \text{)} \quad (\text{eq 4.2a})$$

$$R_m = \frac{16M_P}{L} = \frac{(16)3.64S}{18} = 173, \therefore S = 53.5 \text{ in.}^3$$

Try 16 W 36,

$$S = 56.3 \text{ in.}^3, \quad Z = 64 \text{ in.}^3, \quad 0.5(S + Z) = 60.1 \text{ in.}^3, \quad I = 446 \text{ in.}^4$$

$$M_P = f_{dy} \left( \frac{S + Z}{2} \right) = \frac{41.6(60.1)}{12} = 208 \text{ kip-ft (eq 4.2)}$$

$$R_m = \frac{16M_P}{L} = \frac{16(208)}{18} = 185 \text{ kips}$$

$$k_E = \frac{307EI}{L^3} = \frac{(307)30(10)^3 446}{18^3(144)} = 4900 \text{ kips/ft}$$

$$y_E = \frac{R_m}{k_E} = \frac{185}{4900} = 0.0378 \text{ ft}$$

$$y_m = \alpha\beta y_E = 6(0.0378) = 0.2268 \text{ ft (par. 6-26)}$$

$$\text{Weight} = \left[ \frac{3.75(150)}{(12)} + 6.0 \right] \frac{6.67(18)}{1000} + \frac{36(18)}{1000} = 6.35 + 0.65 = 7.0 \text{ kips}$$

$$\text{Mass } m = \frac{7.0}{32.2} = 0.217 \text{ kip-sec}^2/\text{ft}$$

d. First Trial - Equivalent System Properties.

$$R_{me} = K_L R_m = 0.57(185) = 105.5 \text{ kips (eq 6.12)}$$

$$H_e = K_L H = 0.57(32.9) = 18.75 \text{ kip-sec (eq 6.2)}$$

$$m_e = K_M m = 0.42(0.217) = 0.091 \text{ kip-sec}^2/\text{ft (eq 6.18)}$$

$$W_P = \frac{(H_e)^2}{2m_e} = \frac{(18.75)^2}{2(0.091)} = 1930 \text{ ft-kips}$$

$$T_n = 2\pi \sqrt{K_{LM} m / k_E} = 6.28 \sqrt{0.77(0.217) / 4900} = 0.0368 \text{ sec}$$

e. Work Done vs Energy Absorption Capacity.

$$C_T = T / T_n = 0.38 / 0.0368 = 10.3$$

$$C_R = R_m / B = 185 / 173 = 1.07 \text{ (eqs 6.15, 6.16)}$$

$$t_m / T = 0.12 \text{ (fig. 5.29)}$$

$$t_m = (0.12)0.38 = 0.0455 \text{ sec}$$

Idealized load-time curve is satisfactory at  $t = 0.045 \text{ sec}$  (par. 5-13)

$$C_W = 0.008 \text{ (fig. 5.27)}$$

$$W_m = C_W W_P = 0.008(1930) = 15.4 \text{ ft-kips (eq 6.17)}$$

$$E = R_{me}(y_m - 0.5y_E) = 105.5 [0.2268 - 0.5(0.0378)] \\ = 22.0 \text{ ft-kips (eq 6.18)}$$

$E > W$ , therefore the selected proportions are satisfactory as a preliminary design.

Try to bring  $E$  closer to  $W$  by another trial that follows.

f. Second Trial - Actual Properties.

$$R_m = \frac{0.25W_m + 0.75E}{K_L(y_m - 0.5y_E)} = \frac{0.25(15.4) + 0.75(22.0)}{(0.57)[0.2268 - 0.5(0.0378)]} = 172 \text{ kips (eq 6.19)}$$

Since  $R_m = 172 \approx 173$  from first trial, try

$$R_m = \frac{0.5W_m + 0.5E}{K_L(y_m - 0.5y_E)} = \frac{0.5(15.4 + 22.0)}{0.57(0.208)} = 158 \text{ kips}$$

$$R_m = \frac{16M_P}{L} = \frac{16(3.64S)}{18} = 158, \therefore S = 48.5$$

Try 14 WF 34,

$$S = 48.5 \text{ in.}^3, \quad Z = 54.5 \text{ in.}^3, \quad 0.5(S + Z) = 51.5 \text{ in.}^3, \quad I = 339.2 \text{ in.}^4$$

$$M_P = f_{dy} \frac{(S + Z)}{2} = \frac{41.6(51.5)}{12} = 178 \text{ kip-ft (eq 4.2)}$$

$$R_m = \frac{16M_P}{L} = \frac{16(178)}{18} = 159 \text{ kips}$$

$$k_E = \frac{307EI}{L^3} = \frac{307(30)10^3(339.2)}{18^3(144)} = 3720 \text{ kips/ft}$$

$$y_E = \frac{R_m}{k_E} = \frac{159}{3720} = 0.0427 \text{ ft}$$

$$y_m = \alpha \beta y_E = 6(0.0427) = 0.2562 \text{ (par. 6-26)}$$

g. Second Trial - Equivalent System Properties.

$$R_{me} = K_L R_m = 0.57(159) = 90.5 \text{ kips (eq 6.12)}$$

$$k_e = K_L k_E = 0.57(3720) = 2120 \text{ kips/ft (eq 6.6)}$$

$$H_e = K_L H = 0.57(32.9) = 18.75 \text{ kip-sec (eq 6.2)}$$

$$m_e = K_M m = 0.42(0.217) = 0.091 \text{ kip-sec}^2/\text{ft (eq 6.8)}$$



$$W_P = \frac{H_e^2}{2m_e} = \frac{(18.75)^2}{2(0.091)} = 1930 \text{ ft-kips}$$

$$T_n = 2\pi\sqrt{K_{LM}/k_E} = 6.28\sqrt{0.77(0.217)/3720} = 0.042 \text{ sec}$$

h. Work Done vs Energy Absorption Capacity.

$$C_T = T/T_n = 0.38/0.042 = 9.05$$

$$C_R = R_m/B = 159/173 = 0.92$$

$$C_W = 0.022$$

$$W_m = 0.022(1930) = 42.5; \text{ too large, use initial trial 16 W 36}$$

i. Preliminary Design for Shear Stress.

$$\text{Estimated } V_{\max} = 0.5R_m = 0.5(185) = 92.5 \text{ kips}$$

$$v = \frac{V}{t_w d} = \frac{92,500}{0.299(15.85)} = 19,500 \text{ psi}$$

$$\text{Allowable } v = 21,000 \text{ psi; OK (par. 4-05c)}$$

j. Determination of Maximum Deflection and Dynamic Reactions by

Numerical Integration.

16 W 36,

$$S = 56.3 \text{ in.}^3, \quad Z = 64 \text{ in.}^3, \quad 0.5(S + Z) = 60.1 \text{ in.}^3, \quad I = 446 \text{ in.}^4$$

$$M_P = 208 \text{ kip-ft}$$

$$\text{Weight} = 7.0 \text{ kips}$$

$$\text{Mass } m = 0.217 \text{ kip-sec}^2/\text{ft}$$

Elastic range:

$$R_{lm} = \frac{12M_P}{L} - \text{weight} = \frac{12(208)}{18} - 7.0 = 132.0 \text{ kips}$$

$$k_1 = \frac{384EI}{L^3} = \frac{(384)30(10)^3 446}{18^3(144)} = 6130 \text{ kips/ft}$$

$$y_e = \frac{R_{lm}}{k_1} = \frac{132.0}{6130} = 0.0215 \text{ ft}$$

Elasto-plastic range:

$$R_m = \frac{16M_P}{L} - \text{weight} = \frac{16(208)}{18} - 7.0 = 178.0 \text{ kips}$$

$$k_{ep} = \frac{384EI}{5L^3} = \frac{1}{5} k_1 = 1226 \text{ kips/ft}$$

$$y_{ep} = y_e + \frac{R_m - R_{lm}}{k_{ep}} = 0.0215 + \frac{178 - 132}{1226} = 0.0215 + 0.0375 = 0.059 \text{ ft}$$

Plastic range:

$$R_m = \frac{16M_P}{L} - \text{weight} = 178.0 \text{ kips}$$

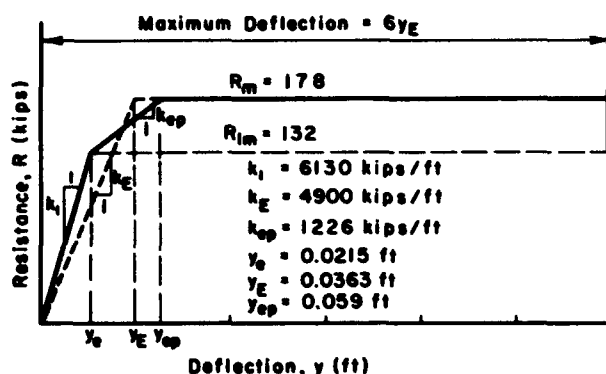


Figure 7.18. Resistance function for 16 W 36 purlin spanning 18 ft

$$k_E = \frac{307EI}{L^3} = 4900 \text{ kips/ft}$$

$$y_E = \frac{R_m}{k_E} = \frac{178.0}{4900} = 0.0363 \text{ ft}$$

$$y_m = \alpha \beta y_E = 6(0.0363) = 0.2178$$

The basic equation for the numerical integration in tables 7.6, 7.7, and 7.8 is  $y_{n+1} = y_n(\Delta t)^2 + 2y_n - y_{n-1}$  (table 5.3) where

Table 7.6. Determination of Maximum Deflection and Dynamic Reactions for Purlin (A) (Zone 3 Local Roof Overpressure)

t (sec)	$P_n$ (kips)	$R_n$ (kips)	$P_n - R_n$ (kips)	$\ddot{y}_n(\Delta t)^2$ (ft)	$y_n$ (ft)	$V_n$ (kips)
0	0	0	3.3	0.00018		0
0.003	20.7	1.1	19.6	0.00106	0.00018	3.3
0.006	73.8	9.0	64.8	0.00349	0.00142	13.6
0.009	129.6	38.8	90.8	0.00489	0.00615	32.1
0.012	180.0	99.5	80.5	0.00434	0.01577	61.0
0.015	207.0	142.1	64.9	0.00345	0.02973	78.1
0.018	212.4	163.4	49.0	0.00260	0.04714	87.1
0.021	192.6	178.0	14.6	0.00092	0.06715	90.8*
0.024	176.4	178.0	-1.6	-0.00010	0.08808	88.8
0.027	163.8	178.0	-14.2	-0.00089	0.10891	87.3
0.030	160.2	178.0	-17.8	-0.00112	0.12885	85.8
0.033	158.4	178.0	-19.6	-0.00123	0.14767	86.6
0.036	156.6	178.0	-21.4	-0.00134	0.16526	86.4
0.039	154.8	178.0	-23.2	-0.00146	0.18151	86.2
0.042	151.2	178.0	-26.8	-0.00168	0.19630	85.8
0.045	145.8	178.0	-32.2	-0.00202	0.20941	85.1
0.048	131.4	178.0	-46.6	-0.00293	0.22050	83.4
0.051	118.8	178.0	-59.2	-0.00372	0.22866	81.9
0.054	104.4	178.0	-73.6	-0.00463	0.23310	80.2
0.057	93.6	176.8	-83.2	-0.00448	0.23291	76.8
0.060	81.0	147.3	-66.3	-0.00357	0.22824	64.4
0.063	68.4	95.3	-26.9	-0.00145	0.22000	43.9
0.066	55.8	34.2			0.21031	27.9
0.069	45.0					22.5

\*  $(V_n)_{max} = 90.8 \text{ kips.}$

**Table 7.7. Determination of Maximum Deflection and Dynamic Reactions for Purlin (B)**  
(Zone 3 Local Roof Overpressure)

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> - R <sub>n</sub> (kips)	$\ddot{y}_n(\Delta t)^2$ (ft)	y <sub>n</sub> (ft)	V <sub>n</sub> (kips)
0	0	0	3.3	0.00018	0	0
0.003	20.7	1.1	19.6	0.00106	0.00018	3.3
0.006	73.8	9.0	64.8	0.00349	0.00142	13.6
0.009	129.6	38.8	90.8	0.00489	0.00615	32.1
0.012	180.0	99.5	80.5	0.00434	0.01577	61.0
0.015	207.0	142.1	64.9	0.00345	0.02973	78.1
0.018	212.4	163.4	49.0	0.00260	0.04714	87.1
0.021	192.6	178.0	14.6	0.00092	0.06715	90.8*
0.024	176.4	178.0	-1.6	-0.00010	0.08808	88.8
0.027	163.8	178.0	-14.2	-0.00089	0.10891	87.3
0.030	160.2	178.0	-17.8	-0.00112	0.12885	86.8
0.033	158.4	178.0	-19.6	-0.00123	0.14767	86.6
0.036	156.6	178.0	-21.4	-0.00134	0.16526	86.4
0.039	154.8	178.0	-23.2	-0.00146	0.18151	86.2
0.042	153.5	178.0	-24.5	-0.00154	0.19630	86.1
0.045	152.1	178.0	-25.9	-0.00163	0.20955	85.9
0.048	150.7	178.0	-27.3	-0.00172	0.22117	85.7
0.051	149.3	178.0	-28.7	-0.00180	0.23107	85.6
0.054	147.9	178.0	-30.1	-0.00189	0.23917	85.4
0.057	146.5	178.0	-31.5	-0.00198	0.24538	85.2
0.060	145.1	178.0	-32.9	-0.00207	0.24961	85.1
0.063	143.7	178.0	-34.3	-0.00216	0.25177	84.9
0.066	142.3	178.0	-35.7	-0.00224	0.25177	84.7
0.069	140.9	163.9	-23.0	-0.00124	0.24953	78.7
0.072	139.5	141.9			0.24605	70.6

\* (V<sub>n</sub>)<sub>max</sub> = 90.8 kips.

**Table 7.8. Determination of Maximum Deflection and Dynamic Reactions for any Purlin**  
(Incident Overpressure)

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> - R <sub>n</sub> (kips)	$\ddot{y}_n(\Delta t)^2$ (ft)	y <sub>n</sub> (ft)	V <sub>n</sub> (kips)
0	0	0	2.8	0.00015	0	
0.003	18.0	0.9	17.1	0.00092	0.00015	
0.006	50.4	7.5	42.9	0.00231	0.00122	
0.009	97.2	28.2	69.0	0.00372	0.00460	
0.012	147.6	71.7	75.9	0.00409	0.01170	
0.015	183.6	133.7	49.9	0.00265	0.02289	
0.018	201.6	150.7	50.9	0.00271	0.03673	
0.021	194.4	171.0	23.4	0.00124	0.05328	
0.024	183.6	178.0	5.6	0.00035	0.07107	
0.027	172.8	178.0	-5.2	-0.00033	0.08921	
0.030	162.0	178.0	-16.0	-0.00101	0.10702	
0.033	160.6	178.0	-17.4	-0.00109	0.12382	
0.036	159.2	178.0	-18.8	-0.00118	0.13953	
0.039	157.8	178.0	-20.2	-0.00127	0.15406	
0.042	156.5	178.0	-21.5	-0.00135	0.16732	
0.045	155.1	178.0	-22.9	-0.00144	0.17923	
0.048	153.7	178.0	-24.3	-0.00153	0.18970	
0.051	152.3	178.0	-25.7	-0.00161	0.19864	
0.054	150.9	178.0	-27.1	-0.00170	0.20597	
0.057	149.5	178.0	-28.5	-0.00179	0.21160	
0.060	148.2	178.0	-29.8	-0.00187	0.21544	
0.063	146.8	178.0	-31.2	-0.00196	0.21741	
0.066	145.4	178.0	-32.6	-0.00205	0.21742	
0.069					0.21538	

89.7

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(\Delta t)^2}{K_{IM}(m)}$$

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)9(10^{-6})}{0.77(0.217)} = 5.386(10^{-5})(P_n - R_n) \text{ ft, elastic range}$$

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)9(10^{-6})}{0.78(0.217)} = 5.317(10^{-5})(P_n - R_n) \text{ ft, elasto-plastic range}$$

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)9(10^{-6})}{0.66(0.217)} = 6.284(10^{-5})(P_n - R_n) \text{ ft, plastic range}$$

The time interval  $\Delta t = 0.003$  sec is approximately  $T_n/10 = 0.00368$  sec (par. 5-08).

The dynamic reaction equations are listed in paragraph 7-23b. The  $P$  values for tables 7.6 and 7.7, second column, are obtained from figure 7.17, multiplying by 18 to account for the length of the purlin. For table 7.8 the  $P_n$  values in the second column are obtained from figure 7.17a multiplying by  $2 \times 18 = 36$  to account for the two slabs loading the purlin.

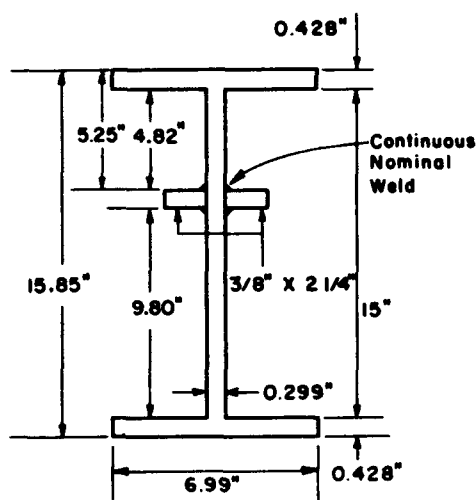
The maximum deflection  $(y_n)_{\max}$  computed in table 7.7 is 0.252 ft. Thus  $\alpha\beta = \frac{0.252}{0.0363} = 6.9 > 6$ . This is satisfactory for this purpose. All other purlins are less critical.

k. Shear Stress Check.

$$V_{\max} = 90.8 \text{ kips (tables 7.6 and 7.7)}$$

$$v = \frac{V}{dt_w} = \frac{90,800}{15.85(0.299)} = 19,200 \text{ psi}$$

Allowable  $v = 21,000$  psi; OK (par. 4-05c)



1. Check Proportions for Local Buckling. (par. 4-06d)

$$16 \text{ WF } 36, \quad b = 6.992, \quad t_f = 0.428,$$

$$a = 15.0, \quad t_w = 0.299$$

$$b/t_f = 6.99/0.428 = 16.4 > 14.0, \text{ OK;}$$

slab provides support

$$a/t_w = 15.0/0.299 = 50.2 > 30, \text{ NG; longitudinal stiffeners required}$$

$$t_s = 3/8 > 0.299, \text{ OK}$$

$$b_s/t_s = 6; \quad b_s = 6t_s = 6(3/8) = 2.25 \text{ in.}$$

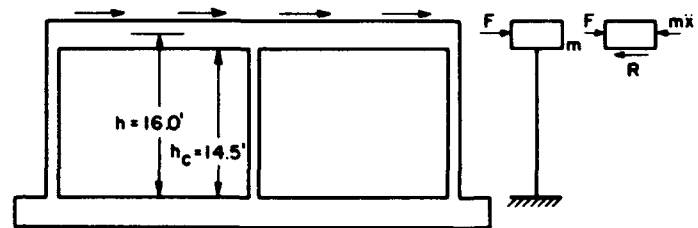
Use two plates  $3/8$  in. by  $2-1/4$  in. full length.

$$c/t_w = (5.25 - 0.43)/0.299 = 16 < 22; \text{OK}$$

$$e/t_w = (10.25 - 0.43)/0.299 = 32.8 < 40; \text{OK}$$

7-24 PRELIMINARY DESIGN OF COLUMNS. A single-story frame subject to lateral load behaves essentially as a single-degree-of-freedom system with the columns displaying the spring properties. It is therefore unnecessary to substitute an equivalent system for the original structure and the mass and load factors which are necessary in the design of beams and slabs are not used in the design of single-story frames.

The preliminary plastic design procedure of paragraph 6-11 is the basis for determining the preliminary column size. The equations of paragraph 7-06 are incorporated into the procedure of paragraph 6-11 replacing some of the factors that are used to determine equivalent systems.



For purposes of preliminary design the frame girders are assumed to be infinitely rigid thus simplifying the determination of the column spring constant. For spring constant computation the effective column height  $h$  is 16 ft based on an assumed girder depth of 3 ft and a clear height  $h_c$  of 14.5 ft. The clear height is used in determining the resistance of the columns (par. 7-06).

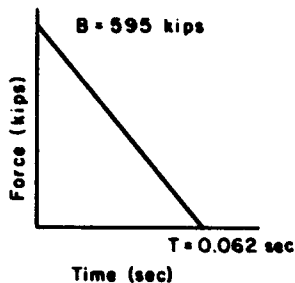
In the preliminary design of steel columns it is desirable that the energy absorption capacity be greater than the work done on the frame as an allowance for the factors which are neglected. These factors are: (1) the effect of direct stress on the plastic hinge moment, (2) the effect of lateral deflection of the column on its resistance, and (3) the effect of girder flexibility.

a. Design Loading. The design lateral load on the frame is obtained from the dynamic reactions at the top of the front wall slab. However, for preliminary design computations, it is satisfactory to use the net lateral overpressure curve (fig. 7.11).

The net lateral load is assumed to be reacted equally by the frame

and the foundation. This results in a conservative load for the frame since the dynamic reaction equations for the wall slab (par. 7-21b) result in footing reactions that are larger than the roof reactions.

The design load as idealized from the computed loading as shown by figure 7.11 is defined by



$$B = 25.3 \text{ psi} = \frac{25.3(144)18\left(\frac{17.5}{2} + 0.31\right)}{1000} = 595 \text{ kips}$$

$$T = 0.062 \text{ sec}$$

$$H = \frac{BT}{2} = \frac{595(0.062)}{2} = 18.45 \text{ kip-sec}$$

b. Mass Computation.

$$\text{Walls } \frac{11(150)18(2)17.5}{12(1000)} = 86.5 \text{ kips}$$

$$\text{Slab and roofing } \left(\frac{3.75(150)}{12} + 6.0\right) \frac{18(43.5)}{1000} = 41.4 \text{ kips}$$

$$\text{Purlins } \frac{42.4(18)7}{1000} = 5.3 \text{ kips}$$

$$\text{Girder (estimate)} \frac{160(41)}{1000} = 6.5 \text{ kips}$$

$$\text{Connections (allow 10\%)} = 0.1(5.3 + 6.5) = 1.2 \text{ kips}$$

$$\text{Columns (estimate)} \frac{140(3)14.5}{1000} = 6.1 \text{ kips}$$

Mass of single-degree-of-freedom system = total roof + 1/3 (columns and walls)

$$m = \frac{41.4 + 5.3 + 6.5 + 1.2 + 0.33(86.5 + 6.1)}{32.2} = \frac{85.2}{32.2} = 2.64 \text{ kip-sec}^2/\text{ft}$$

c. First Trial - Actual Properties.

$$\text{Let } \alpha\beta = 12 \text{ (par. 6-26)}$$

$$\text{Assume } C_R = 0.5 \text{ (experience)}$$

$$R_m = C_R B = 0.5(595) = 297 \text{ kips}$$

$$M_P = 1.05Sf_{dy} = \frac{1.05(41.6)}{12} S = 3.64S \text{ kip-ft (S in in.}^3\text{)} \text{ (eq 4.2a)}$$

$$R_m = (2nM_P)/h_c = [2(3)3.64S]/14.5 = 1.51S = 297, \therefore S = 197 \text{ in.}^3$$

Smallest column that satisfies buckling criteria is 14 W 136

(par. 4-06d)

$$S = 216.0 \text{ in.}^3, Z = 242.7 \text{ in.}^3, I = 1593 \text{ in.}^4, 0.5(S + Z) = 229.3 \text{ in.}^3$$

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$$a/t_w = 12.63/0.66 = 19.2 < 22; \text{ OK}$$

$$b/t_f = 14.74/1.063 = 13.9 < 14.0; \text{ OK (par. 4-06d)}$$

$$M_P = 0.5(S + Z)f_{dy} = \frac{229.3(41.6)}{12} = 795 \text{ kip-ft}$$

$$R_m = \frac{2nM_P}{h_c} = \frac{2(3)795}{14.5} = 328 \text{ kips}$$

$$k = \frac{12EI_n}{h^3} = \frac{12(30)10^3(1593)3}{16^3(144)} = 2910 \text{ kips/ft}$$

$$x_e = \frac{R_m}{k} = \frac{328}{2910} = 0.113 \text{ ft}$$

$$x_m = \alpha \beta x_e = 12(0.113) = 1.35 \text{ ft}$$

$$T_n = 2\pi\sqrt{m/k} = 6.28\sqrt{2.64/2910} = 0.189 \text{ sec}$$

d. First Trial - Work Done vs Energy Absorption Capacity.

$$T/T_n = 0.062/0.189 = 0.328$$

$$C_R = R_m/B = 328/595 = 0.55$$

$$t_m/T = 1.3, t_m = 1.3(0.062) = 0.0805 \text{ (fig. 5.29)}$$

The original load-time curve should be revised to obtain a closer approximation to the total impulse up to time  $t_m$  (par. 5-13). The impulse up to  $t = 0.09$  sec in figure 7.11 is

$$H = 1.00 \text{ psi-sec (obtained by graphical integration)}$$

$$T = 2H/B = 2(1.0)/25.3 = 0.079 \text{ sec}$$

$$T/T_n = 0.079/0.188 = 0.42$$

$$t_m/T = 1.2, t_m = 1.2(0.079) = 0.095 \text{ (fig. 5.29)}$$

Try again for impulse up to  $t = 0.10$  sec,  $H = 1.026$

$$T = 2(1.026)/25.3 = 0.081 \text{ sec}$$

$$T/T_n = 0.081/0.189 = 0.43$$

$$t_m/T = 1.2, t_m = 1.2(0.081) = 0.097 \approx t = 0.10; \text{ OK (fig. 5.29)}$$

$$C_W = 0.71 \text{ (fig. 5.27)}$$

$$W_P = \frac{H^2}{2m} = \left(\frac{BT}{2}\right)^2 / 2m = \frac{(595)^2(0.081)^2}{8(2.64)} = 111 \text{ ft-kips}$$

$$W_m = C_W W_P = 0.71(111) = 78.7 \text{ ft-kips}$$

$$E = R_m \left( x_m - \frac{x_e}{2} \right) = 328 [1.35 - 0.5(0.113)] = 426 \text{ ft-kips}$$

This column section (14 WF136) is more than ample. Try to use a smaller column size.

e. Second Trial - Actual Properties.

$$R_m = \frac{0.5(W_m + E)}{y_m} = \frac{0.5(78.7 + 426)}{1.35} = \frac{252}{1.35} = 187 \text{ kips}$$

$$R_m = \frac{2nM_P}{h_c} = \frac{2(3)3.64S}{14.5} = 187, \therefore S = 124 \text{ in.}^3$$

Try 12 W92 to satisfy buckling criteria (par. 4-04d).

$$S = 125.0 \text{ in.}^3, Z = 130.0 \text{ in.}^3, I = 788.9 \text{ in.}^4, 0.5(S + Z) = 127.5 \text{ in.}^3$$

$$a/t_w = 10.91/0.545 = 20 < 22$$

$$b/t_f = 12.155/0.856 = 14.2 \approx 14.0; \text{ OK}$$

$$M_P = 0.5(S + Z)f_{dy} = \frac{127.5(41.6)}{12} = 441 \text{ kip-ft}$$

$$R_m = \frac{2nM_P}{h_c} = \frac{2(3)441}{14.5} = 182 \text{ kips}$$

$$k = \frac{12EI_n}{h^3} = \frac{12(30)10^3(788.9)3}{16^3(144)} = 1440 \text{ kips/ft}$$

$$x_e = R_m/k = 182/1440 = 0.126 \text{ ft}$$

$$x_m = \alpha_B x_e = 12(0.126) = 1.52 \text{ ft}$$

$$T_n = 2\pi\sqrt{m/k} = 6.28\sqrt{2.64/1440} = 0.268 \text{ sec}$$

f. Second Trial - Work Done vs Energy Absorption Capacity.

$$T/T_n = 0.081/0.268 = 0.302$$

$$C_R = R_m/B = 182/595 = 0.306$$

$$t_m/T = 1.8, t_m = 1.8(0.081) = 0.146 \text{ sec (fig. 5.29)}$$

Revise the load-time curve as above (par. 5-13).

Impulse up to  $t = 0.18 \text{ sec}$  in figure 7.11 is  $H = 1.196 \text{ psi-sec}$

$$T = 2H/B = 2(1.196)/25.3 = 0.094 \text{ sec}$$

$$T/T_n = 0.094/0.268 = 0.351$$

$$t_m/T = 1.8, t_m = 1.8(0.094) = 0.17 \text{ sec; OK (fig. 5.29)}$$

$$C_W = 0.82 \text{ (fig. 5.27)}$$

$$W_P = \frac{H^2}{2m} = \left(\frac{BT}{2}\right)^2 / 2m = \frac{(595)^2 0.094^2}{8(2.64)} = 149.0 \text{ ft-kips}$$

$$W_m = C_W W_P = 0.82(149) = 122 \text{ ft-kips}$$

$$E = R_m \left( x_m - \frac{x_e}{2} \right) = 182 \left( 1.52 - \frac{0.126}{2} \right) = 266 \text{ ft-kips}$$



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This column section is ample. Another trial may be made to reduce the size further.

g. Third Trial - Actual Properties.

$$R_m = \frac{0.5(W_m + E)}{y_m} = \frac{0.5(122 + 266)}{1.52} = 128 \text{ kips}$$

$$R_m = \frac{2nM_P}{h_c} = \frac{2(3)3.64(S)}{14.5} = 128, \therefore S = 85 \text{ in.}^3$$

$$\text{Try 10 W F 77, } S = 86.1 \text{ in.}^3, Z = 97.6 \text{ in.}^3, 0.5(S + Z) = 91.8 \text{ in.}^3, \\ I = 457.2 \text{ in.}^4, A = 22.67 \text{ in.}^2$$

$$a/t_w = 8.89/0.535 = 16.6 < 22; \text{ OK}$$

$$b/t_f = 10.195/0.868 = 11.75 < 14.0; \text{ OK}$$

$$M_P = 0.5(S + Z)r_{dy} = \frac{91.8(41.6)}{12} = 318 \text{ kip-ft}$$

$$R_m = \frac{2nM_P}{h_c} = \frac{2(3)318}{14.5} = 131 \text{ kips}$$

$$k = \frac{12EI_n}{h^3} = \frac{12(30)10^3(457.2)3}{16^3(144)} = 835 \text{ kips/ft}$$

$$x_e = R_m/k = 131/835 = 0.157 \text{ ft}$$

$$x_m = \alpha\beta x_e = 12x_e = 12(0.157) = 1.88 \text{ ft}$$

$$T_n = 2\pi\sqrt{m/k} = 6.28\sqrt{2.64/835} = 0.353 \text{ sec}$$

h. Third Trial - Work Done vs Energy Absorption Capacity.

$$T/T_n = 0.094/0.353 = 0.27$$

$$C_R = R_m/B = 131/595 = 0.22$$

$$t_m/T = 2.5, t_m = 2.5(0.094) = 0.235 \text{ sec (fig. 5.29)}$$

Revise the load-time curve as above (par. 5-13).

Impulse up to time = 0.25 sec is 1.30 psi-sec (fig. 7.11)

$$T = 2H/B = 2(1.30)/25.3 = 0.103 \text{ sec}$$

$$T/T_n = 0.103/0.353 = 0.292$$

$$t_m/T = 2.5 \text{ (fig. 5.29), } t_m = 2.5(0.103) = 0.257 \text{ sec; OK}$$

$$C_W = 0.88 \text{ (fig. 5.26)}$$

$$W_P = \frac{H^2}{2m} = \left(\frac{BT}{2}\right)^2 / 2m = \frac{(595)^2(0.103)^2}{8(2.64)} = 179 \text{ ft-kips}$$

$$W_m = C_W W_P = 0.88(179) = 157 \text{ ft-kips}$$

$$E = R_m \left( x_m - \frac{x_e}{2} \right) = 131 \left( 1.88 - \frac{0.157}{2} \right) = 236 \text{ ft-kips}$$

This size is accepted for preliminary design. As noted above it is desirable to have E exceed W because of the approximations in the preliminary design procedure.

i. Shear Stress Check of 10 W 77.

$$\text{Estimated } V_{\max} = \frac{R_m}{3} = \frac{(131)}{3} = 44 \text{ kips in one column}$$

$$v = \frac{V}{t_w d} = \frac{44,000}{0.535(10.62)} = 7700 \text{ psi}$$

Allowable  $v = 21,000 \text{ psi}$  (par. 4-05c); OK

j. Slenderness Criterion for Beam Columns. (See par. 4-08.) An approximate evaluation of the column slenderness criterion is made before the column size is accepted for final analysis. The criterion is:

$$\left( \frac{M_D}{M_P} \right) \left( \frac{K' L d}{100 b t} \right) + \left( \frac{P_D}{P_P} \right) \left( \frac{K'' L}{15 r} \right) < 1.0 \text{ (eq 4.10)}$$

$$M_P = 318 \text{ kip-ft}$$

$$M_D = \frac{W_m}{E} M_P = \frac{157}{236} (318) = 212 \text{ kip-ft (a rough estimate)}$$

$$P_P = f_{dy} A = 41.6(22.67) = 945 \text{ kips}$$

$$P_D = \frac{(3.7)(144)(43.5)(18)}{(1000)3} = 139 \text{ kips, at } t_m = 0.257 \text{ sec (fig. 7.12)}$$

$$K' = 0.14 \text{ (table 4.1)}$$

$$K'' = 0.50 \text{ (table 4.2)}$$

$$L = 14.5 \text{ ft (clear height)}$$

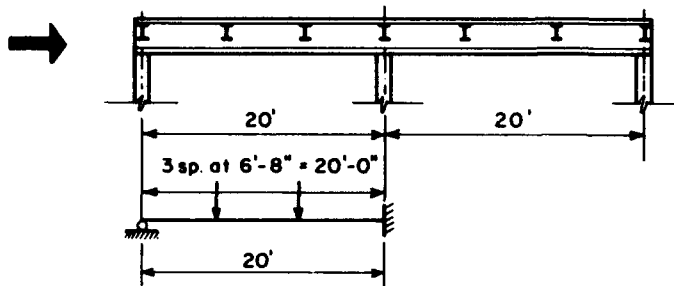
$$r = 2.60 \text{ in., } b = 10.195 \text{ in., } t_f = 0.868 \text{ in., } d = 10.62 \text{ in.}$$

Substituting gives

$$\left( \frac{212}{318} \right) \left[ \frac{0.14(14.5)12(10.62)}{100(10.195)0.868} \right] + \left( \frac{139}{945} \right) \left[ \frac{0.5(14.5)12}{15(2.60)} \right]$$

$$= 0.196 + 0.329 = 0.53 < 1.0; \text{ OK}$$

7-25 DESIGN OF ROOF GIRDER. The girders are rolled structural steel shapes continuous over two spans. Since the procedures of this manual con-



sider only single-span elements, the girders are designed for vertical loads as beams fixed at the interior support and pinned at the exterior support. The vertical loads are of two kinds; uniformly distributed load due

to the girder dead weight and two concentrated loads, static plus blast applied by the purlins at the girder third-points.

The design moment at the interior support is a combination of three superposed bending moments as follows:

- (1) The moment caused by the static loads;
- (2) The moment caused by the vertical blast loads; and
- (3) The moment imposed by the central column in restraining lateral motion of the frame.

The girder is designed for elastic behavior so that at all times it will be capable of providing the full restraint equal to the column plastic moment at each column support. For this two-span girder the column moment in the girder is equal to one-half the plastic hinge moment of the column (par. 7-11).

The basic design procedure is essentially the same as the elastic design procedure illustrated in paragraph 6-12. Although this is an elastic design, the limiting moment is the plastic resisting moment of the section (par. 4-04b).

The preliminary design load is based on an idealized version of the purlin dynamic reactions. After obtaining a satisfactory preliminary design the actual average purlin dynamic reaction is used in a numerical integration analysis to verify the preliminary design.

a. Loading. The critical girder loading results from the blast wave moving parallel to the girder axis. For this condition the average roof loads vary along the axis perpendicular to the girder from a maximum at the ends to a minimum over the central portion of the roof (par. 3-08d). To

obtain the vertical blast load on the girder it is necessary to determine:

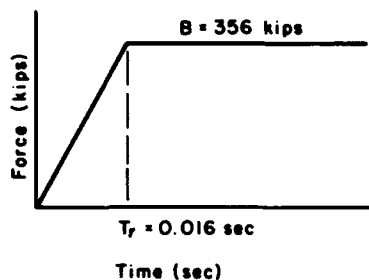
- (1) The variation of Zone 3 local overpressure;
- (2) The slab dynamic reactions for these overpressures; and
- (3) The purlin dynamic reactions when loaded by the slab dynamic reactions.

All this has been done in paragraph 7-23 in designing the purlins. The girder dynamic load is determined in figure 7.19 by adding the dynamic reactions of purlins (A) and (B) plotted with the proper lag time

$$t_{\text{lag}} = \frac{\text{purlin spacing}}{\text{velocity of blast wave}} = \frac{6.67}{1403} = 0.0048 \text{ sec}$$

The dynamic reactions are obtained from tables 7.6 and 7.7 (par. 7-23j).

The design load as idealized from the computed loading shown by figure 7.19



is defined by:

$$B = 4(89) = 356 \text{ kips}$$

$$T_r = 0.016 \text{ sec}$$

b. Elastic Range Dynamic Design Factors.

(Refer to table 6.1.)

Concentrated mass:

$$K_L = 0.81,$$

$$K_M = 0.67,$$

$$K_{IM} = 0.83$$

$$R_m = 6M_P/L,$$

$$k = \frac{132EI}{L^3}$$

$$V_1 = 0.17R + 0.17P,$$

$$V_2 = 0.33R + 0.33P$$

Uniform mass:

$$K_L = 0.81,$$

$$K_M = 0.45,$$

$$K_{IM} = 0.83$$

c. Mass Computation.

$$\text{Slab and roofing} \left[ \frac{3.75(150)}{12} + 6.0 \right] \frac{18(43.5)}{(1000)^2} = 20.7 \text{ kips}$$

$$\text{Purlins} \frac{42.4(18)2}{1000} = 1.5 \text{ kips}$$

$$\text{Girder (estimate)} \frac{160(20)}{1000} = 3.2 \text{ kips}$$

$$\text{Connections (allow 10\%)} = 0.1(3.2 + 1.5) = 0.47 \text{ kips}$$

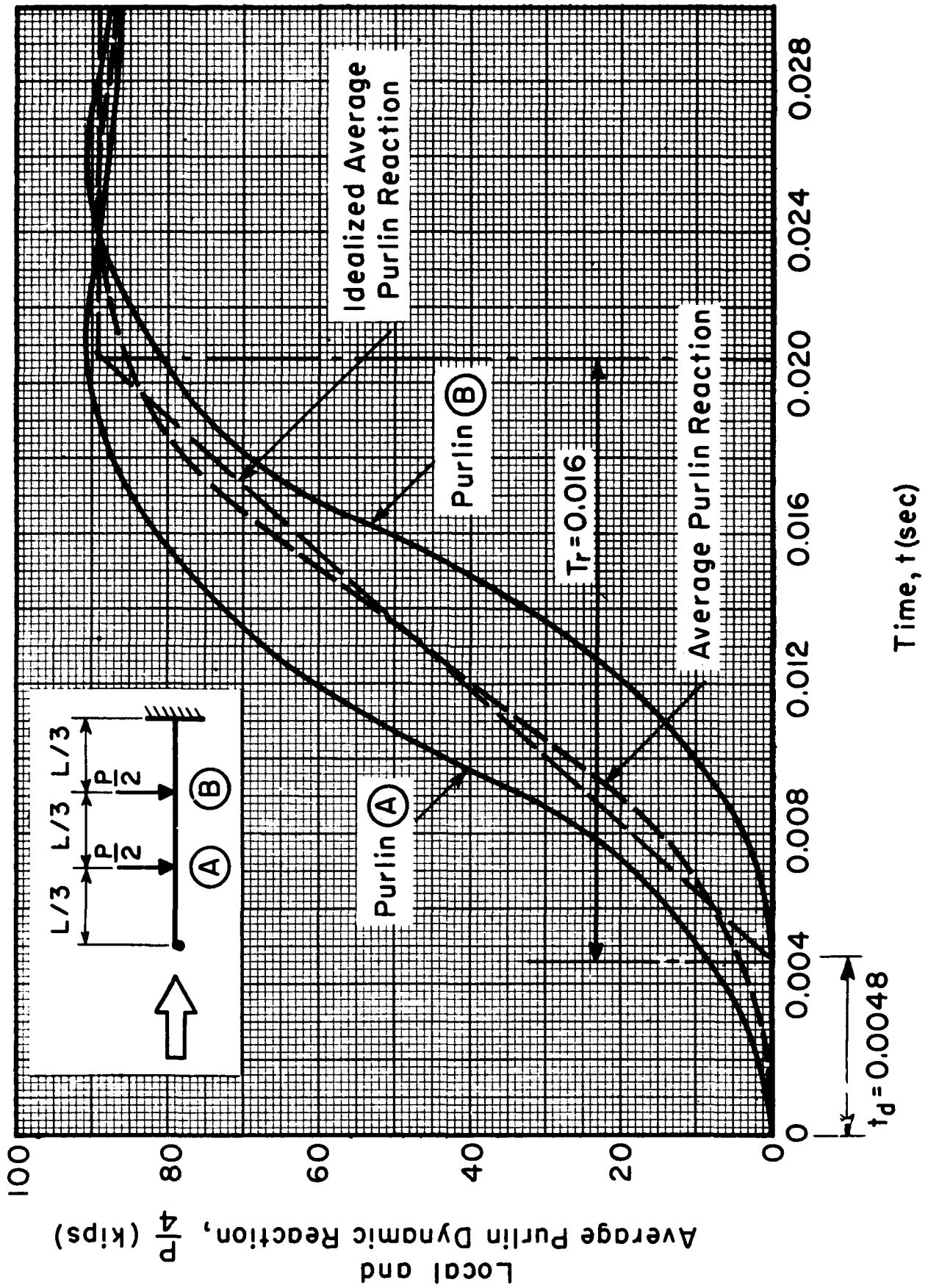


Figure 7.19. Determination of average girder load vs time curve

$$\text{Total concentrated mass } m_c = \frac{(2/3)(20.7) + (1.5)}{32.2} = \frac{15.3}{32.2} \\ = 0.475 \text{ kip-sec}^2/\text{ft}$$

$$\text{Total uniform mass } m_u = \frac{3.2 + 0.47}{32.2} = 0.115 \text{ kip-sec}^2/\text{ft}$$

d. First Trial - Actual Properties.

Assume D.L.F. = 1.4 (experience)

$$R_m = \text{D.L.F.}(B) = 1.4(356) = 500 \text{ kips (par. 6-11)}$$

$$M_P = 1.05Sf_{dy} = \frac{1.05(41.6)S}{12} = 3.64S \text{ kip-ft (S in in.}^3 \text{) (eq 4.2a)}$$

$$R_m = \frac{6M_P}{L} = \frac{6(3.64S)}{20} = 500, \therefore S = 458 \text{ in.}^3$$

Try 36 W 150,

$$S = 503 \text{ in.}^3, Z = 581 \text{ in.}^3, 0.5(S + Z) = 542 \text{ in.}^3, I = 9012 \text{ in.}^4$$

$$M_P = f_{dy} 0.5(S + Z) = \frac{41.6(542)}{12} = 1880 \text{ kip-ft (eq 4.2)}$$

$$k = \frac{132EI}{L^3} = \frac{132(30)10^3(9012)}{20^3(144)} = 30,800 \text{ kips/ft}$$

e. First Trial - Equivalent Properties.

$$\text{Concentrated } m_e = K_M(m_c) = 0.67(0.475) = 0.318 \text{ kip-sec}^2/\text{ft}$$

$$\text{Uniform } m_e = K_M(m_u) = 0.45(0.115) = 0.052 \text{ kip-sec}^2/\text{ft}$$

$$\text{Total } m_e = 0.318 + 0.052 = 0.37 \text{ kip-sec}^2/\text{ft}$$

$$k_e = K_L k = 0.81(30,800) = 25,000 \text{ kips/ft}$$

$$T_n = 2\pi\sqrt{m_e/k_e} = 6.28\sqrt{0.37/25,000} = 0.0241 \text{ sec}$$

$$T_r/T_n = 0.016/0.0241 = 0.665$$

$$\text{D.L.F.} = 1.45 \text{ (fig. 5.21)}$$

$$\text{Required } R_m = \text{D.L.F.}(B) = 1.45(356) = 516 \text{ kips}$$

$$\text{Required } M_P = R_m L/6 = 516(20)/6 = 1720 \text{ kip-ft}$$

$$0.5M_P \text{ for column} = 0.5(318) = 159 \text{ kip-ft (pars. 7-12 and 7-21g)}$$

Moment in girder at interior support due to static loads

$$M_s = 3.7(20)/8 + 15.3(0.33)(0.67)(6.67 + 20)0.25 + \\ 15.3(0.33)(0.67)(13.3 + 20)0.25 = 9.25 + 22.7 + 28.3 = 60 \text{ kip-ft}$$

Girder resistance available for vertical blast loads

$$M = 1880 - 60 - 159 = 1660 \text{ kip-ft} < 1720 \text{ kip-ft}$$

Consider larger sizes

f. Second Trial - Actual Properties. Try 36 W 160

$$S = 541 \text{ in.}^3, \quad Z = 622 \text{ in.}^3, \quad 0.5(S + Z) = 581 \text{ in.}^3, \quad I = 9739 \text{ in.}^4$$

$$M_P = f_{dy} 0.5(S + Z) = 41.6(581) = 2020 \text{ kip-ft}$$

$$k = \frac{132EI}{L^3} = \frac{132(30)10^3(9739)}{20^3(144)} = 33,300 \text{ kips/ft}$$

g. Second Trial - Equivalent Properties. The change in the mass is neglected.

$$k_e = K_L k = 0.81(33,300) = 27,000 \text{ kips/ft}$$

$$T_n = 2\pi\sqrt{m_e/k_e} = 6.28\sqrt{0.37/27,000} = 0.0232 \text{ sec}$$

$$T_r/T_n = 0.016/0.0232 = 0.69$$

$$D.L.F. = 1.38 \text{ (fig. 5.21)}$$

$$\text{Required } R_m = D.L.F.(B) = 1.38(356) = 491 \text{ kips}$$

$$\text{Required } M = R_m L/6 = 491(20)/6 = 1640 \text{ kip-ft}$$

Girder resistance available for vertical blast loads

$$M_P - M_S - 0.5(M_P)_{\text{column}} = 2020 - 60 - 159 = 1800 \text{ kip-ft; OK}$$

$$t_m/T_r = 1.19 \text{ (fig. 5.21)}$$

$$t_m = 0.016(1.19) = 0.019 \text{ sec; OK}$$

h. Preliminary Design for Shear Stress.

$$\text{At } t_m = 0.019 \text{ sec, } t = 0.019 + 0.0048 = 0.0238 \text{ sec}$$

$$P = 4(89) = 356 \text{ kips (fig. 7.19)}$$

$$V = 0.33R_m + 0.33P = 0.33(491 + 356) = 280 \text{ kips}$$

$$v = \frac{V}{t_w d} = \frac{280,000}{553(36.0)} = 11,700 \text{ psi}$$

$$\text{Allowable} = 21,000 \text{ psi (par. 4-05c)}$$

i. Determination of Maximum Deflection and Dynamic Reactions by Numerical Integration.

$$R_m = \frac{6M_P}{L} = \frac{6(1800)}{20} = 540 \text{ kips}$$

$$k = 33,300 \text{ kips/ft}$$

$$y_e = \frac{R_m}{k} = \frac{540}{33,300} = 0.0162 \text{ ft}$$

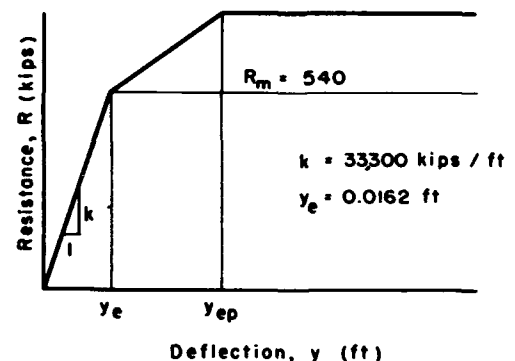


Figure 7.20. Resistance function for 36 W 160 girder spanning 20 ft, fixed at one end and pinned at the other

Table 7.9. Determination of Maximum Deflection and Dynamic Reactions for Roof Girder

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> - R <sub>n</sub> (kips)	$\ddot{y}_n(\Delta t)^2$ (ft)	y <sub>n</sub> (ft)	V <sub>2n</sub> (kips)
0	0	0	0.65	0.000006	0	0
0.002	4.0	0.2	3.8	0.000033	0.000006	1.4
0.004	11.2	1.5	9.7	0.000085	0.000045	4.2
0.006	30.0	5.6	24.4	0.000214	0.000169	11.7
0.008	56.0	17.0	39.0	0.000341	0.000507	24.0
0.010	110.0	39.0	71.0	0.000621	0.001186	49.0
0.012	160.0	83.0	77.0	0.000674	0.002486	80.0
0.014	208.0	149.0	59.0	0.000516	0.004460	118.0
0.016	266.0	231.0	35.0	0.000306	0.006950	164.0
0.018	312.0	325.0	-13.0	-0.000114	0.009746	210.0
0.020	337.0	414.0	-77.0	-0.000674	0.012428	248.0
0.022	351.0	481.0	-130.0	-0.001137	0.014436	275.0
0.024	356.0	510.0	-154.0	-0.001347	0.015307	286.0
0.026	356.0	494.0			0.014831	281.0

The basic equation for the numerical integration in table 7.9 is

$$y_{n+1} = \ddot{y}_n(\Delta t)^2 + 2y_n - y_{n-1} \text{ (table 5.3)}$$

where

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(\Delta t)^2}{K_{LM}(m)} = \frac{(P_n - R_n)(0.002)^2}{(0.55)(0.115) + (0.83)(0.475)}$$

$$= 8.75 \times 10^{-6} (P_n - R_n) \text{ ft}$$

The time interval  $\Delta t = 0.002$  sec is approximately equal to  $T_n/10 = 0.00232$  sec (par. 5-08). The dynamic reaction equations are listed in paragraph 7-25b.

The  $P_n$  values for the second column in table 7.9 are obtained from figure 7.19 multiplying by 4 to obtain the total concentrated load applied to the girder by the two purlins.

j. Shear Stress Check.

$$V = 286 \text{ kips (table 7.9)}$$

$$v = \frac{V}{dt_w} = \frac{286,000}{36.0(0.653)} = 12,150 \text{ psi}$$

$$\text{Allowable } v = 21,000 \text{ psi; OK (par. 4-05c)}$$



k. Check Proportions of 36 WF160 for Local Buckling.

$$b/t_f = 12.0/1.02 = 11.8 < 14; \text{ OK}$$

$$a/t_w = 33.96/0.653 = 52 > 30; \text{ NG, longitudinal stiffeners required}$$

(see par. 4-06d)

$$t_s = 3/4 > 0.653; \text{ OK}$$

$$b_s/t_s = 6; b_s = 6t_s = 6(3/4) = 4.5 \text{ in.}$$

Use longitudinal stiffeners, 2 plates

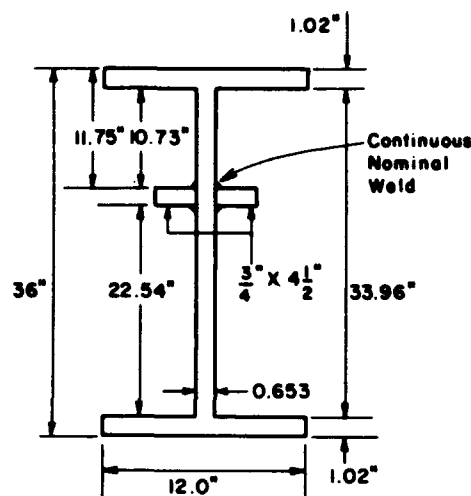
3/4 in. by 4-1/2 in. by full length.

$$c/t_w = 10.73/0.653 = 16.5 < 22; \text{ OK}$$

$$e/t_w = 22.54/0.653 = 34.5 < 40; \text{ OK}$$

Weight of added plate stiffeners

equals 23 lb/ft.



7-26 FINAL DESIGN OF COLUMN. The column design was begun in paragraph 7-24. The final steps in the column design are illustrated by this paragraph. The steps which follow are preliminary to the numerical analysis to determine the lateral deflection of the top of the columns. In the preliminary design (par. 7-24) some of the factors which affect the response are neglected to simplify the computations. In this paragraph these factors are considered: the variation of plastic hinge moment with direct stress, the variation of column resistance with lateral deflection, the effect of girder flexibility on the stiffness of the columns, and the difference between the load on the wall slab and the dynamic reactions from the wall which are used as the lateral design load for the frame columns.

a. Mass Computation. (Refer to par. 7-24b.)

Walls = 86.5 kips, roof slab = 41.4 kips, purlins = 5.3 kips

$$\text{Girder} = \frac{160(41)}{1000} = 6.5 \text{ kips}$$

$$\text{Columns} = \frac{77(3)14.5}{1000} = 3.35 \text{ kips}$$

Connections = 1.2 kips

Mass of single-degree-of-freedom system = total roof + 1/3 (columns and walls)

$$m = \frac{41.4 + 5.3 + 6.5 + 0.33(86.5 + 3.35)}{32.2} = \frac{83.2}{32.2} = 2.6 \text{ kip-sec}^2/\text{ft}$$

b. Column Properties.

10 W 77

$$I = 457.2 \text{ in.}^4, S = 86.1 \text{ in.}^3, Z = 97.6 \text{ in.}^3, 0.5(S + Z) = 91.8 \text{ in.}^3$$

$$A = 22.67 \text{ in.}^2, b = 10.195 \text{ in.}, t_f = 0.868 \text{ in.}$$

$$a = 8.89 \text{ in.}, t_w = 0.535 \text{ in.}, d = 10.62 \text{ in.}$$

c. Column Interaction Design Data. The plastic hinge moment, plastic axial load, and the values of  $M_1$  and  $P_1$  are computed below from the column properties.

$$M_P = f_{dy}(S + Z)0.5 = \frac{41.6}{12} (91.8) = 318 \text{ kip-ft (eq 4.2)}$$

$$P_P = f_{dy}A = 41.6(22.67) = 945 \text{ kips (eq 4.7)}$$

$$P_1 = \frac{f_{dy}}{d} \left[ 2bt_f^2 + \frac{t_w d^2}{2} - 2t_w t_f^2 \right] \text{ (eq 4.12)}$$

$$P_1 = \frac{41.6}{10.62} \left[ 2(10.195)(0.868)^2 + \frac{0.535(10.62)^2}{2} - 2(0.535)(0.868)^2 \right]$$

$$P_1 = 3.91 [15.4 + 30.2 - 0.80] = 175 \text{ kips}$$

$$M_1 = \frac{f_{dy}}{3d} \left[ 4t_w \left( \frac{d}{2} - t_f \right)^3 + bt_f (3d^2 - 6dt_f + 4t_f^2) \right] \text{ (eq 4.11)}$$

$$= \frac{41.6}{(12)3(10.62)} \left\{ 4(0.535) \left( \frac{10.62}{2} - 0.868 \right)^3 + 10.195(0.868) [3(10.62)^2 - 6(10.62)(0.868) + 4(0.868)^2] \right\}$$

$$M_1 = \frac{1.30}{12} \{ 187 + 8.85 [339 - 55.5 + 3.0] \} = \frac{1.3}{12} (2727) = 296 \text{ kip-ft}$$

For  $P_D > P_1$

$$M_D = \left( \frac{P_P - P_D}{P_P - P_1} \right) M_1 = \frac{945 - P_D}{945 - 175} (296) = 364 - 0.384P_D \text{ (eq 4.13)}$$

For  $P_D < P_1$

$$M_D = M_P - \frac{P_D}{P_1} (M_P - M_1) \text{ (eq 4.14)}$$

$$= 318 - \frac{(318 - 296)P_D}{175} = 318 - 0.125 P_D$$

d. Effect of Girder Flexibility. The relative flexibility of the girders reduces the spring constant  $k$  in the elastic range from the value

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obtained in paragraph 7-24 for infinitely stiff girders (par. 7-08). To obtain this revised value of  $k$  a simple sidesway analysis of the frame is made. From the sidesway analysis the magnitude of the lateral force required to cause a unit displacement is determined.

In figure 7.22 the initial slope, which is the proper spring constant until formation of the first hinge, is assumed to hold up to the plastic resistance  $R_m$ . The true resistance curve is represented approximately by the dashed line up to  $R_m$ .

For infinitely rigid girders  $k = 835$  kips/ft (par. 7-24g).

The elastic sidesway analysis in figure 7.21 is started with -1000 kip-ft at top and bottom of each column. This is equivalent to a lateral displacement of the frame,

$$\begin{aligned} X &= (F.E.M.)h^2/6EI \\ &= \frac{(1000)(16)^2 144}{6(30)10^3(457.2)} \\ &= 0.448 \text{ ft} \end{aligned}$$

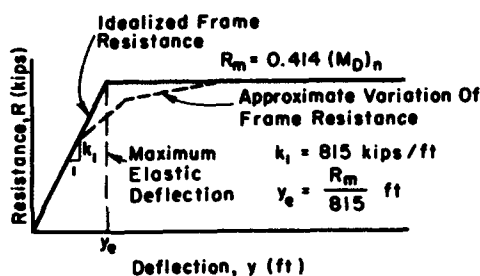


Figure 7.22. Resistance diagram for 10 W 77 columns

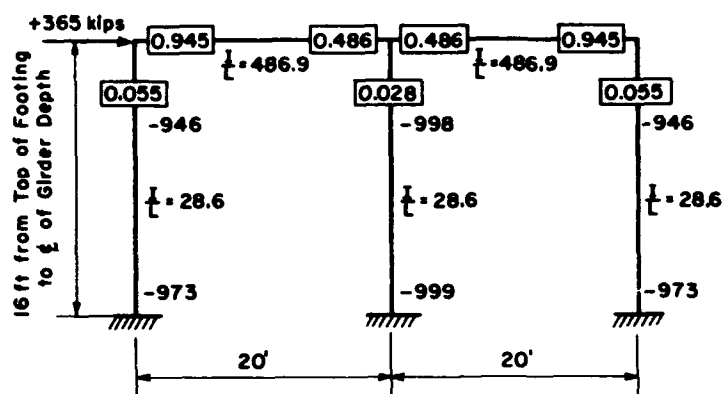


Figure 7.21. Sidesway frame analysis by moment distribution

In the elastic sidesway analysis the conventional procedure of using centerline dimensions is adopted although in all other computations the clear height of the column is used.

$$\begin{aligned} R &= \frac{\Sigma M}{h} = \frac{2(946 + 973) + (999 + 998)}{16} \\ &= 365 \text{ kips} \end{aligned}$$

$$k = \frac{R}{x} = \frac{365}{0.448} = 815 \text{ kips/ft}$$

e. Loading. Both horizontal and vertical loads are considered. The lateral load for the numerical integration  $F_n$  (fig. 7.23) is obtained from the  $V_{lh}$  dynamic reaction column of the numerical integration analysis of the front wall slabs in paragraph 7-21g (table 7.2). The dynamic reaction values for a one-foot width are multiplied by the width of one bay. After

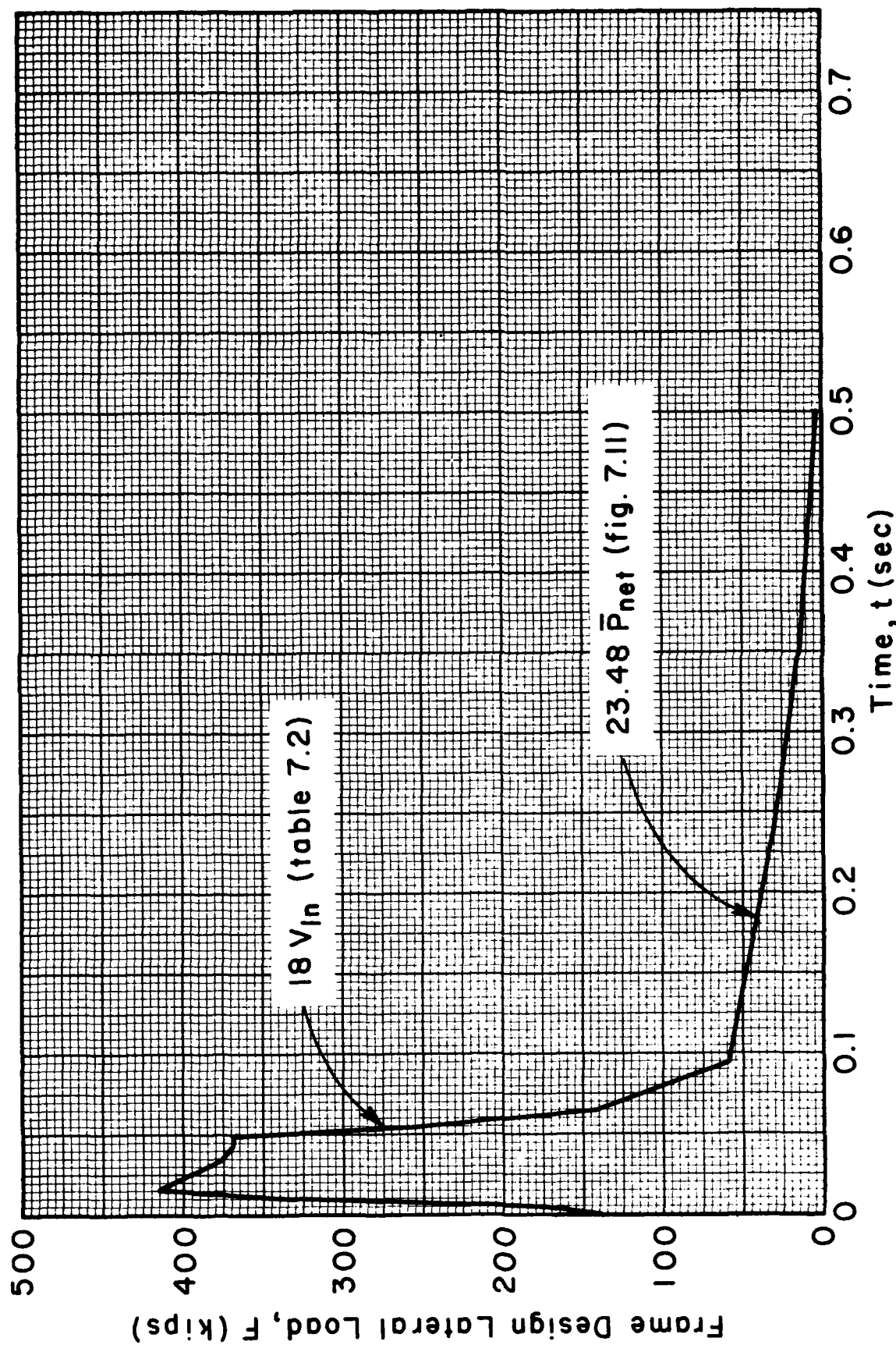


Figure 7.23. Frame design lateral load

the dynamic reaction of the front wall slab has decreased to the level of the applied load (at  $t = 0.055$  sec) the  $F_n$  values are determined from the net lateral load curve (fig. 7.11) where  $F_n = [144(18)9.06]/1000 \bar{P}_{net} = 23.48 \bar{P}_{net}$  kips ( $\bar{P}_{net}$  is in psi). These data are plotted in figure 7.23 to give the frame lateral design load for use in table 7.10.

The effective height of the loaded area is obtained from one-half the wall clear height plus the thickness of the roof slab,

$$h = \frac{17.5}{2} + \frac{3.75}{12} = 9.06 \text{ ft}$$

f. Computation of Deflection of Frame by Numerical Integration.

(Refer to table 7.10.) The total vertical load  $P_n$  in the second column is obtained by multiplying the average roof overpressure (fig. 7.12) by  $[43.5(18)144]/1000 = 112.8$  and adding the dead weight of the roof system. The  $(P_D)_n$  values are the average axial column loads and are obtained by dividing the total vertical load by the number of columns. The  $(P_D)_n$  column is used in the plastic range to obtain the value of  $(M_D)_n$  from the interaction design equations (par. 7-26c). The value of  $(M_D)_n$  is used to obtain the maximum resistance at any time by the relation  $(R_m)_n = [2n(M_D)_n]/h_c = [2(3)(M_D)_n]/14.5 = 0.414(M_D)_n$ .

$R_n$  is equal to  $kx_n = 815x_n$  in the elastic range. In the plastic range  $R_m$  is the limiting value of  $R$ . The expression  $(P_n x_n)/h_c$  indicates the decrease in resistance corresponding to the increase in moment resulting from the eccentric loading. The basic equation for the numerical integration in table 7.10 is

$$x_{n+1} = \ddot{x}_n (\Delta t)^2 + 2x_n - x_{n-1} \text{ (table 5.3)}$$

where

$$\ddot{x}_n (\Delta t)^2 = \frac{\left[ F_n - R_n + \frac{P_n}{h_c} (x_n) \right] (\Delta t)^2}{m}$$

$$\ddot{x}_n (\Delta t)^2 = \frac{(0.02)^2 \left[ F_n - R_n + \frac{P_n}{h_c} (x_n) \right]}{2.6} = 1.54 \times 10^{-4} \left[ F_n - R_n + \frac{P_n}{h_c} (x_n) \right]$$

To check the slenderness criteria (par. 4-08) the following equation is evaluated for each time interval.

Table 7.10. Determination of Column Adequacy

t (sec)	P <sub>n</sub> (kips)	(P <sub>D</sub> ) <sub>n</sub> (kips)	(M <sub>D</sub> ) <sub>n</sub> (kip-ft)	(R <sub>n</sub> ) <sub>n</sub> (kips)	$\frac{P_n}{h_c}$ (kip-ft)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	$\frac{P_n x_n}{h_c}$ (kips)	$F_n - R_n + \frac{P_n x_n}{h_c}$ (kips)	$x_n (\Delta t)^2$ (ft)	x <sub>n</sub> (ft)	$0.292 \frac{(M_D)_n}{M_P}$	$2.231 \frac{(P_D)_n}{P_P}$	Slenderness Criterion
0	55	18	316	131	3.8	139.0	0	0	69.5	0.0106	0	0.29	0.04	0.33
0.02	698	233	275	114	48.1	405.0	8.6	0.5	396.9	0.0607	0.0106	0.25	0.55	0.80
0.04	998	333	236	98	68.8	371.0	66.7	5.6	309.9	0.0474	0.0819	0.217	0.786	1.003
0.06	913	304	247	102	63.0	207.0	102.0	12.0	117.0	0.0179	0.2006	0.23	0.72	0.95
0.08	836	279	257	106	57.7	150.0	106.0	19.0	13.0	0.0020	0.3372	0.24	0.66	0.90
0.10	769	256	266	110	53.0	58.0	110.0	25.0	-27.0	-0.0041	0.4758	0.24	0.60	0.84
0.12	711	237	273	113	49.0	54.0	113.0	30.0	-29.0	-0.0044	0.6103	0.25	0.56	0.81
0.14	663	221	279	116	45.7	50.0	116.0	34.0	-32.0	-0.0049	0.7404	0.26	0.52	0.78
0.16	621	207	285	118	42.8	46.0	118.0	37.0	-35.0	-0.0054	0.8656	0.26	0.49	0.75
0.18	584	195	289	120	40.3	42.0	120.0	40.0	-38.0	-0.0056	0.9854	0.27	0.46	0.73
0.20	551	184	293	121	38.0	38.0	121.0	42.0	-41.0	-0.0063	1.0994	0.27	0.43	0.70
0.22	523	174	296	123	36.1	34.0	123.0	44.0	-45.0	-0.0069	1.2071	0.27	0.41	0.68
0.24	496	165	297	123	34.2	30.0	123.0	45.0	-48.0	-0.0073	1.3079	0.27	0.39	0.66
0.26	471	157	298	123	32.5	27.0	123.0	46.0	-50.0	-0.0076	1.4014	0.27	0.37	0.64
0.28	447	149	299	124	30.8	24.0	124.0	46.0	-54.0	-0.0083	1.4873	0.27	0.35	0.62
0.30	424	141	300	124	29.2	22.0	124.0	46.0	-56.0	-0.0086	1.5649	0.28	0.33	0.61
0.32	401	134	301	125	27.7	19.0	125.0	45.0	-61.0	-0.0093	1.6333	0.28	0.32	0.60
0.34	380	127	302	125	26.2	16.0	125.0	44.0	-65.0	-0.0099	1.6936	0.28	0.30	0.58
0.36	359	120	303	125	24.8	14.0	125.0	43.0	-68.0	-0.0104	1.7434	0.28	0.28	0.56
0.38	337	112	304	126	23.2	12.0	126.0	41.0	-73.0	-0.0112	1.7828	0.28	0.26	0.54
0.40	318	106	305	126	21.9	11.0	126.0	40.0	-75.0	-0.0115	1.8110	0.28	0.25	0.53
0.42	297	99	306	127	20.5	9.5	127.0	37.0	-80.0	-0.0122	1.8277	0.28	0.23	0.51
0.44	278	92	307	127	19.2	8.0	127.0	35.0	-84.0	-0.0128	1.8323	0.28	0.22	0.50
0.46											1.8241			

$$\frac{M_D}{M_P} \left[ \frac{K' L_d}{100 b t_f} \right] + \frac{P_D}{P_P} \left[ \frac{K'' L}{15 r} \right] \leq 1 \quad (\text{eq 4.10})$$

$$K' = 0.14, \quad K'' = 0.50, \quad L = h_c = 14.5 \text{ ft} = 174 \text{ in. (tables 4.1 and 4.2)}$$

$$\frac{M_D}{M_P} \left[ \frac{0.14(174)10.62}{100(10.195)(0.868)} \right] + \frac{P_D}{P_u} \left[ \frac{0.5(174)}{15(2.60)} \right] \leq 1$$

$$0.292 \frac{M_D}{M_P} + 2.231 \frac{P_D}{P_P} \leq 1$$

The time interval  $\Delta t$  used in table 7.10 is based on the natural period

$$T_n = 2\pi \sqrt{m/k} = 6.28 \sqrt{2.6/815} = 0.353.$$

$$t = 0.02 < \frac{T_n}{10} = 0.035 \quad (\text{par. 5-08})$$

The allowable maximum displacement =  $\frac{\alpha \beta R_m}{k} = \frac{12(102)}{815} = 1.5 \text{ ft}$  (pars. 6-26 and 7-24c). The computations in table 7.10 show that the slenderness criterion is satisfied because the combined ratio is only very slightly above 1.0 (0.3%). The computed maximum displacement = 1.56 ft, thus

$$\alpha \beta = \frac{1.56}{1.5} (12) = 12.5; \text{ OK.}$$

7-27 FOUNDATION DESIGN. a. General. The foundation of this building is designed by both of the methods described in paragraph 6-31 primarily to illustrate the procedures.

The first design is according to the Quasi-Static Foundation Design Procedure in paragraph 6-31c, and the second is according to the Dynamic Foundation Design Procedure in paragraph 6-31d. A preliminary plan of one bay of the foundation is presented by figure 7-24.

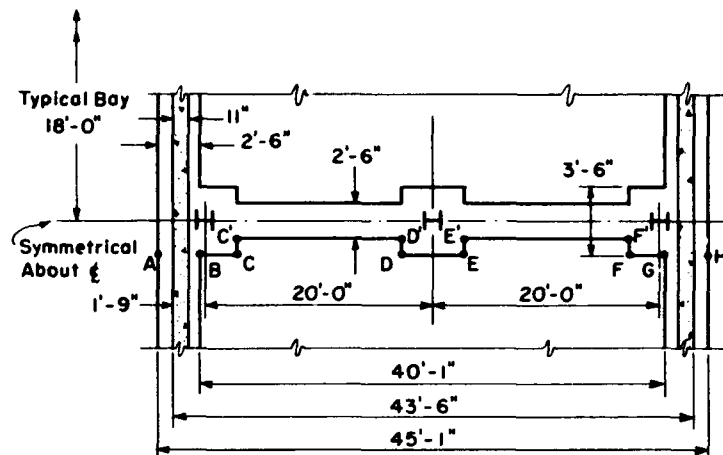
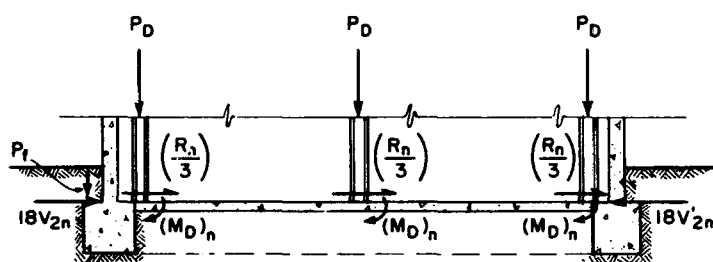


Figure 7.24. Preliminary foundation plan for one bay

b. Loads. The first requirement of the quasi-static analysis is a tabulation of all the lateral and vertical loads on the foundation. Because each numerical integration



performed previously is based on a separate value of  $\Delta t$ , it is necessary to plot curves of the variation of each of the forces with time. The locations of the forces on the

foundation are shown by the sketch on the left. The vertical loads include the column and footing blast loads and the dead load of the entire structure.

Figure 7.25 is a curve showing the time variation of the total column (blast plus static) load  $P_n = 3(P_D)_n$  obtained from data in table 7.10. Figure 7.26 is a curve of the time variation of the blast load on the projecting front footing. The front wall footing is estimated at 2 ft 6 in. for the preliminary analysis so that the projecting area is  $(2.5 - \frac{11}{12})0.5 = 0.79$  ft. The vertical blast load on the footing is obtained by multiplying the projecting area by the front face overpressure (fig. 7.9) thus obtaining

$$0.79(18)\bar{P}_{\text{front}} (144/1000) = 2.05\bar{P}_{\text{front}}$$

Figure 7.27 is the total vertical load curve obtained by adding the ordinates of figures 7.25 and 7.26. The blast load on the rear footing is neglected herein.

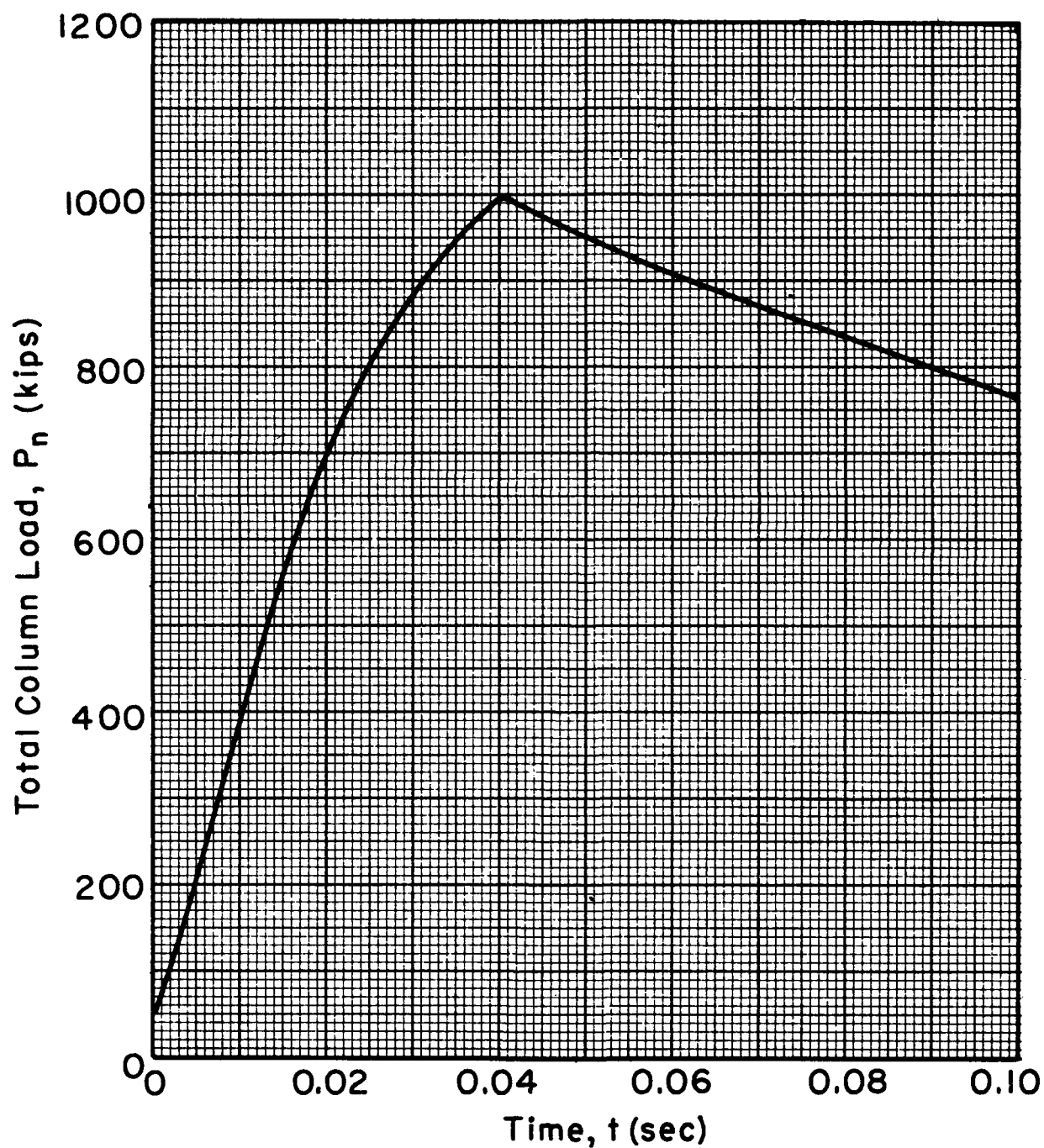


Figure 7.25. Total column load vs time



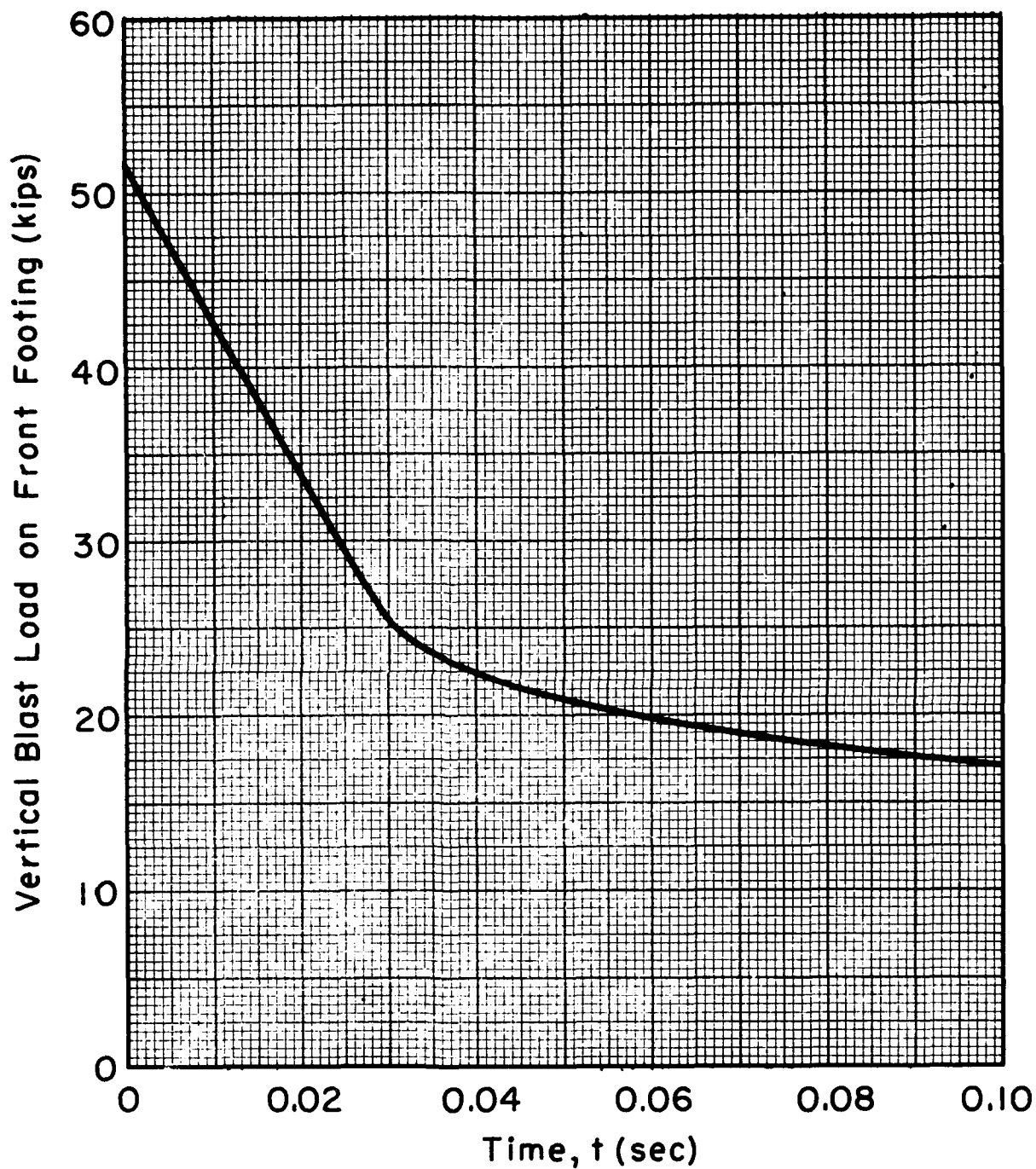


Figure 7.26. Vertical load on front wall footing projection vs time

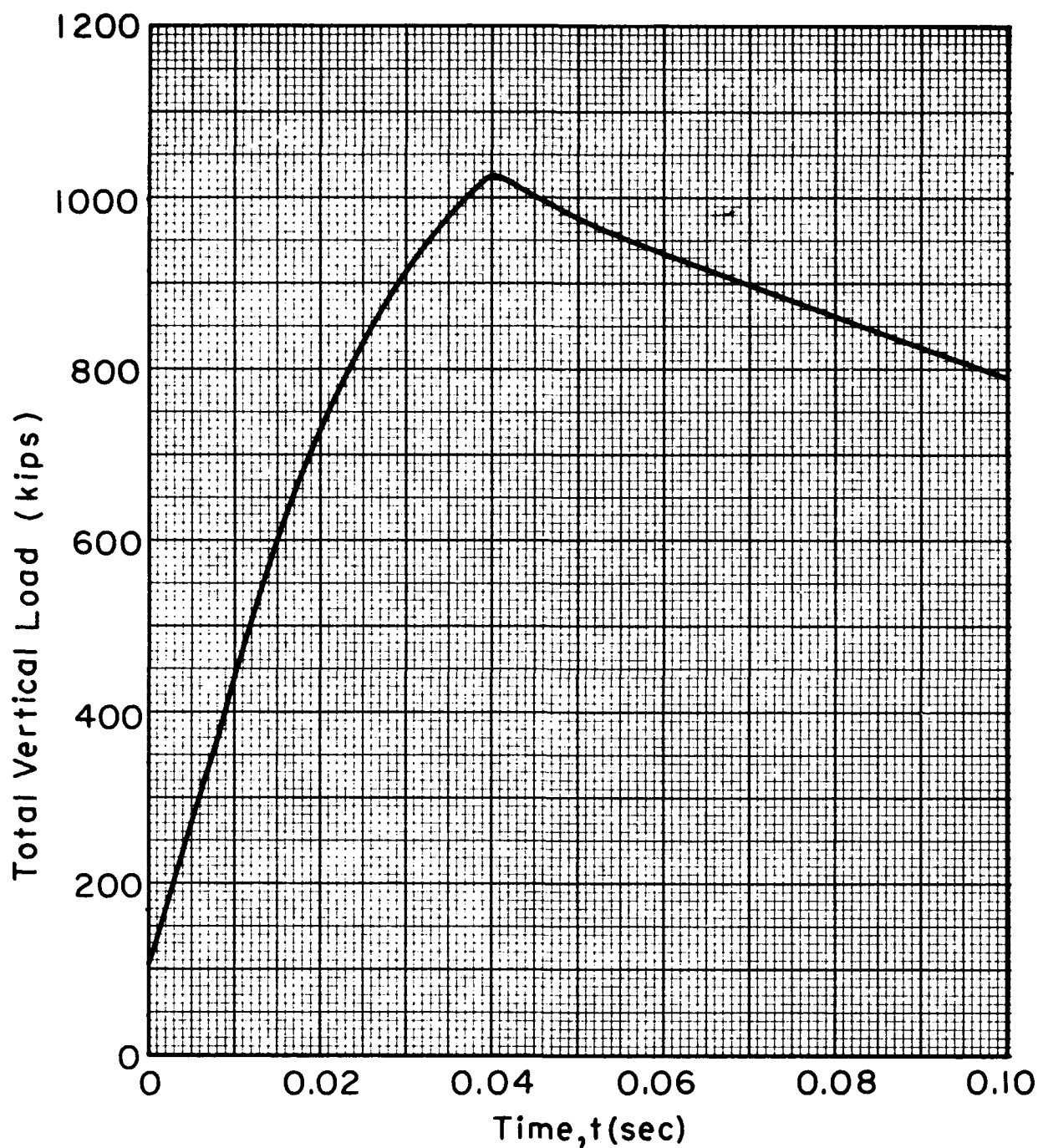


Figure 7.27. Total vertical load vs time

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The lateral loads include the shear in the columns and the front and rear wall slab reactions at the wall footings. The rear wall slab reactions are obtained in table 7.11 by a numerical integration using the rear wall overpressure data in figure 7.10 and the following data from paragraph 7-21g.

$$P_n = 2.52 \bar{P}_{\text{back}}$$

$$y_e = 0.0344 \text{ ft}$$

$$y_{ep} = 0.090 \text{ ft}$$

$$k_1 = 793 \text{ kips/ft}$$

$$k_{ep} = 329 \text{ kips/ft}$$

$$R_m = 45.6 \text{ kips}$$

$$\Delta t = 0.005 \text{ sec}$$

$$\ddot{y}_n(\Delta t)^2 = 4.29(10^{-4})(P_n - R_n) \text{ ft, elastic range}$$

$$\ddot{y}_n(\Delta t)^2 = 4.29(10^{-4})(P_n - R_n) \text{ ft, elasto-plastic range}$$

$$\ddot{y}_n(\Delta t)^2 = 5.07(10^{-4})(P_n - R_n) \text{ ft, elasto-plastic range}$$

$$\ddot{y}_n(\Delta t)^2 = 5.07(10^{-4})(P_n - R_n) \text{ ft, plastic range}$$

Table 7.11. Determination of Dynamic Reactions for Back Wall Slab

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> - R <sub>n</sub> (kips)	$\ddot{y}(\Delta t)^2$	y <sub>n</sub> (ft)	V <sub>1n</sub> (kips)	V <sub>2n</sub> (kips)
0.030	0.216	0	0.216	0.000093	0	0.0	0.0
0.035	1.210	0.073	1.137	0.000488	0.000093	0.2	0.3
0.040	2.520	0.534	1.986	0.000852	0.000674	0.4	0.7
0.045	3.654	1.671	1.983	0.000851	0.002107	0.9	1.4
0.050	4.788	3.482	1.306	0.000560	0.004391	1.5	2.4
0.055	6.048	5.737	0.311	0.000133	0.007235	2.2	3.6
0.060	7.207	8.098	-0.891	-0.000382	0.010212	3.0	4.9
0.065	8.392	10.156	-1.764	-0.000757	0.012807	3.6	6.0
0.070	9.576	11.613	-2.037	-0.000874	0.014645	4.2	6.8
0.075	10.786	12.378	-1.592	-0.000683	0.015609	4.5	7.4
0.080	12.096	12.601	-0.505	-0.000217	0.015890	4.7	7.7
0.085	13.306	12.652	+0.654	+0.000281	0.015954	4.9	8.0
0.090	14.440	12.925	+1.515	+0.000650	0.016299	5.1	8.3
0.095	15.624	13.714	+1.910	+0.000819	0.017294	5.4	8.9
0.100	15.498	15.153	+0.345	+0.000148	0.019108	5.8	9.5

Figure 7.28 is a plot of  $18(V_{2n} - V'_{2n})$  where  $V_{2n}$  and  $V'_{2n}$  are, respectively, the front and rear wall slab footing reactions from tables 7.2 and 7.11. After  $t = 0.07$  sec, the curve is obtained from the net lateral load curve (fig. 7.11):

$$\frac{144(18)17.5}{1000(2)} \bar{P}_{net} = 22.6 \bar{P}_{net} \text{ kips } (\bar{P}_{net} \text{ is in psi})$$

where 17.5 is the wall clear height.

The time variation of the column shear is plotted in figure 7.29 using data from table 7.10.

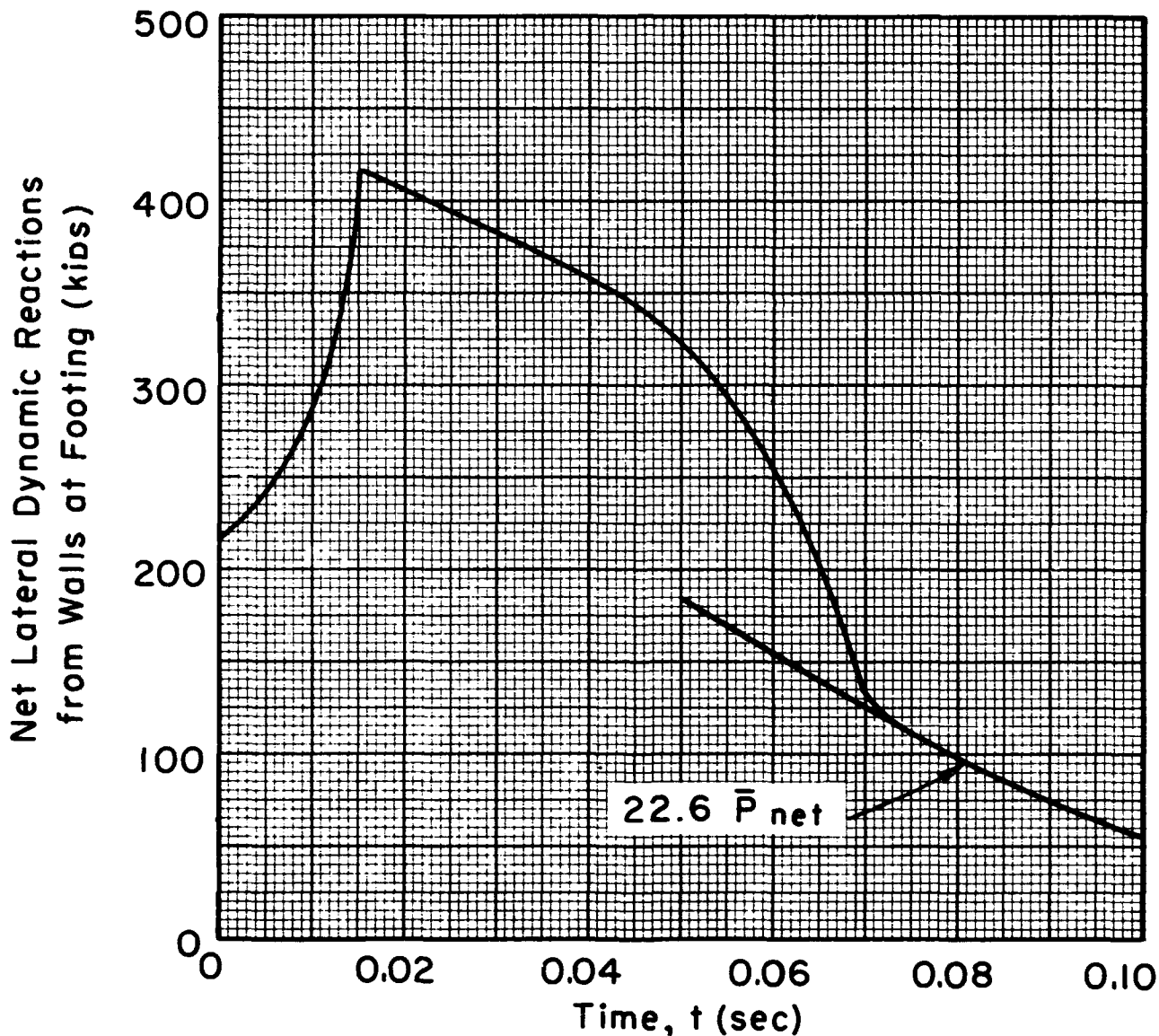


Figure 7.28. Wall reactions at foundation vs time

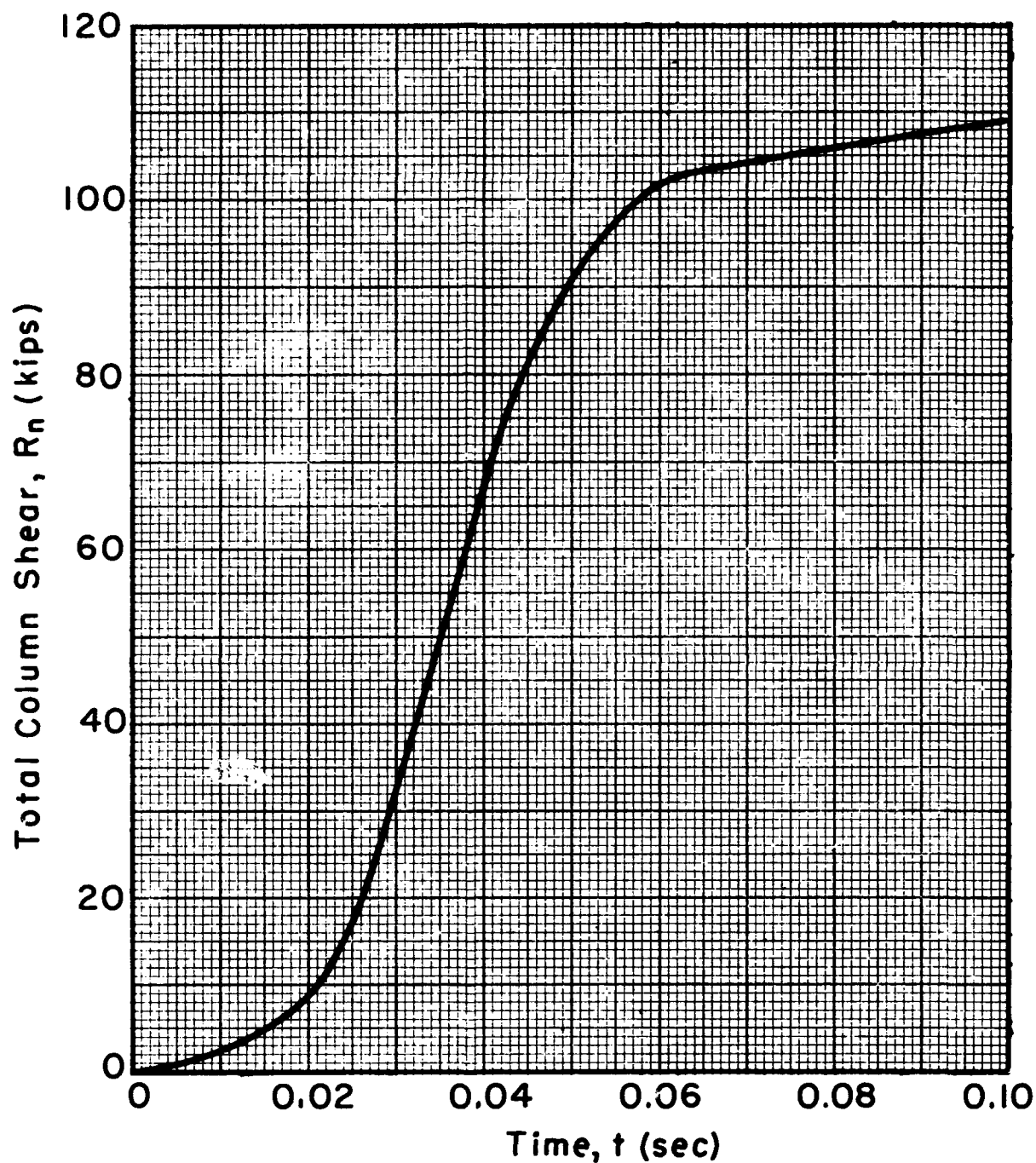


Figure 7.29. Column shear vs time

Figure 7.30 indicates the total lateral load-time variation on the foundation and is obtained by adding the ordinate values of figures 7.28 and 7.29.

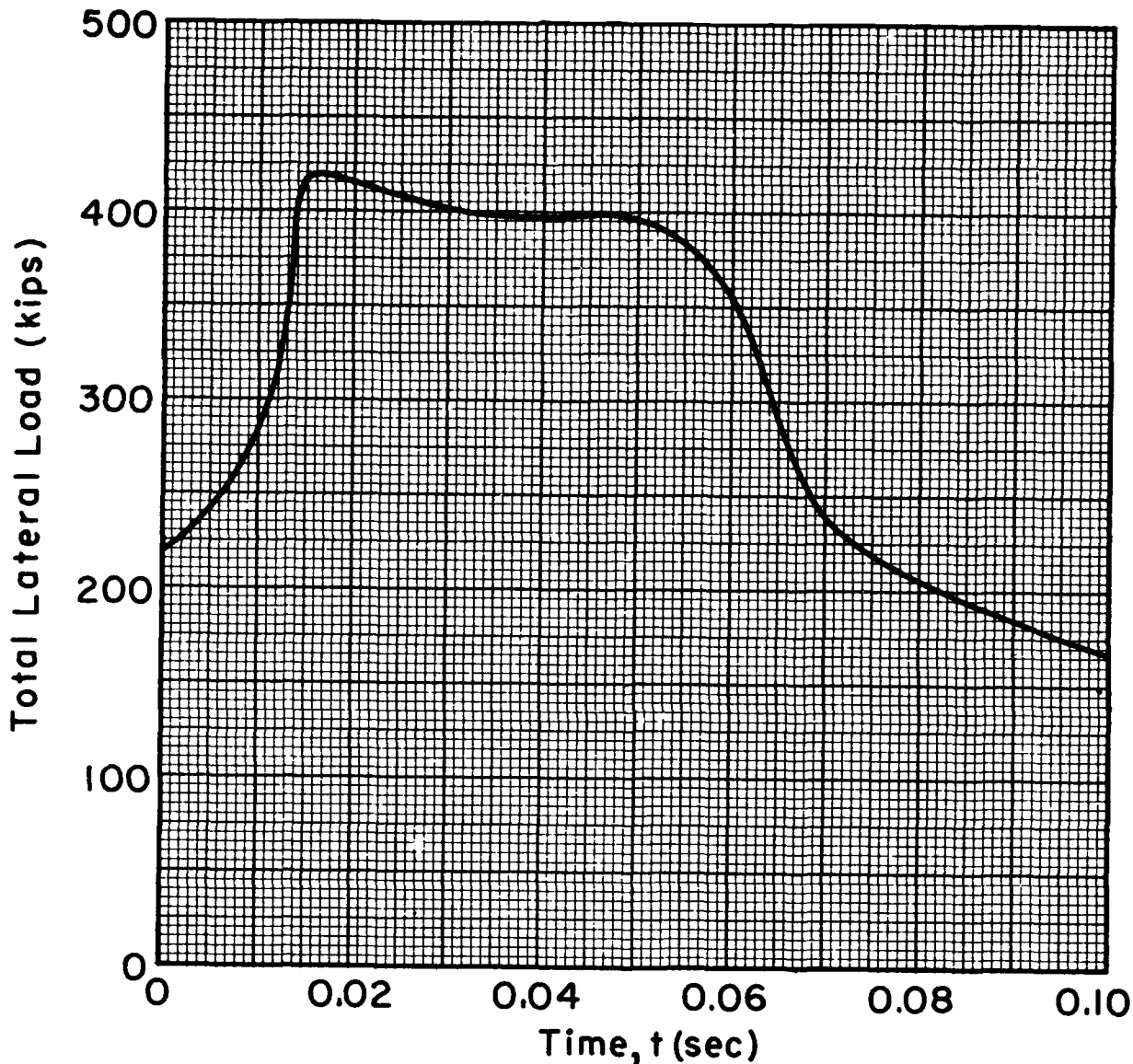


Figure 7.3C. Total lateral load on foundation vs time

Although not needed until a later step, the time variation of the column base moments is plotted in figure 7.31 using data obtained from table 7.10. From  $t = 0$  to  $t = 0.08$  sec the columns are elastic so that the moment from one column on the foundation is

$$M = \frac{1}{3} \frac{h}{2} (R_n) = 2.42R_n$$

From  $t = 0.08$  to 0.20 sec, the columns are plastic, hence  $(M_D)_n$  applies.

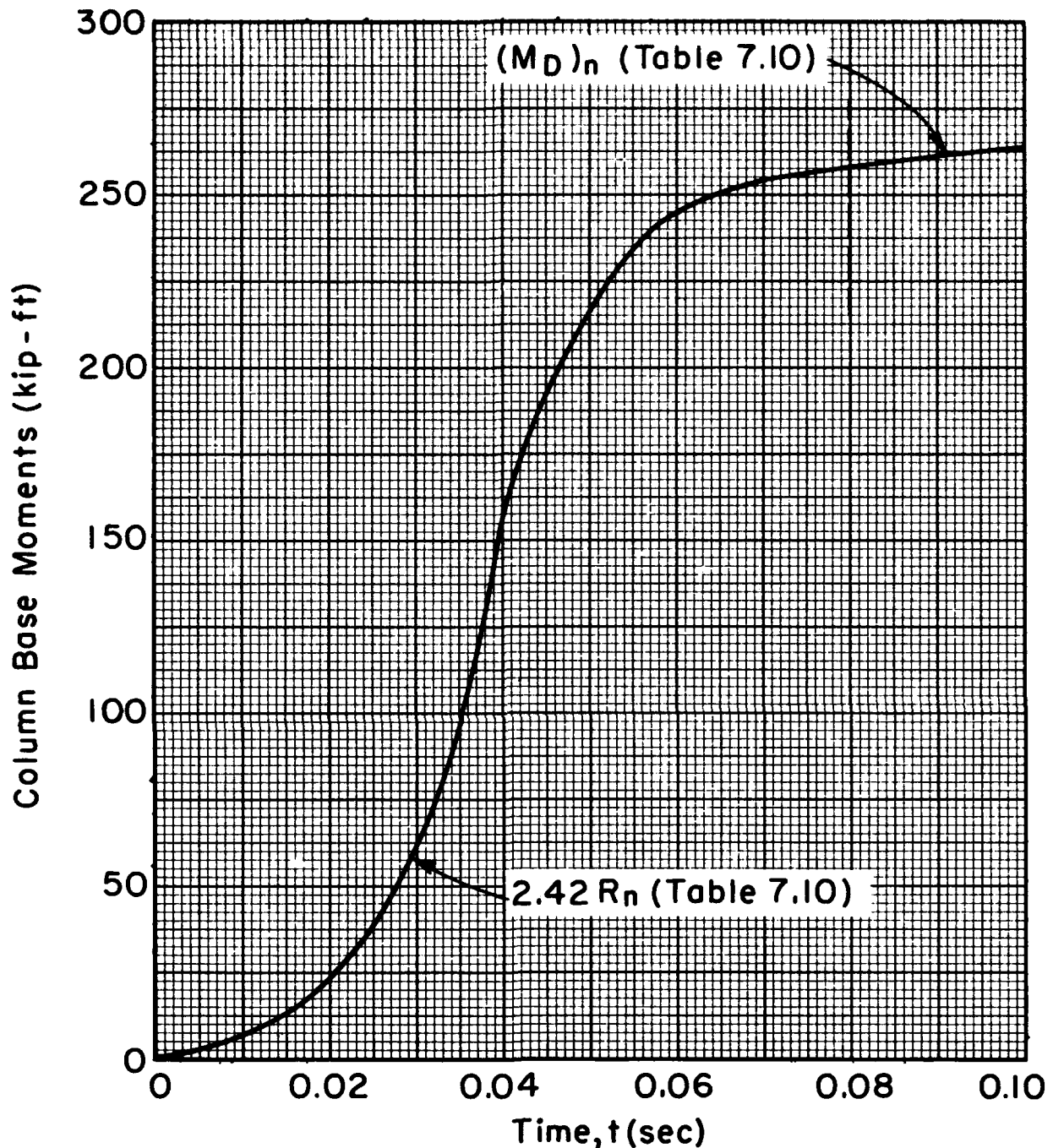


Figure 7.31. Column base moments vs time

c. Preliminary Sizes of Members. The preliminary column footing sizes are determined on the basis of the maximum column vertical load (fig. 7.25) and the dynamic design bearing capacity, 30 ksf

$$\text{Required area} = \frac{(P_D)_{\max}}{30} = \frac{1000}{3(30)} = 11.1 \text{ sq ft}$$

Try column footings, 3 ft 6 in. square, area = 12.25 sq ft.

The size of the wall footing is determined by using the maximum value of blast load on the footing projection and adding to it the dead load of wall and overburden.

Dead load of exterior wall and footing: (assume wall height = 18.0, footing width and depth = 2.5 ft, neglect overburden)

$$\text{Wall} = (2)(150/1000)(11/12)18(18.0) = 89.0 \text{ kips}$$

$$\text{Footing} = (2)(150/1000)(2.5)2.5(18.0) = 33.8 \text{ kips}$$

$$\text{Total wall dead load} = 89.0 + 33.8 = 122.8 \text{ kips}$$

$$\text{Maximum value of blast load} = 54 \text{ kips (fig. 7.26)}$$

$$\text{Theoretical width} = \frac{\text{total exterior wall load}}{\text{length (allowable bearing capacity)}}$$

$$\text{Width} = \frac{(122.8 + 54)}{2(18)30} = 0.164 \text{ ft}$$

This value is too small for practical purposes, hence an arbitrary width of 2 ft 6 in. will be used for subsequent analyses.

d. Preliminary Depth of Foundation. The time variation of the unbalanced load is obtained in table 7.12 by subtracting the available

Table 7.12. Determination of Unbalanced Lateral Load

①	②	③	④	⑤	⑥
Time (sec)	Lateral Load (kips)	Vertical Load (kips)	Total Vertical Load (kips)	Friction (kips)	Unbalanced Lateral Load (kips)
	fig. 7.30	fig. 7.27	③ + 163.6	0.5 ④	② - ⑤
0	218	105	268	134	84
0.005	243	270	433	217	26
0.010	288	440	603	302	None
0.015	420	600	763	382	
0.020	414	730	893	447	
0.030	403	905	1068	534	
0.040	397	1020	1183	592	
0.050	397	971	1134	567	
0.060	355	930	1093	547	



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frictional force from the total lateral load. The lateral load causing sliding is obtained from figure 7.30. The frictional resistance to sliding is obtained by multiplying the total vertical load by the coefficient of friction,  $\mu = 0.50$  (par. 7-18). The total vertical load at any time is obtained by adding the dead load of the total structure to the vertical loads in figure 7.27.

Columns = 3.35 kips (par. 7-23)

Column strap footings =  $2.5(2.5)40(0.15) = 37.5$  kips

Total dead load =  $122.8 + 3.35 + 37.5 = 163.6$  kips

The unbalanced lateral load is then the total lateral load minus the frictional resistance to sliding. The maximum value of this unbalance is 84 kips.

The procedure recommended in paragraph 6-31c requires that the depth of footing be that necessary to develop sufficient passive pressure resistance to equal twice the unbalanced lateral load at any time. The effect of the blast pressures acting on the surface of the ground at the back of the building should be included in determining the passive pressure resistance. In this example the back wall loading does not begin until 0.030 sec (table 7.11) and there is no unbalanced lateral load after  $t = 0.005$  sec (table 7.12).

The required depth of footing is determined by:

$$F_P = K_{P0} \frac{\gamma H^2}{2} \quad (\text{eq 4.58})$$

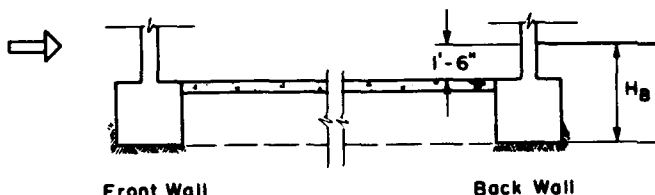
where

$F_P$  = normal component of total passive resistance

=  $2(\text{maximum unbalance}) = 2(84) = 168$  kips

The resistance to lateral motion is provided by the lesser of: (1) combined passive pressure resistances of front and back footings or (2) frictional resistance of earth between footings plus passive pressure resistance of back footing.

Considering case (1) first, the length of back wall capable of resisting passive pressure is 18 ft. The effective height of soil is  $H_B$ .



The length of front wall capable of resisting passive pressure is  $(18 - 2.5) = 15.5$  ft. The effective height of soil is  $(H_B) - (1.5 \text{ ft}) +$  (increase for height of concrete floor)  $= H_B - 1.5 + \frac{(150 - 100)}{100} (0.5)$   
 $= H_B - 1.25$  ft.

$F_P$  = front wall resistance + back wall resistance

$$F_P = 15.5 K_{P\phi} \frac{\gamma(H_B - 1.25)^2}{2} + 18 K_{P\phi} \frac{\gamma(H_B)^2}{2}$$

$$168 = 15.5(10)0.100 \frac{(H_B - 1.25)^2}{2} + 18(10) \frac{0.100(H_B)^2}{2}$$

$$H_B = 3.68 \text{ ft (compare available } H_B = 4.31)$$

On this basis, the back footing is capable of providing a passive resistance equal to

$$18 K_{P\phi} \frac{\gamma(H_B)^2}{2} = \frac{18(10)0.100(3.68)^2}{2} = 121 \text{ kips}$$

The potential frictional resistance of soil between footings equals

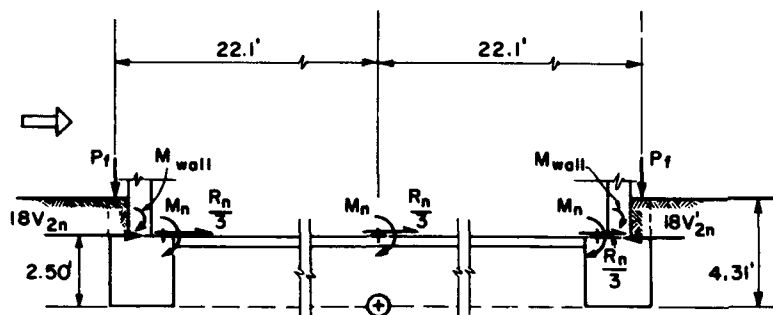
$$\mu W = \mu(\gamma)15.5(H_B - 1.25)41.7 = 0.50(0.100)15.5(2.43)41.7 = 78 \text{ kips}$$

$$78 + 121 = 199 > 168, \text{ case (2) is OK.}$$

e. Overturning Moment. The sketch below indicates the position of

loads considered in the overturning computations.

In table 7.3, the time variation of the overturning moment on the structure is determined. The wall reaction overturning



moment is obtained by multiplying the net wall reaction from figure 7.28 by 2.50 ft.

The wall support moments  $M_{wall}$  are obtained from data in tables 7.2 and 7.11. In the elastic range  $M_{wall} = (R_n L)/8$ , for one bay  $M_{wall} = [(18)/8](17.5)R_n = 39.3R_n$ . In the elasto-plastic and plastic ranges  $M_{wall} = 18(59.5) = 1070 \text{ kip-ft}$ . The column base moments are obtained by multiplying the column moments (fig. 7.31) by the number of columns in one

Table 7.13. Determination of Overturning Moment

①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪
Time (sec)	Net Column and Wall Reaction (kips)	Wall Reaction Overturning Moment (ft-kips)	Front Wall Support Moment (ft-kips)	Back Wall Support Moment (ft-kips)	Column Base Moments (ft-kips)	M <sub>o</sub> (ft-kips)	Blast Load on Front Footing Projection (kips)	Blast Load on Back Footing Projection (kips)	Net Footing Projection Moments (ft-kips)	Net Overturning Moment (ft-kips)
Ref	fig. 7.30	② 2.50	table 7.2	table 7.11	fig. 7.31	③ - ⑥	fig. 7.26	fig. 7.32	22.1 [⑧ + ⑨]	[⑦ - ⑩]
0	218	545	0	0	0	545	52	0	1175	-630
0.005	240	600	425	0	4	1029	48	0	1085	-56
0.010	285	712	1070	0	15	1797	43	0	971	826
0.015	420	1050	1070	0	30	2150	39	0	883	1267
0.020	415	1038	1070	0	60	2168	34	0	770	1398
0.025	410	1025	1070	0	120	2215	30	0	680	1535
0.030	400	1000	1070	0	187	2257	26	0	589	1668
0.035	400	1000	1070	2.6	300	2373	24	1.0	520	1853
0.040	395	988	1070	21.0	480	2559	23	2.1	472	2087
0.045	395	988	1070	65.0	585	2708	22	2.9	431	2277
0.050	395	988	1070	136.0	651	2845	21	3.9	386	2459
0.055	380	950	944	225.0	705	2824	21	4.9	362	2462
0.060	355	888	583	317.0	741	2529			362	2167

bay. The footing projection righting moment is obtained by multiplying the blast load on front footing projection from figure 7.26 by 22.1 ft. The back footing overturning moment is obtained by multiplying the blast load on back footing projection from figure 7.32 by 22.1. The vertical blast load on the rear footing (fig. 7.32) is obtained by multiplying the projecting footing area by the rear face overpressure (fig. 7.10) obtaining

$$0.79(18)\bar{P}_{\text{back}} \frac{144}{1000} = 2.05\bar{P}_{\text{back}}$$

The overturning moment due to the passive pressure is neglected conservatively.

f. Combined Axial Load and Overturning Moment. From the equation for combined stresses it is possible to write

$$\text{Ultimate bearing capacity} = \frac{P}{A} + \frac{Mc}{I}$$

From which by substitution is obtained

$$30 = \frac{P}{197.5} + \frac{M(22.54)}{55,290}$$

$$P = 5929 - 0.0806M$$

where

$$\text{Ultimate bearing capacity} = 30 \text{ kips/ft}^2 \text{ (par. 7-18)}$$

P = total vertical load on one bay of foundation

A = total area of one bay of foundation = 197.5 ft<sup>2</sup>

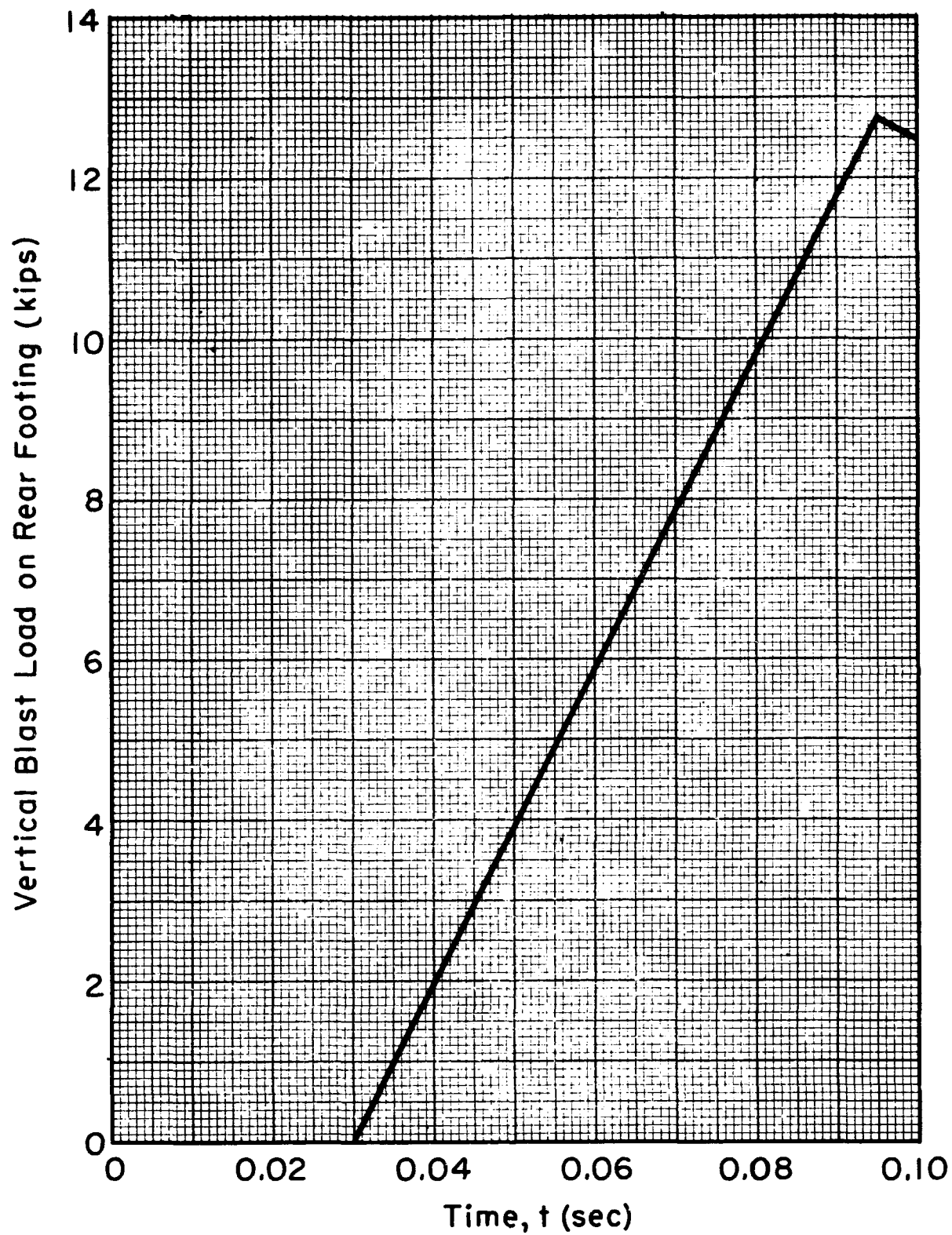


Figure 7.32. Vertical blast load on rear footing

$M$  = total overturning moment on foundation

$c$  = distance to extreme point on foundation from center

$I$  = moment of inertia of foundation = 55,290 ft<sup>4</sup>

Introducing the  $P$  and  $M$  data from table 7.13 into the equation for  $P$  (above) shows that the soil pressures and the preliminary sizes of the foundation elements are satisfactory.

g. Design of Foundation Members. In a complete design study it would be necessary to determine stresses at various times in order to determine the critical condition. To illustrate the procedure it is sufficient to compute stresses at only one time.

The column strap footing is designed for the stress conditions at  $t = 0.055$  sec. One bay of the foundation 18 ft wide centered on a column strap footing is considered (fig. 7.33).

$$\frac{P}{A} = \frac{1113}{197.5} = 5.63 \text{ ksf}$$

(tables 7.12 and 7.13)

$$\frac{Mc}{I} = \frac{2462(22.54)}{55,290} = 1.00 \text{ ksf}$$

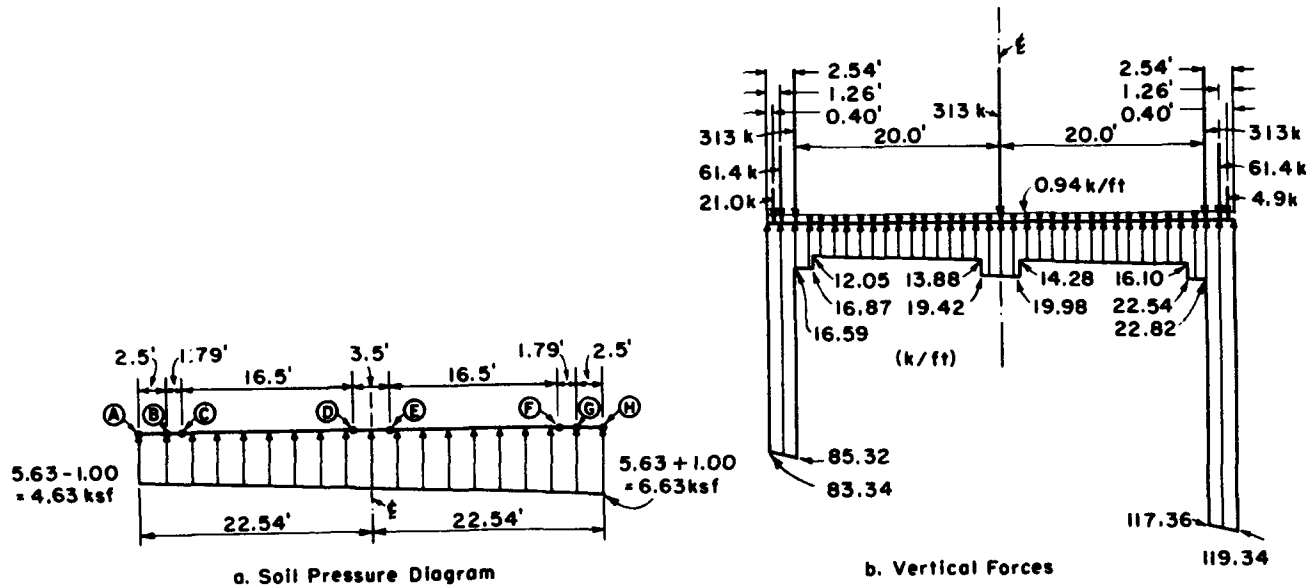


Figure 7.33. Vertical forces and soil pressure on foundation

The soil pressure variation is shown in figure 7.33a. The soil pressure multiplied by the footing width gives the soil load per foot (fig. 7.33b).

In figures 7.33 and 7.34 all the forces and moments acting on the foundation at time  $t = 0.055$  sec are shown. They are as follows:

Front wall = rear wall =  $122.8/2 = 61.4$  kips (par. 7-27c)

Column strap footing =  $37.5/40 = 0.94$  kip-ft (par. 7-27d)

Front footing blast load = 21.0 kips (table 7.13)

Rear footing blast load = 4.9 kips (fig. 7.32)

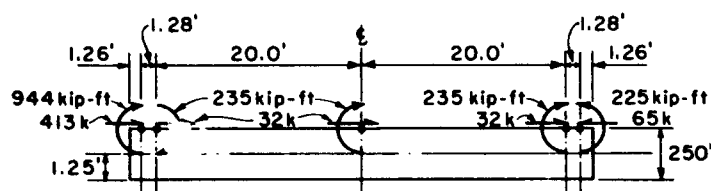
Column load  $(3.35 + 936)/3 = 313$  kips (table 7.10)(fig. 7.12)

Moment at base of front wall = 944 kip-ft (table 7.13)

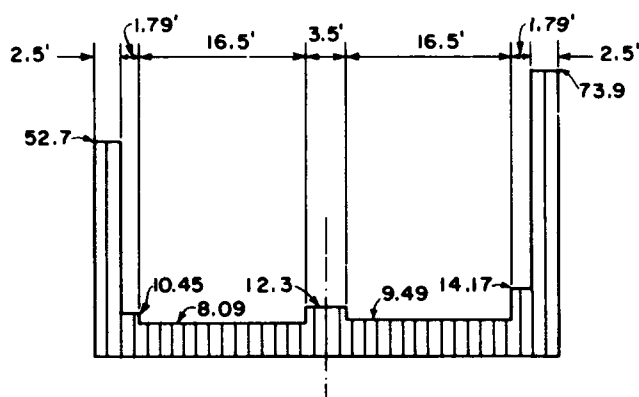
Moment at base of rear wall = 225 kip-ft (table 7.13)

Shear at base of front wall =  $18V_{2n} = 18(22.9) = 413$  kips (table 7.2)

Friction at base of footing = 557 kips (table 7.12)



a. Applied Moments and Lateral Shears



b. Moment (kip-ft/ft) about Centerline of Footing Due to Soil Friction

Figure 7.34. Applied moments and shears

The moments at the base of the front and rear walls are assumed to be transmitted to the column strap footing. This is a conservative procedure for the strap footing since the fixity at the base of the wall must be provided continuously along the wall.

The friction force is assumed to be distributed along the base proportionally to the total vertical soil load. For example between points A and B

$$W = \frac{83.34 + 85.32}{2} (2.5) = 210.8 \text{ kips}$$

$$\text{Friction} = \frac{210}{1113} (557) = 105.4 \text{ kips (table 7.12)}$$

The critical sections are considered below.

#### Shear Point E

$\Sigma$  loads down -  $\Sigma$  loads up (fig. 7.33b)

$$\frac{V}{1.5} = 4.9 + 61.4 + 313 + 18.29(0.94) - \left( \frac{119.34 + 117.36}{2} \right) 2.5 + \left( \frac{22.82 + 22.54}{2} \right) 1.79 + \left( \frac{16.10 + 14.28}{2} \right) 16.5$$

$$\frac{V}{1.5} = 396.5 - 587.1 = -190.6 \text{ kips}$$

$$V = -190.6(1.5) = -286 \text{ kips}$$

Shear Point G

$$\frac{V}{1.5} = (4.9 + 614) - \left( \frac{117.36 + 119.34}{2} \right) 2.5$$

$$\frac{V}{1.5} = 66.3 - 295.9 = -229.6 \text{ kips}$$

$$V = 1.5(-229.6) = -344 \text{ kips}$$

Moment Point C (figs. 7.33 and 7.34)

$$\begin{aligned} \frac{M}{1.5} = & 944 + 413(1.25) + 235 + 32(1.25) + 52.7(2.5) + \\ & \left( \frac{83.34 + 85.32}{2} \right) 2.5 \left( \frac{2.5}{2} \right) - 21(2.14) - 61.4(1.28) \end{aligned}$$

$$\frac{M}{1.5} = 2007$$

$$M = 3010 \text{ kip-ft}$$

Moment Point E (figs. 7.33 and 7.34)

$$\begin{aligned} \frac{M}{1.5} = & -225 - 65(1.25) + 235 + 32(1.25) + 73.9(2.5) + \\ & 14.17(1.79) + 9.49(16.5) + 4.9(20.39) + 61.4(19.53) + \\ & 313(18.25) + 0.94(18.29)(9.14) - \left( \frac{117.36 + 119.34}{2} \right) (2.5)(19.54) - \\ & \left( \frac{22.82 + 22.54}{2} \right) (1.79)(17.39) - \left( \frac{14.28 + 16.10}{2} \right) (16.5)(8.25) \end{aligned}$$

$$\frac{M}{1.5} = 1051$$

$$M = 1577 \text{ kip-ft}$$

The footing strap is checked to determine if the sizes originally assumed can be reinforced sufficiently. The sections are checked for bending moment capacity by equation 4.16 with the values of  $f_{dy}$  and  $f'_{dc}$  replaced by  $f_y$  and  $f'_c$ , respectively, because the loads are considered to be applied statically.

Section at Point E -  $b = 2.5 \text{ ft}$ , assume  $d = 30 \text{ in.}$

$$M_P = p b d f_y d \left( 1 - \frac{p f_y}{1.7 f'_c} \right) \text{ (eq 4.16)}$$

For  $p = 0.02$

$$M_P = 0.02(2.5)30(40)30 \left[ 1 - \frac{0.02(40)}{1.7(3.0)} \right] = 1520 \text{ kip-ft}$$

$$M_p = 1520 \text{ kip-ft} \approx 1577 \text{ kip-ft; OK}$$

Shear at this location = 286 kips

$$\text{Shear stress} = v = \frac{8V}{7bd} \text{ (eq 4.23)}$$

$$v = \frac{8(286)1000}{7(30)30} = 363 \text{ psi}$$

$$\begin{aligned} \text{For no web reinforcement, allowable } v &= 0.04f'_c + 5000p \text{ (eq 4.24a)} \\ &= 0.04(3000) + (5000)0.02 \\ &= 120 + 100 = 220 \text{ psi} \end{aligned}$$

Stirrups are supplied to provide  $(363 - 220 = 143 \text{ psi})$  additional required shear strength.

$$rf_y = 143 \text{ (eq 4.24)}$$

$$r = \frac{143}{40,000} = 0.0036$$

$$r = \frac{A_v}{b_s} = \frac{A_v}{30(12)} = 0.0036$$

$$A_v = 1.29 \text{ sq in./ft}$$

Section at Point C -  $b = 3.5 \text{ ft}$ , assume  $d = 30 \text{ in.}$

$$M_p = pbdf_y d \left( 1 - \frac{pf_y}{1.7f'_c} \right) \text{ (eq 4.16)}$$

$$\text{For } p = 0.03$$

$$M_p = 0.03(3.5)30(40)30 \left[ 1 - \frac{0.03(40)}{1.7(3.0)} \right]$$

$$M_p = 2900 \text{ kip-ft} \approx 3010 \text{ kip-ft; OK}$$

Maximum shear occurs at point G. This location has same cross section as point C. For reversal of load on structure maximum shear would occur at point C ( $V = 344 \text{ kips}$ ).

$$\text{Shear stress} = v = \frac{8V}{7bd} \text{ (eq 4.23)}$$

$$v = \frac{8(344)1000}{7(42)30} = 312 \text{ psi}$$

If no shear reinforcement is used, the allowable  $v = 0.04f'_c + 5000p$  (eq 4.24b).

$$v = 0.04(3000) + 5000(0.03) = 120 + 150 = 270 \text{ psi}$$

Shear reinforcement equal to  $(312 - 270 = 42 \text{ psi})$  is provided.

$$r = \frac{37}{40,000} = 0.000925$$



$$A_v = rbs = 0.000925(42)12 = 0.47 \text{ sq in./ft}$$

Since the critical sections are capable of resisting moments and shears, the entire foundation footing strap could be reinforced in proportion to moments and shears. The wall footings need only nominal reinforcement since dimensions selected are much larger than necessary for calculated soil pressure.

#### h. Dynamic Foundation

Design. The dynamic design procedure (par. 6-31c) usually results in a more economical design than the quasi-static procedure because the dynamic effects are computed rather than conservatively estimated.

The results of the quasi-static design, with certain modifications illustrated in figure 7.35, will be used as a preliminary design in the dynamic design procedure. In this design the wall footings are placed eccentrically to assist in resisting overturning most effectively.

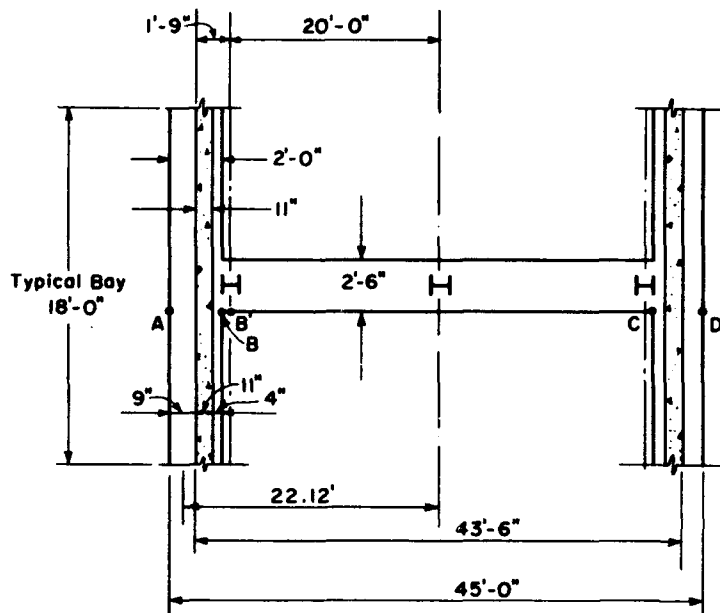


Figure 7.35. Plan of foundation

Using the principles presented in paragraph 6.31e for the sliding and overturning analysis, the horizontal acceleration  $\ddot{x}_o$  and the angular acceleration  $\alpha_o$  about the longitudinal axis through the bottom of the footings are determined. The corresponding horizontal displacement  $x_o$  and the angular displacement  $\theta$  are determined as a function of time by means of a concurrent numerical integration of equations (6.94) and (6.95).

$$\ddot{x}_o = \frac{M_o - F_o \bar{y}}{I_o - m \bar{y}^2} \quad (\text{eq 6.94})$$

$$\ddot{x}_o = \frac{F_o}{m} - \alpha_o \bar{y} \quad (\text{eq 6.95})$$

Element	Dimension	Volume (cu ft)	Weight (kips)	Mass (kip-sec <sup>2</sup> /ft)	$\bar{y}^*$ (ft)	$\bar{m}_y$ (ft)	$d_1^{**}$ (ft)	$\frac{md^2}{12} +$ (kip-sec <sup>2</sup> )	$\frac{m\bar{y}^2}{12}$ (kip-sec <sup>2</sup> )	$I_{xx}$ (kip-sec <sup>2</sup> )	$d_2^{++}$ (ft)	$\frac{md^2}{12} +$ (kip-sec <sup>2</sup> )	$\bar{x}$ (ft)	$\frac{m\bar{x}^2}{12}$ (kip-sec <sup>2</sup> /ft)	$I_{yy}$ (kip-sec <sup>2</sup> /ft)
Roof slab	(See par. 7.21b)		41.4	1.285	20.32	26.11	(0.354)	-----	530.55	530.55	43.5	202.5	20.0	9.6	202.5
Purlin (A)	18 ft long		0.76	0.024	19.48	0.47	0.54	0.07	9.16	9.23	0.12	0	13.34	4.27	9.6
Purlin (B)	18 ft long		0.76	0.024	19.48	0.47	0.54	0.07	9.16	9.23	0.12	0	6.67	1.07	4.27
Purlin (C)	18 ft long		0.76	0.024	19.48	0.47	0.54	0.07	9.16	9.23	0.12	0	0	0	1.07
Purlin (D)	18 ft long		0.76	0.024	19.48	0.47	0.54	0.07	9.16	9.23	0.12	0	6.67	1.07	0
Purlin (E)	18 ft long		0.76	0.024	19.48	0.47	0.54	0.07	9.16	9.23	0.12	0	13.34	4.27	1.07
Purlin (F)	18 ft long		0.76	0.024	19.48	0.47	0.54	0.07	9.16	9.23	0.12	0	20.0	9.6	4.27
Purlin (G)	18 ft long		0.76	0.024	19.48	0.47	0.54	0.07	9.16	9.23	0.12	0	20.0	9.6	9.6
Girder	40 ft 10 in. long		7.50	0.233	18.65	4.35	1.19	0.33	101.40	101.77	0.20	0.01	20.0	14.0	0.01
Column 1	14.5 long		1.12	0.035	9.90	0.35	14.5	0.61	3.47	4.08	0.37	0	20.0	14.0	14.0
Column 2	14.5 long		1.12	0.035	9.90	0.35	14.5	0.61	3.47	4.08	0.37	0	20.0	14.0	0
Column 3	14.5 long		1.12	0.035	9.90	0.35	14.5	0.61	3.47	4.08	0.37	0	20.0	14.0	0
Front wall	(0.92)(17.15)(18.0)	289.8	43.47	1.350	11.42	15.42	17.50	34.45	176.06	210.76	0.92	0.10	21.29	611.91	612.01
Back wall	(0.92)(17.15)(18.0)	289.8	43.47	1.350	11.42	15.42	17.50	34.45	176.06	210.76	0.92	0.10	21.29	611.91	612.01
Column strap footing	(2.5)(2.67)(41.6)	277.7	41.66	1.244	1.34	1.73	2.67	0.76	2.32	3.08	41.66	187.1	21.5	207.09	187.1
Front wall footing	(2.67)(2.0)(18.0)	96.12	14.42	0.448	1.34	0.60	2.67	0.27	0.80	0.05	2.0	0.15	21.5	207.09	207.24
Back wall footing	(2.67)(2.0)(18.0)	96.12	14.42	0.448	1.34	0.60	2.67	0.27	0.80	0.05	2.0	0.15	21.5	207.09	207.24
Floor	(0.5)(41.66)(15.5)	322.8	48.42	1.504	2.42	3.64	0.5	0.03	8.81	9.12	41.66	217.5	21.5	207.09	217.5
Totals			8.18		8.83	72.21				1144.25					2303.49

\* Mass center of element from bottom of footing.

\*\*  $d_1$  = radius of gyration for W shapes.

+  $md_1^2$  used for W shapes

++  $d_2$  = radius of gyration for W shapes.

+  $md_2^2$  used for W shapes

Table 7.14. Computation of Centroid and Polar Moment of Inertia



$$\frac{I_{net}}{I_{gross}} = \frac{(1/12)bd^3 - (1/12)b_1d_1^3}{(1/12)bd^3} = 1 - \frac{b_1d_1^3}{bd^3}$$

where, from figure 7.35,

$$b = 18.0 \text{ ft}$$

$$d = 45.0 \text{ ft}$$

$$b_1 = 18.0 - 2.5 = 15.5 \text{ ft}$$

$$d_1 = 45.0 - 2(2.0) = 41.0 \text{ ft}$$

$$L = d/2 = 45/2 = 22.5 \text{ ft}$$

$$\frac{I_{net}}{I_{gross}} = 1 - \frac{15.5(41.0)^3}{18.0(45.0)^3} = 0.348$$

For an 18-ft typical bay

$$B_{gross} = \frac{\pi(40)(12)^2}{4(1 - 0.333^2)} (22.5)^2 18 = 46.1(10)^6 \text{ kip-ft/radian}$$

$$B_{net} = (0.348)B_{gross} = (0.348)46.1(10)^6 = 16.1(10)^6 \text{ kip-ft/radian}$$

k. Dynamic Analysis. The dynamic rigid body sliding and overturning analysis is performed in table 7.15. A simultaneous numerical integration is performed to obtain the time history of the rigid body rotation  $\theta$  and the rigid body translation  $x_o$ . The acceleration impulse extrapolation method (eq 5.49) is used for the dynamic analysis in the form

$$(\theta)_{n+1} = 2(\theta)_n - (\theta)_{n-1} + (\alpha_o)_n(\Delta t)^2$$

$$(x_o)_{n+1} = 2(x_o)_n - (x_o)_{n-1} + (\ddot{x}_o)_n(\Delta t)^2$$

where

$$(x_o)_n = \frac{(F_o)_n}{m} - (\alpha_o)_n \bar{y} \text{ (eq 6.95)}$$

While the structure is sliding

$$(\alpha_o)_n = \frac{(M_o)_n - (F_o)_n \bar{y}}{I_o - m\bar{y}^2} \text{ (eq 6.94)}$$

where

$(M_o)_n$  = moment of all external forces about axis of rotation "O" at time n.

$(F_o)_n$  = summation of all external horizontal forces applied to the structure at time n

Table 7.15. Simultaneous Sliding and Overturning Analysis

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
t (sec)	M <sub>o</sub> (kip-ft.)	V <sub>P</sub> (kips)	V <sub>B</sub> (kips)	M <sub>V net</sub> (kip-ft.)	F <sub>P</sub> (kips)	F <sub>P</sub> P <sub>1</sub> (kip-ft.)	F <sub>a</sub> (kips)	F <sub>a</sub> V <sub>a</sub> (kip-ft.)	B <sub>6</sub> (kip-ft.)	M <sub>o</sub> (kip-ft.)	P <sub>n</sub> (kips)
Ref	table 7.13	1.95P <sub>front</sub>	1.95P <sub>back</sub>	-22.12 [3-4]	0.00198 (25)	-1.49 (6)	14.93 (4)	-2.24 (8)	-16.1 (20)	(2) + (5) + (7) + (9) + (10) fig. 7.25	
0	545	50	0	-1106	0	0	0	0	0	-561	55
0.005	1029	47	0	-1140	0.24	0	0	0	+4	+83	210
0.010	1797	41	0	-907	0.68	-1.0	0	0	+219	+1108	390
0.015	2150	37	0	-818	0.93	-1.5	0	0	+175	+1506	560
0.020	2168	32	0	-708	1.31	-2.0	0	0	-35	+1424	700
0.025	2215	29	0	-641	1.31	-2.0	0	0	-491	+1082	810
0.030	2257	25	0	-555	1.31	-2.0	0	0	-1074	+627	890
0.035	2373	23	1.0	-486	1.31	-2.0	14.93	-33	-1731	+122	960
0.040	2559	22	2.0	-442	1.31	-2.0	29.86	-67	-2401	-352	990
0.045	2708	21	3.0	-398	1.31	-2.0	44.79	-100	-3030	-821	975
0.050	2845	20	4.0	-354	1.31	-2.0	59.72	-134	-3564	-1208	950
0.055	2824	20	5.0	-332	1.31	-2.0	74.65	-167	-3957	-1633	930
0.060	2529	19	6.0	-288	1.31	-2.0	89.58	-201	-4158	-2119	910

(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)
V (kips)	V (kips)	H <sub>net</sub> (kips)	F <sub>C</sub> (kips)	F <sub>OY</sub> (kip-ft.)	α <sub>o</sub> (rad/sec <sup>2</sup> )	α <sub>o</sub> (Δt) <sup>2</sup> (10 <sup>-6</sup> rad)	θ (10 <sup>-6</sup> rad)	F <sub>O/m</sub> (ft/sec <sup>2</sup> )	α <sub>o</sub> Y <sub>o</sub> (ft/sec <sup>2</sup> )	Y <sub>o</sub> (ft/sec <sup>2</sup> )	Y <sub>o</sub> (Δt) <sup>2</sup> (10 <sup>-6</sup> ft)	X (10 <sup>-6</sup> ft)
161 + (3) + (4) + (12)	0.50 (13)	(2) x table 7.12	(15) - (14) - (6) - (8)	8.83 (16)	par. 7-24k			0.122 (15)	8.83 (18)	(21) - (22)		
266	133	218	+85.0	+751	-(-0.467)/2	-5.84	0	+10.37	-2.06	(+12.43)/2	155.4	0
418	209	243	+34.0	+300	-0.077	-1.93	-5.84	+4.15	-0.68	+4.83	120.8	155.4
592	296	288	-8.5	-75	+0.421	+10.53	-13.61	+4.15	+3.72	-4.76	-119.0	431.6
758	379	420	+40.0	+353	+0.410	+10.25	-10.85	+4.88	+3.62	+1.26	+31.5	588.8
893	447	414	-34.0	-300	+0.614	+15.35	+2.16	-4.15	+5.42	-9.57	-239.3	777.5
1000	500	409	0	0	+0.314	+7.85	+30.52	0	0	0	0	786.9
1076	538	403	0	0	+0.182	+4.55	+66.73	0	0	0	0	
1145	573	400	0	0	+0.035	+0.88	+107.49	0	0	0	0	
1175	588	397	0	0	-0.102	-2.55	+149.13	0	0	0	0	
1160	580	397	0	0	-0.238	-5.95	+188.22	0	0	0	0	
1135	568	397	0	0	-0.350	-8.75	+221.36	0	0	0	0	
1116	558	382	0	0	-0.474	-11.85	+245.75	0	0	0	0	
1096	548	355	0	0	-0.615	-15.38	+255.45	0	0	0	0	

While the structure is not sliding

$$(\alpha_o)_n = \frac{(M_o)_n}{I_o} \text{ (eq 6.96)}$$

In table 7.15,  $M_o$  is the summation of external moments about the point of rotation excluding horizontal footing projection moments. The blast loads on front and back footing projections are obtained by multiplying pressure values from figures 7.9 and 7.10, respectively, by  $9/12[144\bar{P}(18)/1000] = 1.95\bar{P}$ .

The lateral passive soil pressure force  $F_p$  is considered to vary linearly with displacement from zero to a peak value at  $x = 0.04H_B = 0.04(4.48) = 0.179$  ft.

From paragraph 7-27 the maximum available force is  $F_p = 181 + 102 = 283$  kips, where passive pressure on rear face gives  $[18(10)(0.1)(4.48)^2]/2 = 181$  and friction between wall footings gives  $0.5(0.1)15.5(4.48 - 1.25)41 = 102$ . Thus at any displacement  $x$ , the passive force is  $F_p = (283/0.179)x = 1580x$ . This is the value with no pressure on the soil at the rear of the building. If there is blast pressure on the soil at the rear, the passive force increases. The additional passive force can be determined from the surcharge equivalent of the overpressure, thus  $\Delta F_p = K_{p\phi} \text{ (blast load) } 4.48 = [10(144)\bar{P}_{\text{back}}(18)4.48]/1000 = 116\bar{P}_{\text{back}}$ . In table 7.15 the lateral motion ceases before the blast wave begins to load the soil at the rear of the building and the passive force is considered to remain at a constant value.

The effect of the pressure on the soil is then considered as an active soil pressure force equal to one-fourth the passive force, or

$$\frac{116\bar{P}_{\text{back}}}{4} = 29\bar{P}_{\text{back}}$$

The centroid of  $F_p$  is at  $y_{p1} = 4.48/3 = 1.49$  ft above the base of the footing, and the centroid of the blast pressure induced force is at  $y_a = y_{p2} = 4.48/2 = 2.24$  ft above the base of the footing.

The vertical load is obtained by adding the dead load of the walls, foundation, and columns to the tabulated values of  $P_n$  and the blast loads on front and back footing projections. The partial dead load obtained from table 7.14 is:

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Columns	3.4
Front wall	43.5
Back wall	43.5
Column strap	41.7
Front wall footing	14.4
Back wall footing	14.4
	<hr/>
	160.9 kips

In table 7.15 column 18, when  $x_{n+1} > x_n$ , i.e. structure is sliding,

$$(\alpha_o)_n = \frac{(M_o)_n - (F_o)_n \bar{y}}{I_o - m\bar{y}^2} = \frac{\text{column 11} - \text{column 17}}{3448 - 8.18(8.83)^2}$$

$$(\alpha_o)_n = 0.0003559 (\text{column 11} - \text{column 17})$$

When  $x_{n+1} < x_n$ , i.e. structure is not sliding,

$$(\alpha_o)_n = \frac{(M_o)_n}{I_o} = \frac{\text{column 11}}{3448} = 0.00029 (\text{column 11})$$

The results of the numerical analysis in table 7.15 are:

- (1) Maximum rigid-body translation  $(x_o)_m = 0.000778$  ft,
- (2) Maximum rigid-body rotation  $(\theta)_{\max} = 0.000258$  radians,
- (3) Vertical upward motion at the front =  $0.000258(45/2) = 0.0058$  ft.

These motions are negligible; the next step is to investigate soil pressures and member sizes.

1. Design of Foundation Members. In a refined analysis it would be necessary to determine stresses at various times in order to determine the critical stress condition. However, to illustrate the procedure in this design, stresses are computed at only one time.

The column strap footing is designed for the stress conditions at  $t = 0.060$  sec. One bay of the foundation 18 ft wide centered on a column strap footing is considered.

The soil pressure is computed from  $\frac{P}{A} + \frac{\theta Bc}{I}$

where

$P$  = total vertical load

$A$  = total area of foundation =  $2(2.0)(18) + 2.5(41.0) = 174.5$  sq ft

$\theta B$  = 4158 kip-ft overturning reaction (table 7.15)

$c$  = distance to extreme point on foundation =  $45.0/2 = 22.5$  ft

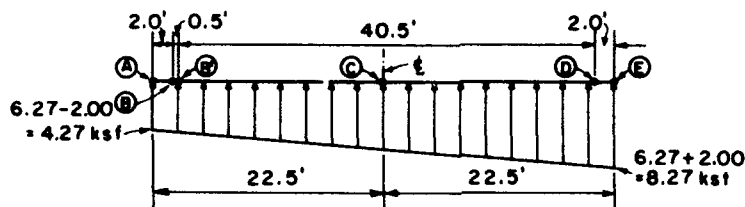
$I$  = moment of inertia of foundation

$$I = 136,000 - 89,200 = 46,800 \text{ ft}^4$$

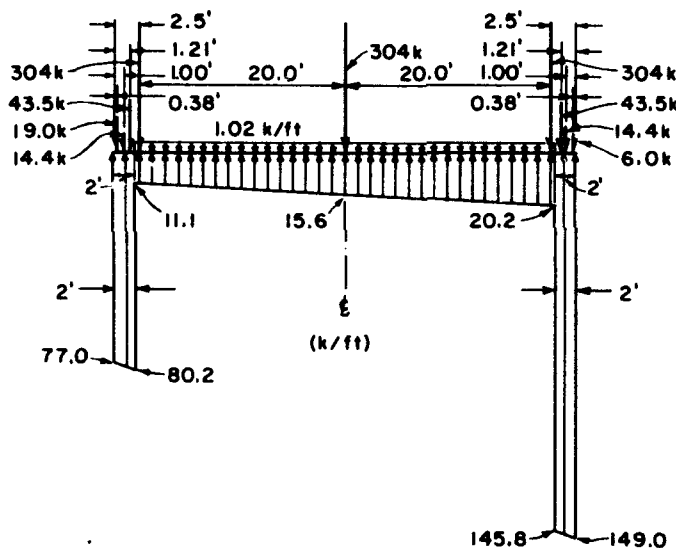
$$\frac{P}{A} = \frac{1096}{174.5} = 6.27 \text{ ksf}$$

table 7.15

$$\frac{6Bc}{I} = \frac{4158(22.5)}{46,800} = 2.00 \text{ ksf}$$



a. Soil Pressure Diagram



b. Vertical Forces

Figure 7.37. Vertical forces and soil pressure on foundation

The soil pressure multiplied by the footing width gives the soil load per foot. In figures 7.37 and 7.38 all the forces and moments acting on the foundation at time  $t = 0.050$  sec are shown. They are as follows:

Front wall = back wall = 43.47 kips (table 7.14)

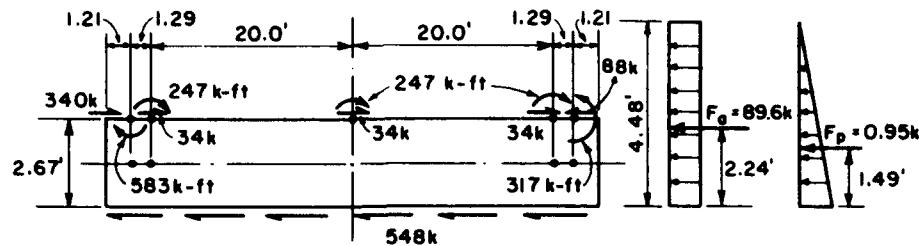
Wall footing = 14.42 kips (table 7.14)

Column strap footing =  $41.66/41.0 = 1.02$  kips/ft (table 7.14)

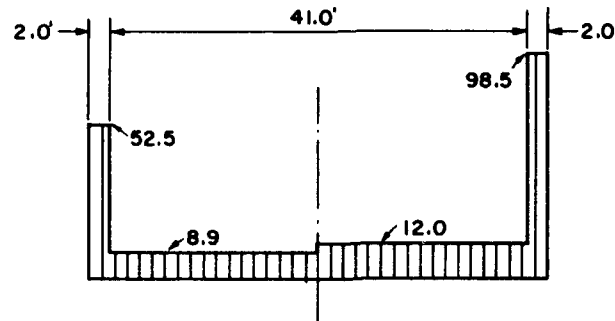
Front footing blast load = 19 kips (table 7.15)

Back footing blast load = 6.0 kips (table 7.15)





a. Applied Moments and Lateral Shears



b. Moment (kip-ft/ft) about Centerline of Footing Due to Soil Friction

Figure 7.38. Applied moments and shears and moment due to friction

Column load =  $(3.35 + 910)/3 = 304$  kips (table 7.15)

Moment at base of front wall = 583 kip-ft (table 7.13)

Moment at base of back wall = 317 kip-ft (table 7.13)

Shear at base of front wall =  $18V_{2n} = 18(18.9) = 340$  kips (table 7.2)

Shear at base of back wall =  $18V'_{2n} = 18(4.9) = 88$  kips (table 7.11)

Friction at base of footing = 548 kips (table 7.15)

Column shear =  $102/3 = 34$  kips (fig. 7.29)

Column base moments =  $741/3 = 247$  kip-ft (table 7.13)

The moments at the base of the front and back walls are assumed to be transmitted to the column strap footing. This is a conservative procedure for the strap footing since the fixity at the base of the wall must be provided continuously along the wall.

The friction force is assumed to be distributed along the base in proportion to the total vertical soil pressure. For example, between points A and B

$$W = \frac{77.0 + 80.2}{2} (2.0) = 157.2 \text{ kips}$$

$$\text{Friction} = \frac{157.2}{1096} (548) = 78.8 \text{ kips}$$

In figure 7.38b the moment about the centerline of footing due to soil friction is equal to  $2.67/2$  (friction force per foot). Thus between A and B the moment equals  $78.8/2 \times 2.67/2 = 52.5 \text{ kip-ft/ft}$ . The critical sections are considered below.

#### Shear Point C

$V = \Sigma \text{ loads down} - \Sigma \text{ loads up (fig. 7.37b)}$

$$V = [6.0 + 14.4 + 43.5 + 304 + 20.5(1.02)] - \left[ \frac{149.0 + 145.8}{2} (2.0) + \frac{15.6 + 20.2}{2} (20.5) \right]$$

$$V = (388.8 - 661.8) = -273 \text{ kips}$$

#### Shear Point D

$$V = [6.0 + 14.4 + 43.5] - \frac{149.0 + 145.8}{2} (2.0)$$

$$= 63.9 - 294.8 = -230.9 \text{ kips}$$

#### Moment Point B'

$$M = 583 + 340(1.34) + 247 + 34(1.34) + 52.5(2.0) + 8.9(0.5) + \frac{77.0 + 80.2}{2} (2.0)(1.5) + 11.1(0.5)0.25 - 19.0(2.12) - 14.4(1.5) - 43.5(1.29)$$

$$M = 1559.8 \text{ kip-ft}$$

#### Moment Point C

$$M = 583 + 340(1.34) + 247 + 34(1.34) + 247 + 34(1.34) + 52.5(2.0) + 8.9(20.5) + \frac{77.0 + 80.2}{2} (2.0)21.5 + \frac{11.1 + 15.6}{2} (20.5) \left( \frac{20.5}{2} \right) - 19(22.12) - 14.4(21.5) - 43.5(21.29) - 304(20.0) - 1.02(20.5) \left( \frac{20.5}{2} \right)$$

$$M = 145.8 \text{ kip-ft}$$

An approximate check is made to determine if the sizes assumed can be reinforced sufficiently. The bending moment capacity of the footing strap is based on equation (4.16) with  $f_{dy}$  and  $f'_{dc}$  replaced by  $f_y$  and  $f'_c$ , respectively, because the soil pressure loads are considered as static loads.

Section at Point B'  $b = 2.5$  ft, assume  $d = 30$  in.

$$M_P = pbd f_y d \left( 1 - \frac{pf_y}{1.7f'_c} \right) \quad (\text{eq 4.16})$$

For  $p = 0.020$

$$M_P = 0.020(2.5)30(40)30 \left[ 1 - \frac{0.020(40)}{1.7(3.0)} \right]$$

$M_P = 1518$  kip-ft,  $1518 \approx 1560$  ft-kips; OK, section can be reinforced

Shear is investigated at point D. The cross section is the same as at point B. If the initial blast load is from the opposite direction the maximum shear occurs at point B ( $V = 231$  kips)

$$\text{Shear stress} = v = \frac{8V}{7bd} \quad (\text{eq 4.23})$$

$$v = \frac{8(231)1000}{7(30)30} = 293 \text{ psi}$$

If no web reinforcement is used the

$$\text{allowable } v = 0.04f'_c + 5000p \quad (\text{eq 4.24b})$$

$= 0.04(3000) + 5000(0.020) = 220$  psi,  $220 < 293$  psi;  $\therefore$  stirrups are needed to supply  $293 - 220 = 73$  psi

$$r = \frac{73}{40,000} = 0.00183$$

$$A_v = rbs = 0.00183(30)12 = 0.659 \text{ sq in./ft}$$

Section at Point C  $b = 2.5$  ft, assume  $d = 30$  in.

$$M_P = pbd f_y d \left( 1 - \frac{pf_y}{1.7f'_c} \right) \quad (\text{eq 4.16})$$

For minimum steel,  $p = 0.006$  (par. 4.10a)

$$M_P = 0.006(2.5)30(40)30 \left[ 1 - \frac{0.006(40)}{1.7(3.0)} \right]$$

$$M_P = 514.6 \text{ kip-ft}$$

$514.6 > 145.8$  kip-ft; OK

Shear at this location = 273 kips

$$\text{Shear stress } v = \frac{8(273)1000}{7(30)30} = 347 \text{ psi}$$

$$\begin{aligned}\text{Allowable } v &= 0.04f'_c + 5000p \text{ (eq 4.24a)} \\ &= 0.04(3000) + 5000(0.006) = 150 \text{ psi}\end{aligned}$$

Stirrups must be provided to supply  $(347 - 150 = 197 \text{ psi})$  sufficient web reinforcement.

$$r = 197/40,000 = 0.00493$$

$$A_v = rbs = 0.00493(30)12 = 1.77 \text{ sq in./ft}$$

Since the footing strap is of constant cross section in this example, other locations of the strap can be reinforced sufficiently. In this particular design the earth passive pressure effect could have been neglected, since a much greater resistance to sliding was supplied by the frictional effect of the soil. In general, however, the passive pressure effect should not be neglected because in certain cases the principal resistance to sliding is supplied by the passive pressure.

The results of the dynamic analysis indicate the development of a maximum soil pressure which is much lower than the permissible value and the requirement of relatively high percentage of reinforcement in the column strap footing. Because of these, another cycle of design for further refinement might include a reduction in plan area of all foundation members and an increased depth of column strap footing. In addition, the depth of foundation below ground surface could be reduced, if need be, because little use is made of the potential passive pressure.

#### NUMERICAL EXAMPLE, DESIGN OF A ONE-STORY STEEL FRAME BUILDING - ELASTIC BEHAVIOR

7-28 GENERAL. This numerical example presents an elastic design of a typical bay of a windowless one-story, steel, rigid-frame building with reinforced concrete walls and roof. Included is the design of a typical wall slab, roof slab, girder, and column of the building (fig. 7.39).

One-way reinforced concrete slabs are used for both the wall and roof deck. The roof slab is supported by structural steel purlins. The girders and columns of the frame are structural steel sections joined by

welding to provide moment-resisting connections.

No foundation design is presented. Reference is made to paragraph 7-27 for an illustration of this design procedure.

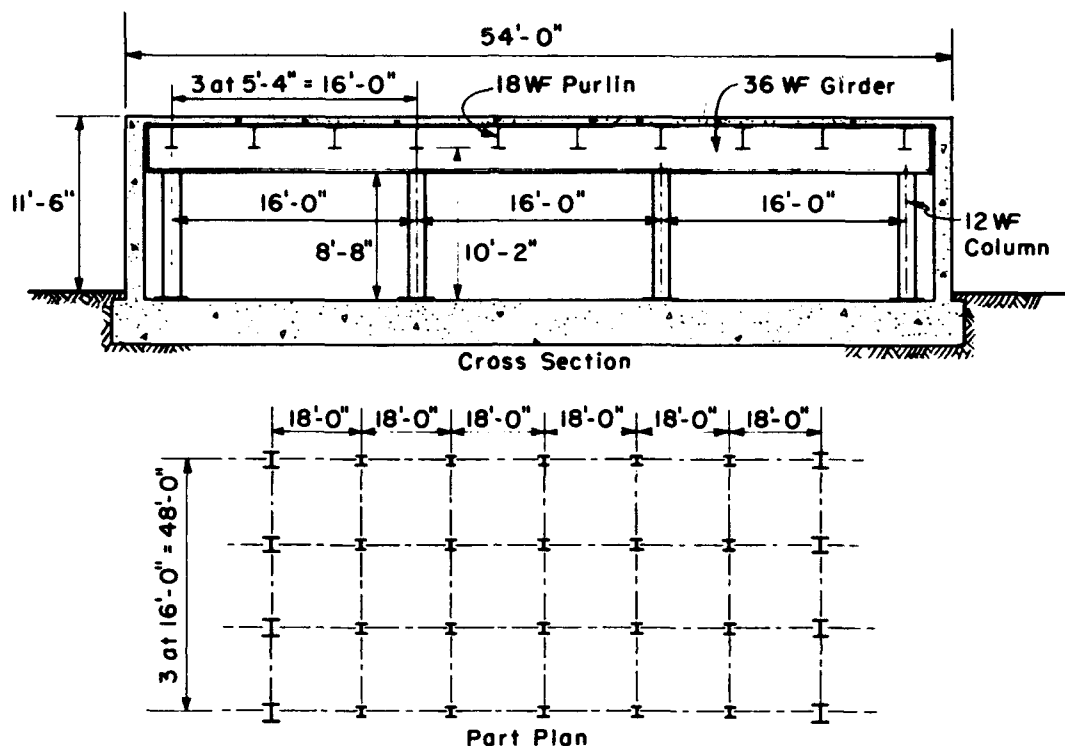


Figure 7.39. Plan and section of building

7-29 DESIGN PROCEDURE. The steps in the design procedure are as follows:

Step 1. Estimate the sizes of all structural elements which determine the over-all dimensions of the building.

Step 2. Compute and plot the time relationship for the incident overpressure, the front face overpressure, the rear face overpressure, the net lateral overpressure, and the average roof overpressure in accordance with EM 1110-345-413 procedures.

Step 3. Design the wall slab using the procedure of paragraph 6-12 and an idealized triangular load vs time curve derived from the front face overpressure vs time curve. Check the design using the actual time variation of front face overpressure in a numerical integration. From this analysis obtain the dynamic reaction on the roof slab.

Step 4. Follow the same procedure as for step 3 for the roof slab and the purlin design. In both preliminary designs the incident overpressure is idealized by a triangular load-time curve. The numerical integration check of the preliminary roof slab is performed using the incident overpressure curve. The numerical integration check of the purlin is based on the dynamic reactions from the slab giving due consideration to the effect of blast wave direction on the purlin loads.

Step 5. Make a preliminary design of the column assuming the girders to be infinitely rigid and neglecting the effect of axial load on the column. As in all other preliminary designs, an idealized load-time curve is used. This load is obtained from the net lateral overpressure-time curve.

Step 6. Design the girder using an idealized load derived from the purlin dynamic reactions and check by a numerical integration using the purlin dynamic reactions directly.

Step 7. Check the column design by a numerical integration considering the relative flexibility of the column and girder and the effect of axial load on the column. For the load use the dynamic reactions from the wall slab.

7-30 LOAD DETERMINATION. The computation of loads is explained in EM 1110-345-413 and illustrated again in paragraph 7-19 for a one-story building. In this example the necessary load curves are presented without any explanation or computation. The design overpressure of 10 psi is selected arbitrarily for this example.

The overpressure vs time curves that are presented are:

- (1) Incident overpressure vs time (fig. 7.40)
- (2) Front face overpressure vs time (fig. 7.41)
- (3) Rear face overpressure vs time (fig. 7.42)
- (4) Net lateral overpressure vs time (fig. 7.43)
- (5) Average roof overpressure vs time (fig. 7.44)

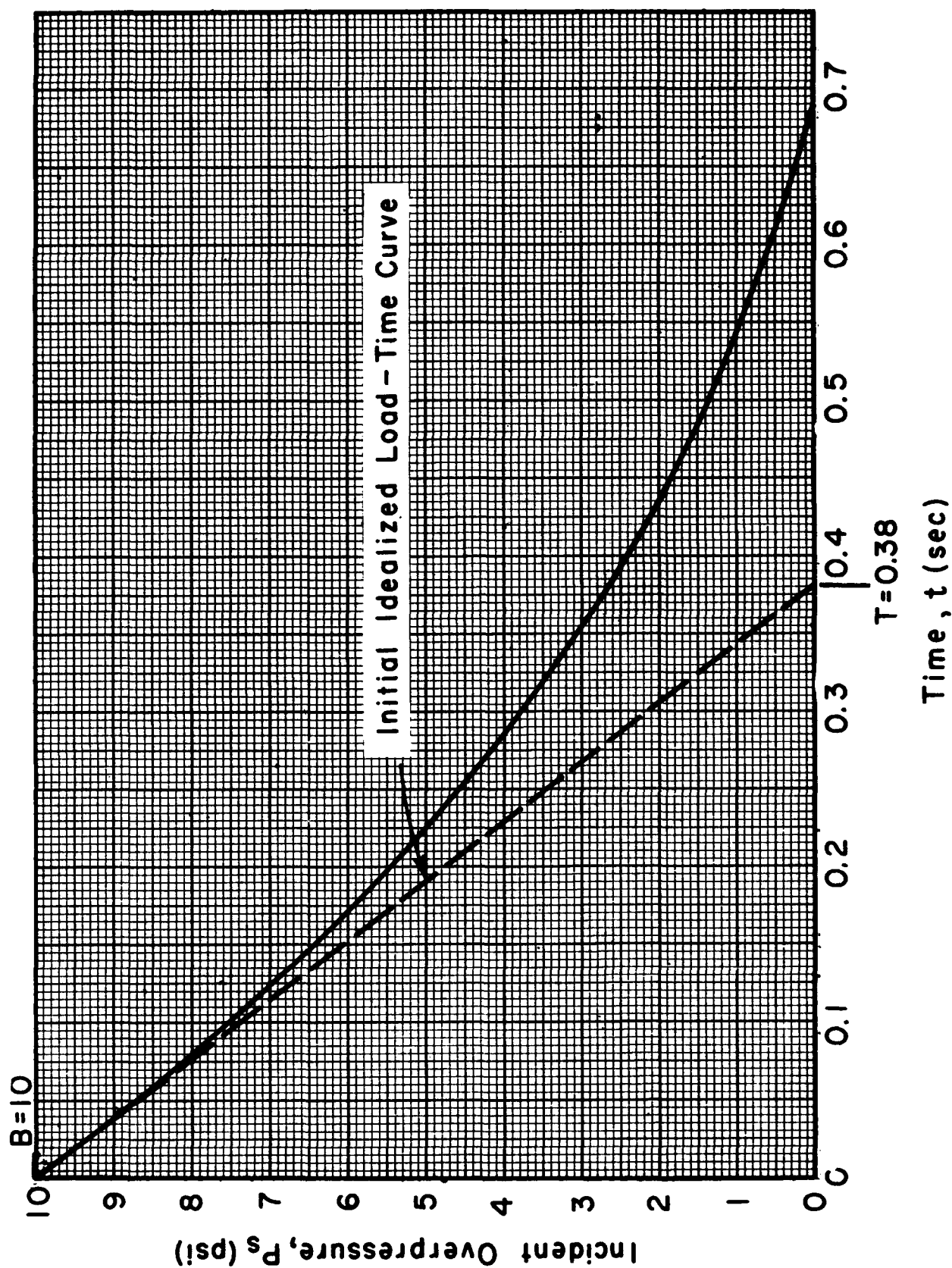


Figure 7.40. Incident overpressure vs time curve

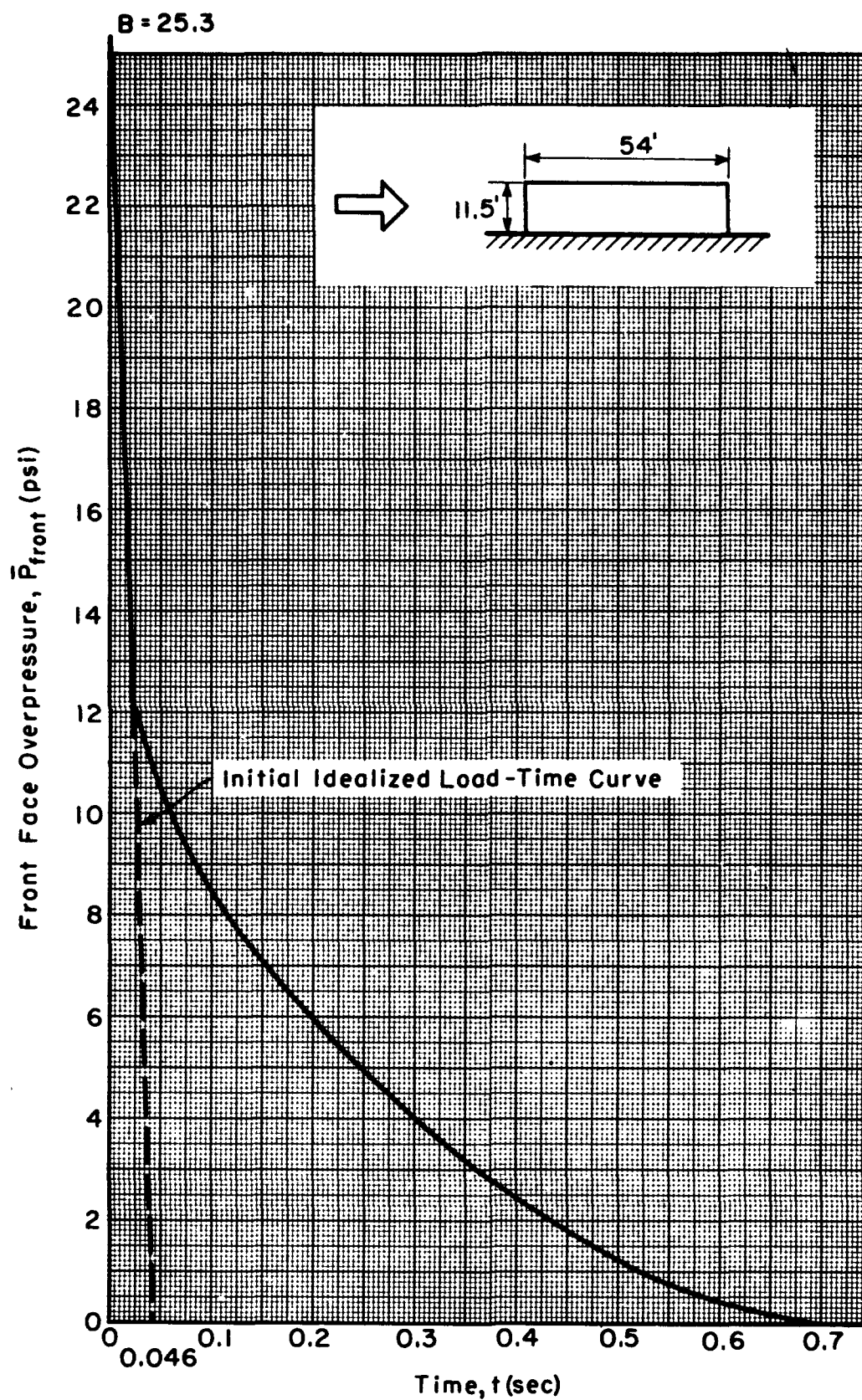


Figure 7.41. Front face overpressure vs time curve



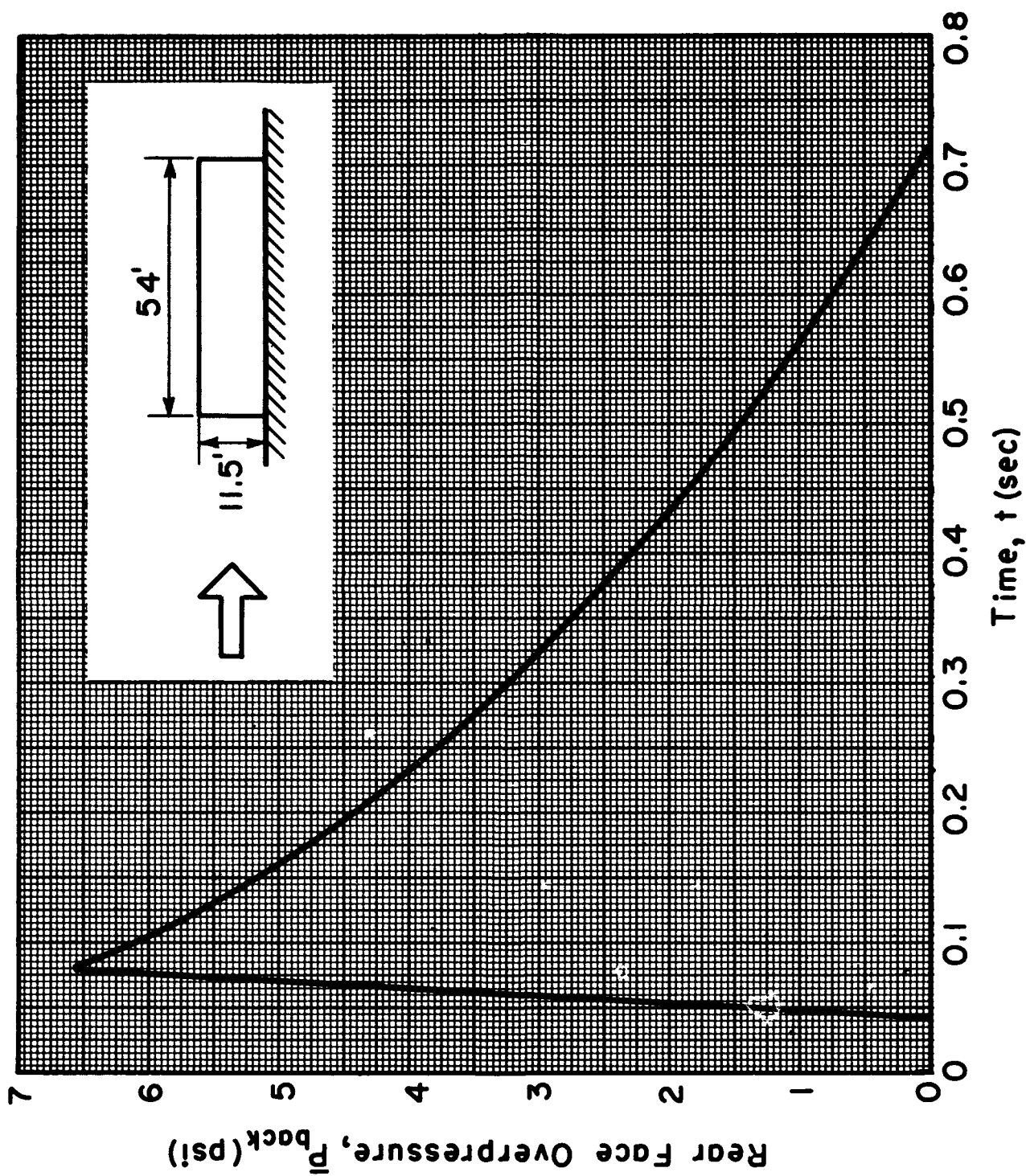


Figure 7.42. Rear face overpressure vs time curve

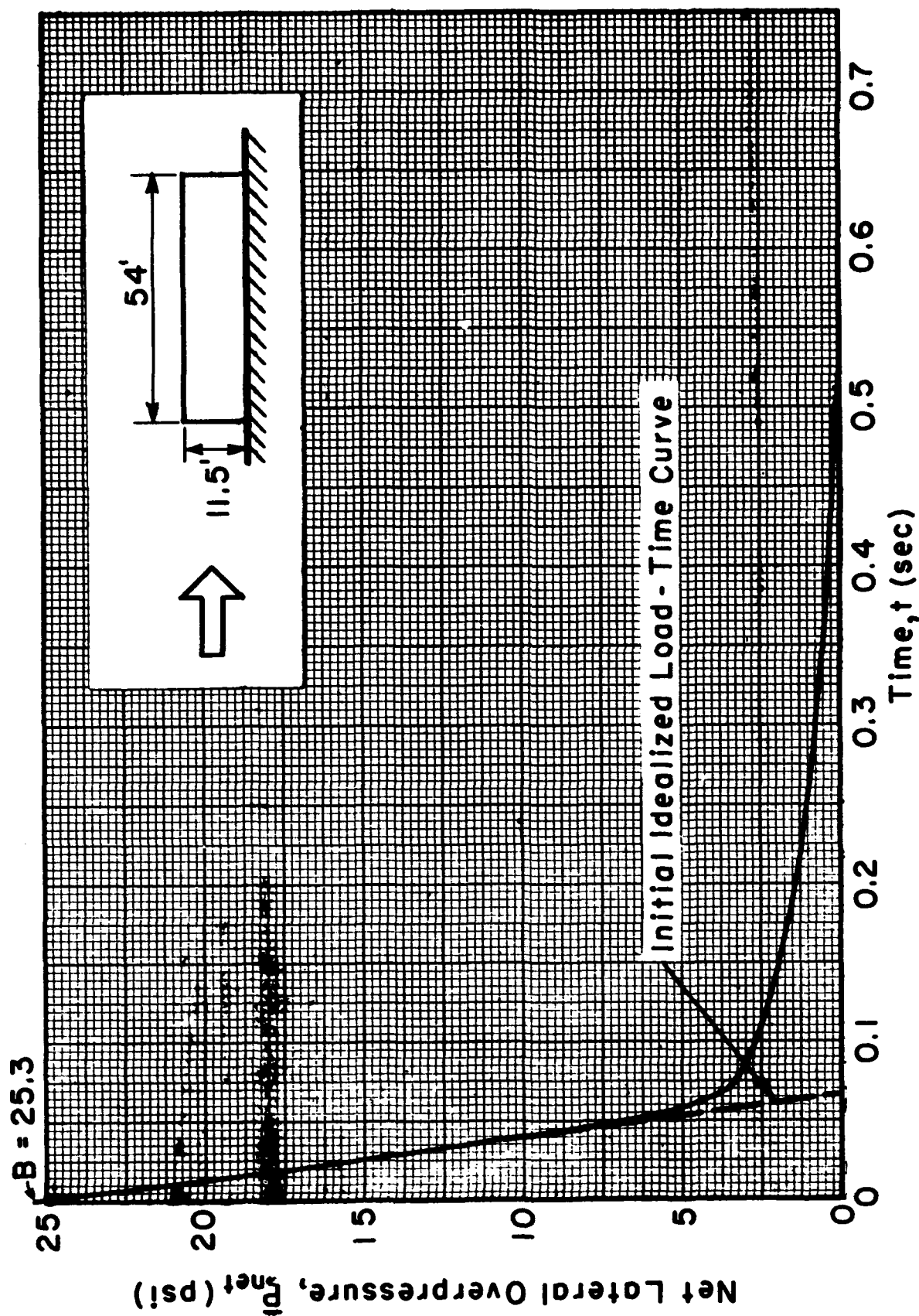


Figure 7.43. Net lateral overpressure vs time curve

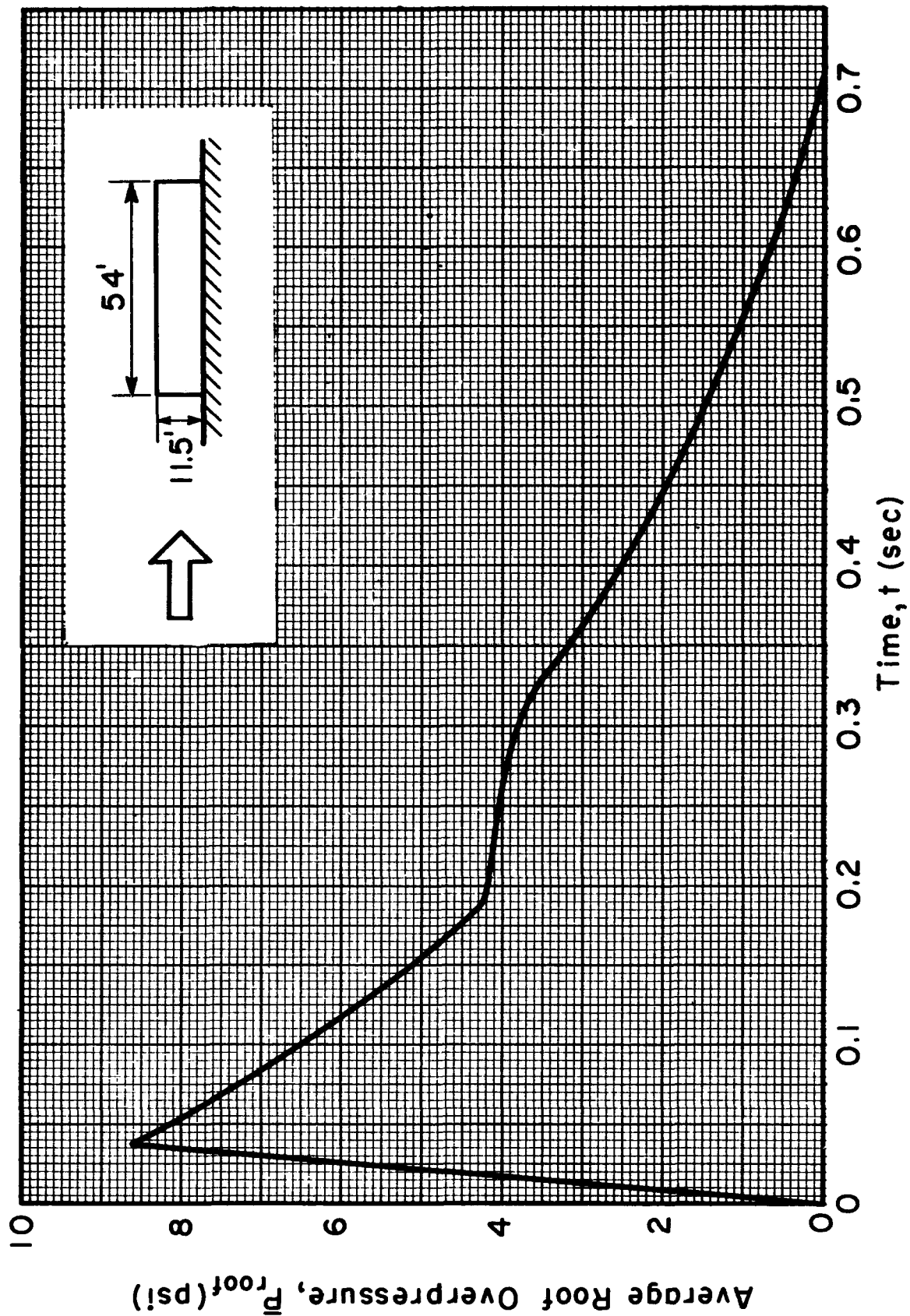
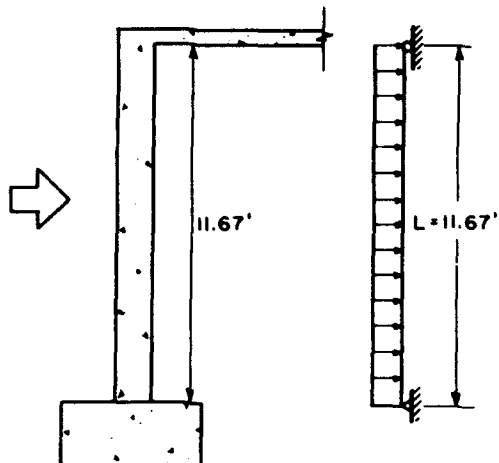


Figure 7.44. Average roof overpressure vs time curve

7-31 DESIGN OF WALL SLAB. The wall slab is designed as a simple beam, one foot in width, spanning vertically between the foundation and the roof slab. The slab is designed so that a plastic hinge will be developed at midspan when the slab is subjected to the design loading. This strain condition is considered to be the limit of the elastic range for purposes of this manual.



The preliminary elastic design procedure is outlined and then illustrated by an example in paragraph 6-12. The dead load stresses are neglected for a vertical wall. The slab thickness is selected so as to illustrate a procedure for designing slabs which are critical for shear stress. To reduce the shear stress a thicker slab might be used.

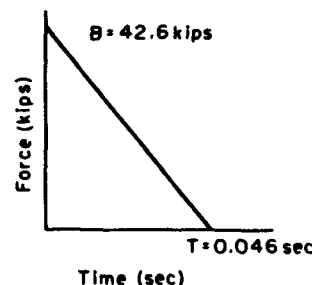
The design span length is equal to the clear height of the wall between foundation and roof.

a. Design Loading. The design load as idealized from the computed loading shown by figure 7.41 is defined by:

$$B = 25.3 \text{ psi} = \frac{25.3(11.67)(144)}{1000}$$

$$= 42.6 \text{ kips}$$

$$T = 0.046 \text{ sec}$$



b. Elastic Range Dynamic Design Factors. (Refer to table 6.1.)

$$K_L = 0.64,$$

$$K_M = 0.50,$$

$$K_{LM} = 0.78$$

$$R_m = \frac{8M_P}{L},$$

$$k = \frac{384EI}{5L^3},$$

$$V = 0.39R + 0.11P$$

c. First Trial - Actual Properties.

Assume D.L.F. = 1.7 (experience)

$$R_m = \text{D.L.F.}(B) = 1.7(42.6) = 72.5 \text{ kips (par. 6-11)}$$

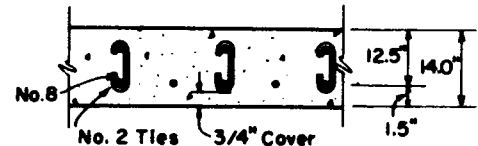
Assume  $p = 0.015$

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$$M_P = p f_{dy} b d^2 \left[ 1 - \frac{p f_{dy}}{1.7 f'_{dc}} \right] \quad (\text{eq 4.16})$$

$$= 0.015(52)(1)d^2 \left[ 1 - \frac{0.015(52)}{1.7(3.9)} \right] = 0.688d^2 \text{ kip-ft (d in inches)}$$

$$R_m = \frac{8M_P}{L} = \frac{(8)0.688d^2}{11.67} = 72.5, \therefore d = 12.4 \text{ in.}$$

Try  $h = 14.0 \text{ in.}$ , $d = 12.5 \text{ in.}$ , $p = 0.015$ , $np = 0.15$ 

$$M_P = 0.688d^2 = 0.688(12.5)^2 = 107.5 \text{ kip-ft}$$

$$R_m = \frac{8M_P}{L} = \frac{8(107.5)}{11.67} = 73.7 \text{ kips}$$

$$I_g = bh^3/12 = (14.0)^3 = 2740 \text{ in.}^4$$

$$I_t = bd^3 \left[ \frac{k^3}{3} + np(1 - k)^2 \right] = 12(d)^3 \left[ \frac{(0.42)^3}{3} + 0.15(1 - 0.42)^2 \right]$$

$$= 0.905d^3 = 0.905(12.5)^3 = 1770 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 0.5(2740 + 1770) = 2255 \text{ in.}^4$$

$$k = \frac{384EI}{5L^3} = \frac{(384)3(10^3)2255}{5(11.67)^3(144)} = 2270 \text{ kips/ft}$$

$$\text{Weight} = \frac{14(150)(11.67)}{12(1000)} = 2.04 \text{ kips}$$

$$\text{Mass } m = \frac{2.04}{32.2} = 0.0634 \text{ kip-sec}^2/\text{ft}$$

d. First Trial - Equivalent Properties.

$$k_e = K_L k = 0.64(2270) = 1450 \text{ kips/ft (eq 6.6)}$$

$$m_e = K_M m = 0.50(0.0634) = 0.0317 \text{ kip-sec}^2/\text{ft (eq 6.8)}$$

$$T_n = 2\pi\sqrt{m_e/k_e} = 6.28\sqrt{0.0317/1450} = 0.0293 \text{ sec (eq 6.14)}$$

e. First Trial - Available Resistance vs Required Resistance.

$$C_T = T/T_n = 0.046/0.0293 = 1.57$$

$$\text{D.L.F.} = 1.7 \text{ (fig. 5.20)}$$

$$t_m/T = 0.3 \text{ (fig. 5.20)}$$

$$t_m = 0.3(0.046) = 0.0138 \text{ sec}$$

Idealized load is satisfactory replacement for actual load-time curve up to  $t = 0.0138$  sec (par. 5-13).

$$\text{Required } R_m = \text{D.L.F.}(B) = 1.7(42.6) = 72.5 \text{ kips}$$

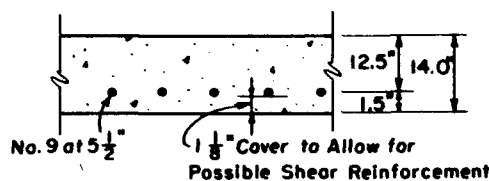
The required  $R_m < \text{available } R_m$ , therefore the selected proportions are satisfactory as a preliminary design.

f. Preliminary Design for Bond Stress.

$$\text{At } t_m = 0.0138 \text{ sec, } P = \frac{(18.5)11.67(144)}{1000} = 31.0 \text{ kips (fig. 7.41)}$$

$$V = 0.39R_m + 0.11P = 0.39(72.5) + 0.11(31.0) = 28.3 + 3.4 = 31.7 \text{ kips}$$

$$\text{Allowable } u = 0.15f'_c = 0.15(3000) = 450 \text{ psi (par. 4-09)}$$



$$\Sigma o = \frac{V}{u j d} = \frac{8(31,700)}{450(7)12.5} = 6.45 \text{ in.}$$

$$\text{Try \#9 at } 5\text{-}1/2 \text{ in., } A = 2.18 \text{ in.}^2$$

$$\Sigma o = 7.72 \text{ in.}$$

$$p = A_s / bd = \frac{2.18}{12(12.5)} = 0.0145, \quad np = 10(0.0145) = 0.145$$

g. Determination of Maximum Deflection and Dynamic Reactions by Numerical Integration.

$$I_g = bh^3/12 = (14.0)^3 = 2740 \text{ in.}^4$$

$$I_t = bd^3 \left[ \frac{k^3}{3} + np(1 - k)^2 \right] = 12(12.5)^3 \left[ \frac{(0.41)^3}{3} + 0.145(1 - 0.41)^2 \right] = 1720 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 0.5(2740 + 1720) = 2230 \text{ in.}^4$$

$$\text{Weight} = \frac{14(150)11.67}{12(1000)} = 2.04 \text{ kips}$$

$$\text{Mass } m = \frac{2.04}{32.2} = 0.0634 \text{ kip-sec}^2/\text{ft}$$

$$\begin{aligned} M_p &= pf_{dy} bd^2 \left[ 1 - \frac{pf_{dy}}{1.7f'_{dc}} \right] \\ &= (0.0145)(52)(1)(12.5)^2 \left[ 1 - \frac{0.0145(52)}{1.7(3.9)} \right] \\ &= 104.5 \text{ kip-ft (eq 4.16)} \end{aligned}$$

$$\begin{aligned} R_m &= \frac{8M_p}{L} - \text{weight} = \frac{8(104.5)}{11.67} - 2.04 \\ &= 71.6 - 2.04 = 69.6 \text{ kips} \end{aligned}$$

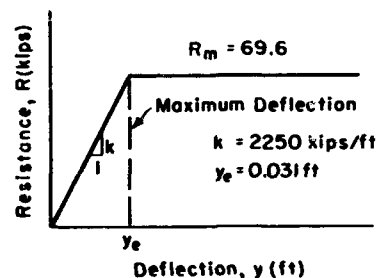


Figure 7.45. Resistance function for 14-in. slab spanning 11.67 ft as a simple beam

$$k = \frac{384EI}{5L^3} = \frac{(384)3(10)^3 2230}{5(11.67)^3 144} = 2250 \text{ kips/ft}$$

$$y_e = R_m/k = \frac{69.6}{2250} = 0.031 \text{ ft}$$

Table 7.16. Determination of Maximum Deflection and Dynamic Reactions for Front Wall Slab

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> - R <sub>n</sub> (kips)	y <sub>n</sub> (Δt) <sup>2</sup> (ft)	y <sub>n</sub> (ft)	V <sub>n</sub> (kips)
0	42.6	0	21.3	0.00388	0	4.9
0.003	39.5	8.7	30.8	0.00561	0.00388	7.7
0.006	36.6	30.1	6.5	0.00118	0.01337	15.7
0.009	33.7	54.1	-20.4	-0.00371	0.02404	24.8
0.012	30.8	69.6	-38.8	-0.00834	0.03100*	30.1
0.015	27.8	66.7	-38.9	-0.00834	0.02962	29.0
0.018	25.0	24.8	+0.2	+0.00004	0.0199	12.4

\* (y<sub>n</sub>)<sub>max</sub> = 0.031 ft.

The basic equation for the numerical integration in table 7.16 is  
 $y_{n+1} = y_n(\Delta t)^2 + 2y_n - y_{n-1}$  (table 5.3) where

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(\Delta t)^2}{K_{IM}(m)} = \frac{(P_n - R_n)(0.003)^2}{0.78(0.0634)} = 1.818(10^{-4})(P_n - R_n) \text{ ft}$$

The time interval  $\Delta t = 0.003$  sec is approximately equal to  $T_n/10$  (par. 5-08). The dynamic reaction equation is given in paragraph 7-31b. The  $P_n$  values for column 2 are obtained from figure 7.41, multiplying by  $144(11.67)/1000 = 1.68$  to obtain load in kips.

The maximum deflection  $(y_n)_{\max}$ , computed in table 7.16, is 0.031 ft which is equal to the allowable  $y_m$  of 0.031 ft.

#### h. Shear Strength and Bond Stress.

$$V_{\max} = 30.1 \text{ kips (table 7.16)}$$

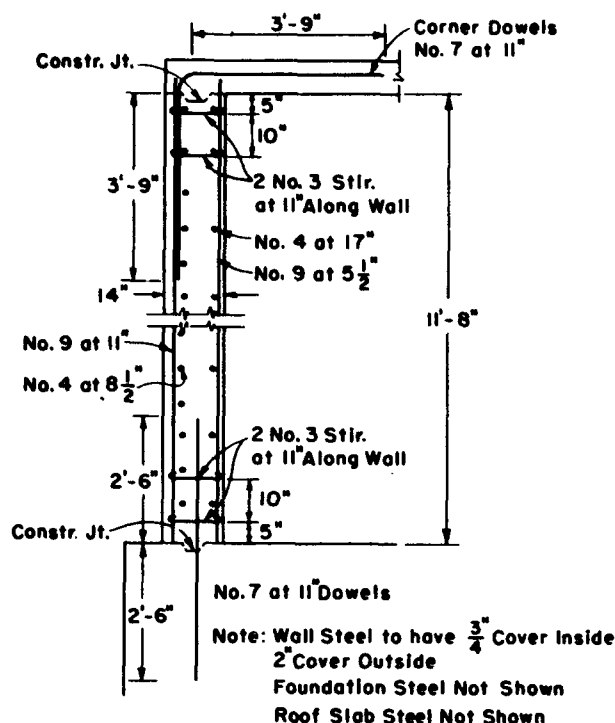
For no shear reinforcement

$$\text{Allowable } v_y = 0.04f'_c + 5000p \text{ (eq 4.24)}$$

$$v_y = 0.04(3000) + 5000(0.0145) = 120 + 72.5 = 192.5 \text{ psi}$$

$$v = \frac{8V}{7bd} = \frac{8(30.1)}{7(12)(12.5)} = 229 \text{ psi, therefore shear reinforcement re-}$$

quired for  $229 - 192.5 = 37$  psi.



$$r = \frac{37}{40,000} = 0.001$$

$$\text{Try 1 \#3, } A_s = 0.11$$

$$r = A_s / bs = \frac{0.11}{11s} = 0.001,$$

$$\therefore s = 10 \text{ in.}$$

$$u = \frac{8V}{\Sigma o d} = \frac{8(30,100)}{7(7.72)12.5}$$

$$= 356 \text{ psi; OK}$$

$$\text{Allowable } u = 0.15f'_c \\ = 0.15(3000) = 450 \text{ psi}$$

#### 1. Summary.

14-in. slab

$$p = 0.0145$$

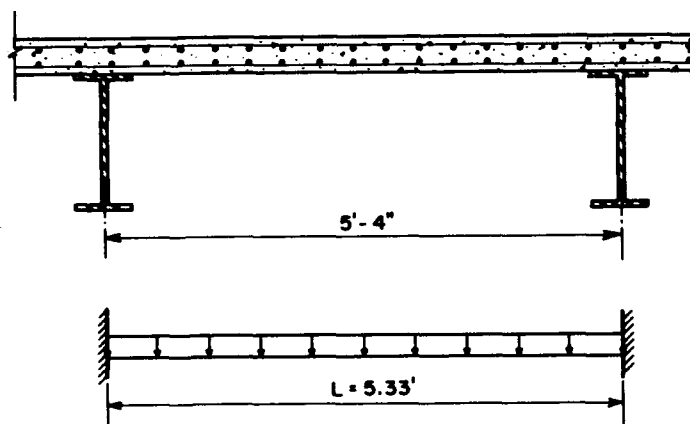
$$\Sigma o = 7.72 \text{ in.}$$

Shear reinforcement #3 ties

at 10-in. alternate bars.

7-32 DESIGN OF ROOF SLAB. The design of the roof slab is similar to the design of the wall slab in paragraph 7-31 except for the load considerations and the inclusion of dead load stresses.

This roof slab is supported on purlins spaced at 5 ft 4 in. so as to load the girder at the third points. The design load is based on the incident overpressure curve (fig. 7.40) which results from the blast wave moving normal to the span direction of the slab.



The load is uniformly distributed along the slab and varying with time. Adjacent slab elements are loaded progressively; however, the shock wave speed is such that the entire slab may be considered to be loaded at once.

The slab is continuous over the purlins; however, it is designed as a single span fixed at both ends according to the procedures of paragraph 7-11. The span length of the beam is the centerline spacing of the purlins. The design is based on a one-foot width of slab.

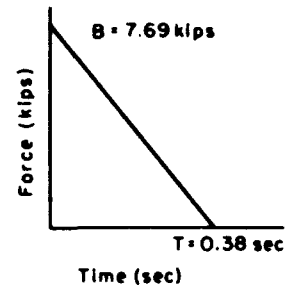


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a. Design Loading. The design load as idealized from the computed loading shown by figure 7.40 is defined by:

$$B = 10 \text{ psi} = \frac{10(144)5.33}{1000} = 7.69 \text{ kips}$$

$$T = 0.38 \text{ sec}$$



b. Elastic Range Dynamic Design Factors. (Refer to table 6.1.)

$$K_L = 0.53,$$

$$K_M = 0.41,$$

$$K_{LM} = 0.77$$

$$R_m = \frac{12M_P}{L},$$

$$k = \frac{384EI}{L^3}$$

$$V = 0.36R_n + 0.14P_n$$

c. First Trial - Actual Properties.

Assume D.L.F. = 2.0 (experience)

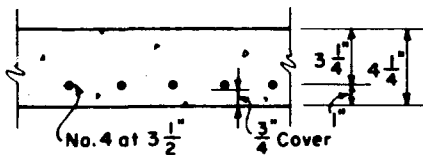
$$R_m = \text{D.L.F.}(B) = 2.0(7.69) = 15.38 \text{ kips (par. 6-11)}$$

Assume  $p = 0.015$

$$M_P = pf_{dy}bd^2 \left( 1 - \frac{pf_{dy}}{1.7f'_{dc}} \right) \text{ (eq 4.16)}$$

$$= 0.015(52)(1)d^2 \left[ 1 - \frac{0.015(52)}{1.7(3.9)} \right] = 0.688d^2 \text{ kip-ft (d in inches)}$$

$$R_m = \frac{12M_P}{L} = \frac{(12)0.688d^2}{5.33} = 15.38, \therefore d = 3.14 \text{ in.}$$



Try  $h = 4\text{-}1/4 \text{ in.}$ ,  $d = 3\text{-}1/4 \text{ in.}$ ,

$p = 0.015$ ,  $np = 0.15$

$$M_P = 0.688d^2 = 0.688(3.25)^2 = 7.26 \text{ kip-ft}$$

$$R_m = \frac{12M_P}{L} = \frac{12(7.26)}{5.33} = 16.35 \text{ kips}$$

$$I_g = bh^3/12 = (4.25)^3 = 76.5 \text{ in.}^4$$

$$I_t = bd^3 \left[ \frac{k^3}{3} + np(1 - k)^2 \right] = 12(d)^3 \left[ \frac{(0.42)^3}{3} + 0.15(1 - 0.42)^2 \right]$$

$$= 0.905d^3 = 0.905(3.25)^3 = 31.0 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 0.5(31.0 + 76.5) = 53.7 \text{ in.}^4$$

$$k = \frac{384EI}{L^3} = \frac{(384)3(10^3)53.7}{(5.33)^3 144} = 2840 \text{ kips/ft}$$

$$\text{Weight} = \left[ \frac{4.25(150)}{12} + 6.0 \right] \frac{5.33}{1000} = 0.314 \text{ kips}$$

$$\text{Mass } m = \frac{0.314}{32.2} = 0.00975 \text{ kip-sec}^2/\text{ft}$$

d. First Trial - Equivalent Properties.

$$k_e = K_L k = 0.53(2840) = 1505 \text{ kips/ft (eq 6.6)}$$

$$m_e = K_M m = 0.41(0.00975) = 0.004 \text{ kip-sec}^2/\text{ft (eq 6.8)}$$

$$T_n = 2\pi \sqrt{m_e/k_e} = 6.28 \sqrt{0.004/1505} = 0.01025 \text{ sec (eq 6.14)}$$

e. First Trial - Available Resistance vs Required Resistance.

$$C_T = T/T_n = 0.38/0.01025 = 37.0$$

$$\text{D.L.F.} = 2.0 \text{ (fig. 5.20)}$$

$$t_m/T \approx 0 \text{ (fig. 5.20)}$$

$$t_m \approx 0 \text{ sec}$$

$$\text{Required } R_m = \text{D.L.F.}(B) = 2.0(7.69) = 15.38 \text{ kips}$$

The required  $R_m \approx$  available  $R_m$ , therefore the selected proportions are satisfactory as a preliminary design.

f. Preliminary Design for Bond Stress.

$$\text{At } t_m = 0, P = \frac{(10)144(5.33)}{1000} = 7.69 \text{ kips (fig. 7.40)}$$

$$V = 0.36R_m + 0.14P = 0.36(16.35) + 0.14(7.69) = 5.9 + 1.08 = 6.98 \text{ kips}$$

$$\text{Allowable } u = 0.15f'_c = 0.15(3000) = 450 \text{ psi (par. 4-09)}$$

$$\Sigma o = \frac{V}{u_j d} = \frac{8(6980)}{450(7)(3.25)} = 5.45 \text{ in.}$$

$$\text{Try \#4 at } 3\text{-}1/2 \text{ in., } A_s = 0.69 \text{ in.}^2, \Sigma o = 5.4 \text{ in.}$$

$$p = A_s/bd = \frac{0.69}{12(3.25)} = .0176, \quad np = 10(.0176) = 0.176$$

g. Determination of Maximum Deflection and Dynamic Reactions by

Numerical Integration.

$$I_g = bh^3/12 = (4.25)^3 = 76.5 \text{ in.}^4$$

$$k = \sqrt{n^2 p^2 + 2np} - np = 0.444$$

$$I_t = bd^3 \left[ \frac{k^3}{3} + np(1 - k)^2 \right] = 12(3.25)^3 \left[ \frac{0.444^3}{3} + 0.176(1 - 0.444)^2 \right]$$

$$= 34.5 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 0.5(76.5 + 34.5) = 55.5 \text{ in.}^4$$

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$$\text{Weight} = 0.314 \text{ kips}$$

$$\text{Mass } m = \frac{0.314}{32.2} = 0.00975 \text{ kip-sec}^2/\text{ft}$$

$$M_P = pf_{dy} bd^2 \left[ 1 - \frac{pf_{dy}}{1.7f'_{dc}} \right]$$

$$= 0.0176(52)(1)(3.25)^2 \left[ 1 - \frac{0.0176(52)}{1.7(3.9)} \right]$$

$$= 835 \text{ kip-ft (eq 4.16)}$$

$$R_m = \frac{12M_P}{L} - \text{weight} = \frac{12(8.35)}{5.33} - 0.314 = 18.5 \text{ kips}$$

$$k = \frac{384EI}{L^3} = \frac{384(3)(10)^3 55.5}{5.33^3 (144)} = 2930 \text{ kips/ft}$$

$$y_e = R_m/k = \frac{18.5}{2930} = 0.0063 \text{ ft}$$

The basic equation for the numerical integration in table 7.17 is  $y_{n+1} = \ddot{y}_n(\Delta t)^2 + 2y_n - y_{n-1}$  (table 5.3) where

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(\Delta t)^2}{K_{IM}(m)} = \frac{(P_n - R_n)10^{-6}}{0.77(0.00975)} = 1.33 (10^{-4})(P_n - R_n) \text{ ft}$$

The time interval  $\Delta t = 0.001 \text{ sec}$  is approximately equal to  $T_n/10 = 0.001025$  (par. 5-08). The dynamic reaction equations are listed in

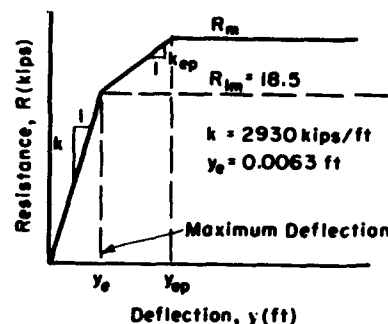


Figure 7.46. Resistance function for a 4-1/4-in. continuous slab spanning 5.33 ft

Table 7.17. Determination of Maximum Deflection and Dynamic Reactions for Roof Slab

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> - R <sub>n</sub> (kips)	y <sub>n</sub> (Δt) <sup>2</sup> (ft)	y <sub>n</sub> (ft)	V <sub>n</sub> (kips)
0	7.69	0	3.84	0.00051	0	1.08
0.001	7.67	1.49	6.18	0.00082	0.00051	1.61
0.002	7.65	5.39	2.26	0.00030	0.00184	3.01
0.003	7.63	10.17	-2.54	-0.00034	0.00347	4.73
0.004	7.61	13.95	-6.34	-0.00084	0.00476	6.08
0.005	7.59	15.27	-7.68	-0.00102	0.00521*	6.56
0.006	7.57	13.60	-6.03	-0.00080	0.00464	5.96
0.007	7.54	9.58			0.00327	4.50
0.008	7.51					

\* (y<sub>n</sub>)<sub>max</sub> = 0.0052 ft.

paragraph 7-32b. The  $P_n$  values for the second column are obtained from figure 7.40, multiplying by  $[144(5.33)(1)]/1000 = 0.769$  to obtain load in kips.

The maximum deflection  $(y_n)_{\max}$ , computed in table 7.17, is 0.0052 ft. This is less than  $y_e = 0.0063$  ft. Design is satisfactory.

h. Shear Strength and Bond Stress.

$$V_{\max} = 6.56 \text{ kips (table 7.17)}$$

For no shear reinforcement

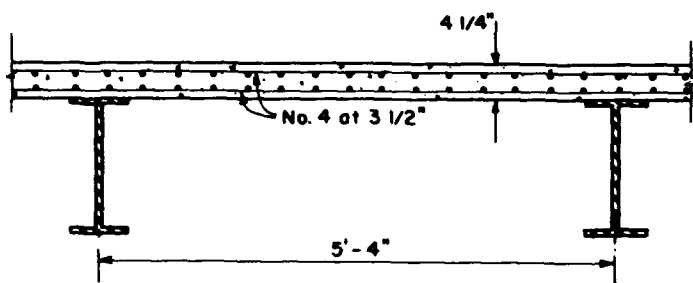
$$\text{Allowable } v_y = 0.04f'_c + 5000p \text{ (eq 4.24)}$$

$$v_y = 0.04(3000) + 5000(0.0176) = 120 + 88 = 208 \text{ psi}$$

$$v = \frac{8V}{7bd} = \frac{8(6560)}{7(12)(3.25)} = 192 \text{ psi; OK, no shear reinforcement required}$$

$$u = \frac{8V}{7\Sigma o d} = \frac{8(6560)}{7(5.4)(3.25)} = 428 \text{ psi}$$

$$\text{Allowable } u = 0.15f'_c = 0.15(3000) = 450 \text{ psi; OK for bond}$$



i. Summary.

4-1/4-in. slab

$$p = 0.0176$$

$$\Sigma o = 5.4 \text{ in.}$$

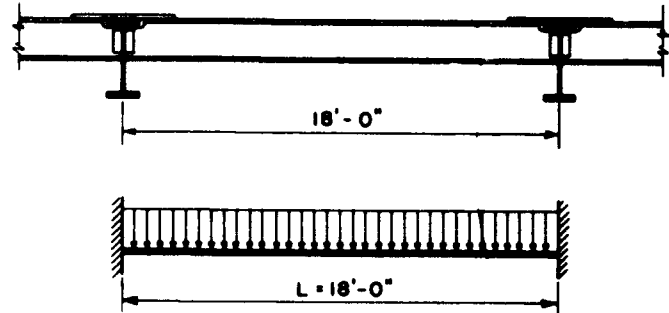
No shear reinforcement

7-33 DESIGN OF ROOF PURLINS. The purlins are framed flush with the top of the girders and are provided with moment-resisting connections. The top flanges of the purlins are anchored to the concrete roof slab to provide lateral support to the top compression flange.

Although composite behavior of the slab and purlin can be expected to develop to a limited extent, preliminary computations showed that design for independent behavior of slab and purlin is more desirable for this arrangement of slab and purlin (pars. 4-12 and 6-23). Accordingly, this purlin design neglects composite behavior.

The purlins are designed to pass into the elasto-plastic range, but not into the plastic range. This means that the deflection is limited to that necessary to just develop the plastic hinges at midspan. This point

is indicated in figure 7.50 (page 130). Since the design procedures of this manual provide only for single-span elements, this purlin is designed as a fixed-end beam spanning 18 ft between girder centerlines.



a. Loading. For the blast wave moving normal to the long axis of the building and thus normal to the axis of the purlin, the loading may be considered to be uniformly distributed along the length of the purlin. For this condition the pressure vs time variation at each point on the roof is a function of its position (par. 3-09). In addition the load on a purlin is a function of the length of the slab spans between purlins because the load on the purlin increases as the blast wave traverses the two adjoining slabs. In the preliminary design of the purlins the design load is obtained from the incident overpressure curve (fig. 7.40) without modification of any sort. The rise time of the load on the slab, the slab dynamic reactions, and local variation in overpressure on the slab are all neglected in this preliminary step.

For the blast wave moving parallel to the long axis of the building and thus parallel to the axis of the purlin, the load varies along the span as a result of the time required for the blast wave to traverse the purlin span. At any point along the purlin the time variation of the load is the same and defined by the incident overpressure vs time curve (fig. 7.40).

In the calculations that follow the load vs time curves for the purlin are obtained first for the blast wave moving parallel to the long axis of the building and then for the blast wave moving parallel to the short axis of the building.

The purlin obtained by the preliminary design procedure is then analyzed for both loads in tables 7.19 and 7.20 (pages 124 and 126).

Blast wave moving parallel to the short axis of the building:

The variation of the local overpressure at the centerlines of purlins (A) and (B) during the critical time period for the purlins and girders of this building is only slightly different from the incident

overpressure curve (fig. 7.40). The detailed computations to show this are not presented but are similar to the computations in paragraph 7-23a. Reference to the example in paragraph 7-23 and comparison of the critical time values of that example with the results of the following numerical analysis (table 7.18) show that the incident overpressure is a satisfactory basis for determining the purlin loading.

Table 7.18. Determination of Dynamic Reactions for Roof Slab, Local Roof Overpressure at Centerline of Purlin (A) (Including Rise Time Correction)

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> - R <sub>n</sub> (kips)	y <sub>n</sub> (Δt) <sup>2</sup> (ft)	y <sub>n</sub> (ft)	V <sub>n</sub> (kips)
0	0	0	0.31	0.000041	0	0
0.001	2.01	0.12	1.89	0.000251	0.000041	0.32
0.002	4.03	0.98	3.05	0.000406	0.000333	0.91
0.003	6.04	3.02	3.02	0.000402	0.001031	1.94
0.004	7.64	6.24	1.40	0.000186	0.002131	3.32
0.005	7.62	10.01	-2.39	-0.000318	0.003417	4.67
0.006	7.60	12.85	-5.25	-0.000698	0.004385	5.69
0.007	7.57	13.64	-6.07	-0.000807	0.004655	5.97
0.008	7.55	12.07	-4.52	-0.000601	0.004118	5.41
0.009	7.53	8.73	-1.20	-0.000160	0.002980	4.19
0.010	7.51	4.93			0.001682	3.76
0.020	7.31					3.65
0.030	7.11					3.55
0.040	6.91					3.45

The roof slab is analyzed in table 7.18 for the modified incident overpressure curve presented in figure 7.47. The analysis in table 7.18 is based on the following data developed in paragraph 7.32.

$$\text{Elastic range: } \ddot{y}_n(\Delta t)^2 = 1.33(10^{-4})(P_n - R_n) \text{ ft}$$

$$k = 2930 \text{ kips/ft}$$

$$y_e = 0.0063 \text{ ft}$$

$$R_m = 18.5 \text{ kips}$$

$$V_n = 0.36R_n + 0.14P_n$$

This computation is performed to obtain the slab dynamic reactions on the purlins. The dynamic reactions of the slab for the overpressure variation at purlins (A) and (B) are the same for the time range considered, because

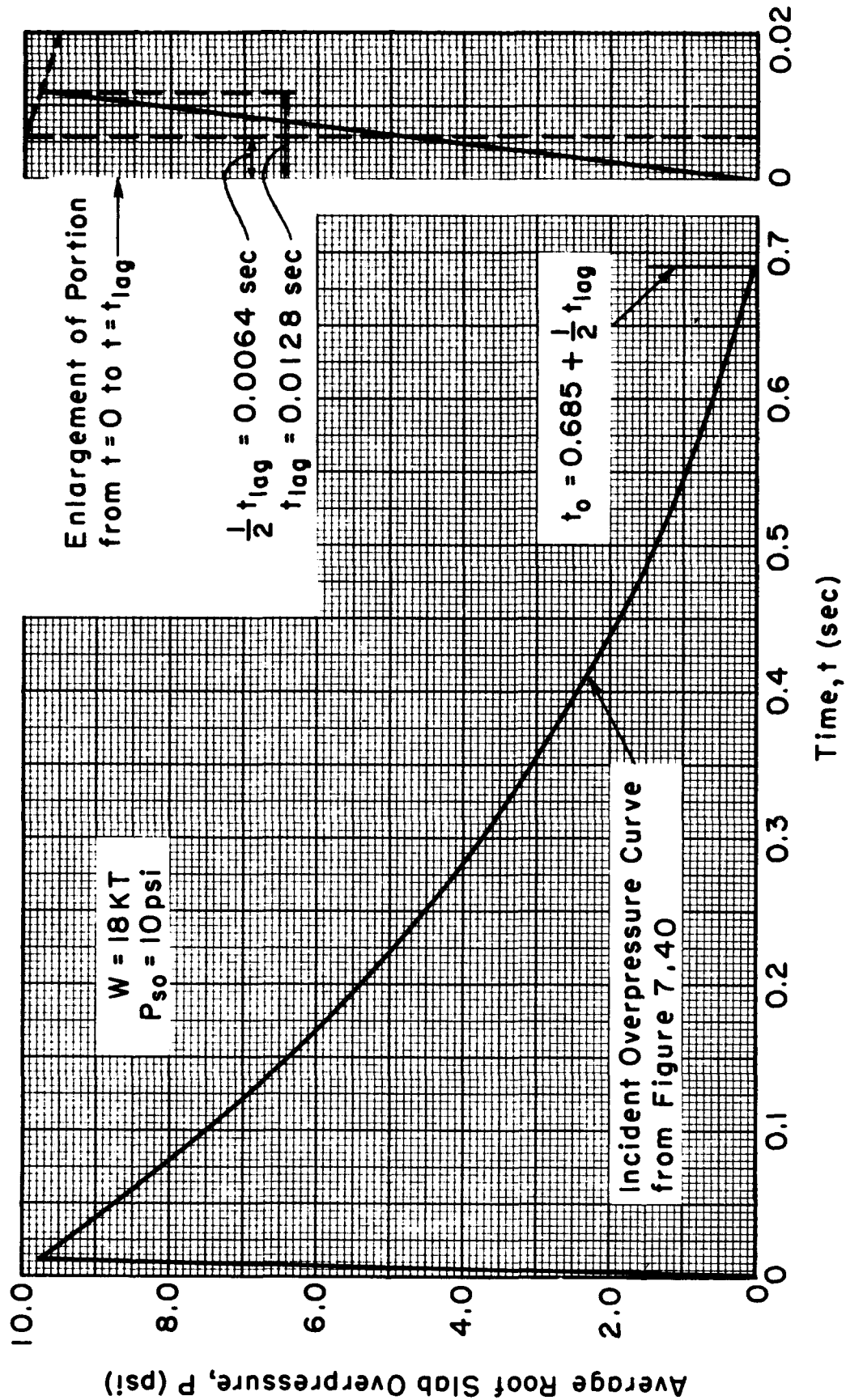


Figure 7.47. Incident overpressure curve modified for time of rise

the maximum response of the slab occurs before there is any appreciable difference in the overpressure at points (A) and (B).

To obtain the design load for purlins (A) and (B) the dynamic reaction data from table 7.18 are plotted in figure 7.48. The total purlin load is equal to the sum of the reactions of the slabs forward and aft of the purlin. In figure 7.48 it may be seen that the same dynamic reactions are plotted with a time lag

$$t_{\text{lag}} = \frac{5.33}{1403} = 0.0038 \text{ sec}$$

The loads from figure 7.48 are used in table 7.19 to check the preliminary purlin design.

Table 7.19. Determination of Maximum Deflection and Dynamic Reactions for Purlins, Blast Wave Perpendicular to Purlin Axis

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> - R <sub>n</sub> (kips)	y <sub>n</sub> (Δt) <sup>2</sup> (ft)	y <sub>n</sub> (ft)	V <sub>n</sub> (kips)
0	0	0	5.6	0.00033	0	0
0.003	36.0	2.3	33.7	0.00198	0.00033	5.9
0.006	120.6	18.7	101.9	0.00600	0.00264	23.6
0.009	162.0	77.7	84.3	0.00496	0.01095	50.7
0.012	162.0	155.6	6.4	0.00037	0.02422	78.5
0.015	133.2	175.0	-41.8	-0.00243	0.03786	82.9
0.018	132.3	190.9	-58.6	-0.00341	0.04907	89.0
0.021	131.0	202.0	-71.0	-0.00413	0.05687	93.2
0.024	129.6	205.0	-75.4	-0.00525	0.06054*	93.4
0.027	128.3	193.8	-65.5	-0.00386	0.05896	87.7
0.030	126.9	155.2	-28.3	-0.00167	0.05352	73.6
0.033	125.6	104.7			0.04641	62.8
0.036	124.2					62.1
0.039	122.9					61.4
*(y <sub>n</sub> ) <sub>max</sub> = 0.060 ft.						

Blast wave moving parallel to the long axis of the building:

The design load on the purlin is determined in figure 7.49 using the slab dynamic reactions obtained in table 7.17 for incident overpressure. The variation of the average load on the purlin with time is found by introducing the time lag



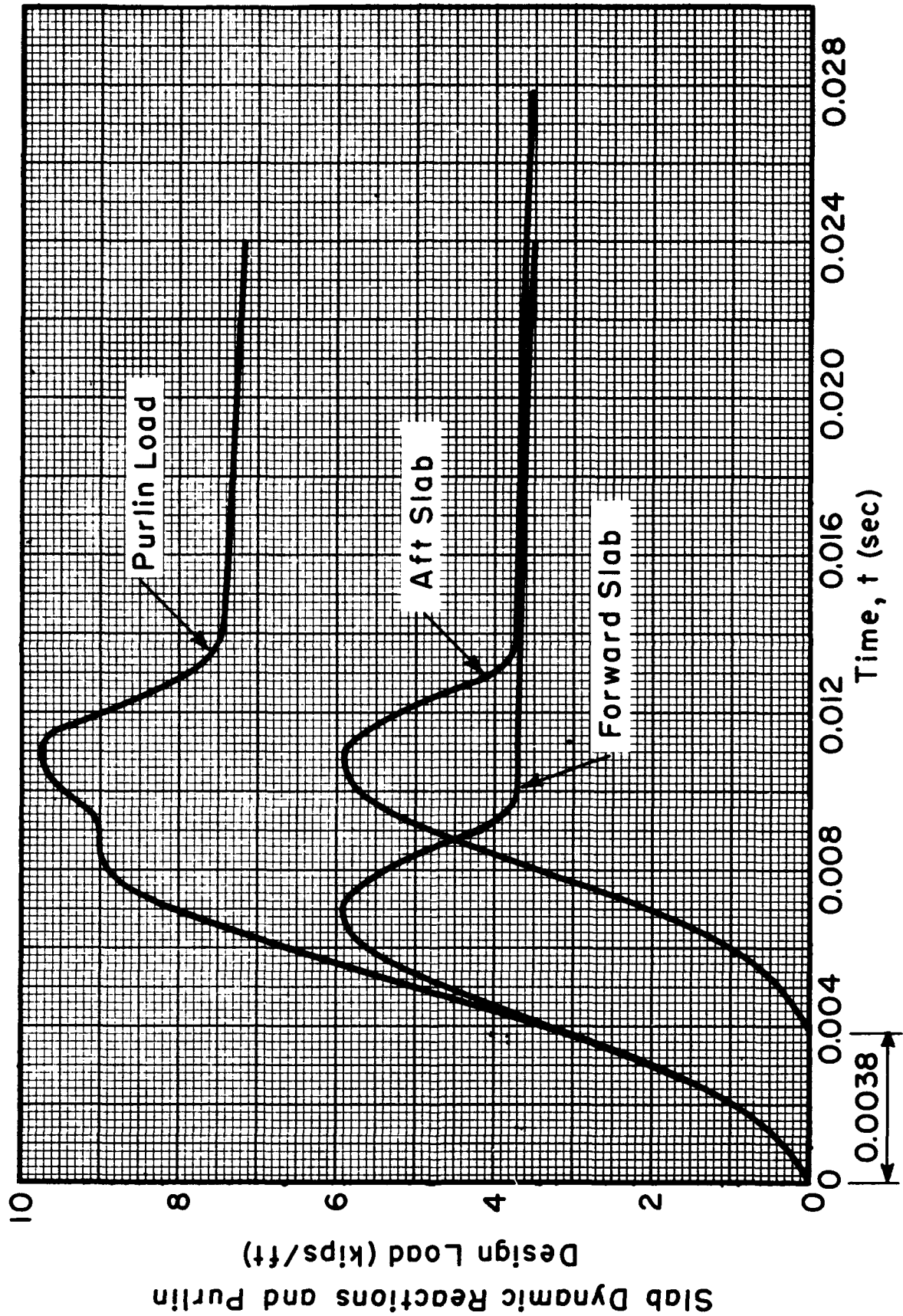


Figure 7.48. Purlin design load for local roof overpressure, blast wave moving perpendicular to purlin axis

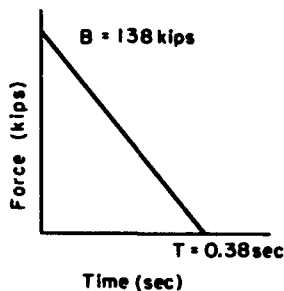
$$t_{\text{lag}} = \frac{18}{1403} = 0.0128 \text{ sec}$$

because the variation in slab dynamic reaction with time is the same at each point along the purlin. The load curve from figure 7.49 is used in table 7.20 to check the preliminary purlin design.

Table 7.20. Determination of Maximum Deflection and Dynamic Reactions for Purlins, Blast Wave Parallel to Purlin Axis

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> -R <sub>n</sub> (kips)	y <sub>n</sub> (Δt) <sup>2</sup> (ft)	y <sub>n</sub> (ft)	V <sub>n</sub> (kips)
0	0	0	3.1	0.00018	0	0
0.003	19.8	1.3	18.5	0.00109	0.00018	3.2
0.006	72.0	10.3	61.7	0.00363	0.00145	13.8
0.009	108.0	45.1	62.9	0.00370	0.00635	31.4
0.012	140.4	106.1	34.3	0.00202	0.01495	57.9
0.015	158.4	157.5	0.9	0.00009	0.02557	78.8
0.018	144.0	172.7	-28.7	-0.00167	0.03628	83.2
0.021	133.9	185.6	-51.7	-0.00301	0.04532	87.1
0.024	133.6	194.1	-60.5	-0.00352	0.05135	90.4
0.027	133.2	197.7	-64.5	-0.00375	0.05386*	91.8
0.030	132.8	188.9	-56.1	-0.00330	0.05262	86.6
*(y <sub>n</sub> ) <sub>max</sub> = 0.054 ft.						

The preliminary design load as idealized from the computed loading as shown by figure 7.40 is defined by



$$B = 10 \text{ psi} = \frac{10(144)5.33(18)}{1000} = 138 \text{ kips}$$

$$T = 0.38 \text{ sec}$$

b. Dynamic Design Factors. (Refer to table 6.1.)

Elastic range:

$$K_L = 0.53,$$

$$K_M = 0.41,$$

$$K_{LM} = 0.77$$

$$R_{lm} = \frac{12M_{Ps}}{L},$$

$$k_1 = \frac{384EI}{L^3}$$

$$V = 0.36R + 0.14P$$

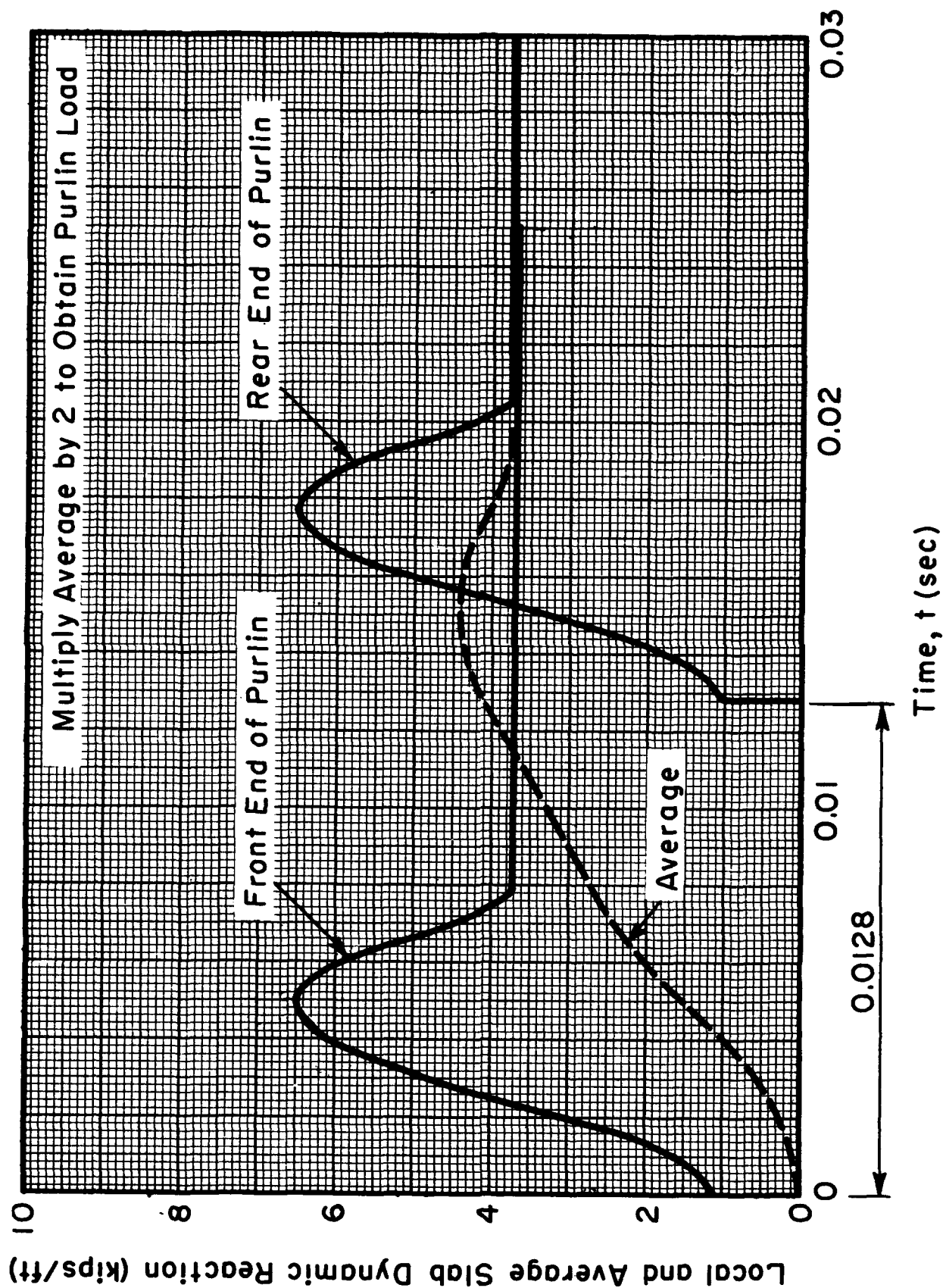


Figure 7.49. Purlin design load for incident overpressure, blast wave moving parallel to purlin axis

Elasto-plastic range:

$$K_L = 0.64, \quad K_M = 0.50, \quad K_{IM} = 0.78$$

$$R_m = \frac{8}{L} (M_{Ps} + M_{Pm}), \quad k_{ep} = \frac{384EI}{5L^3}$$

$$V = 0.39R + 0.11P$$

Average values:

$$K_L = (0.53 + 0.64)0.5 = 0.58$$

$$K_M = (0.41 + 0.50)0.5 = 0.45$$

$$K_{IM} = (0.77 + 0.78)0.5 = 0.77$$

$$R_m = \frac{22M_P}{L}$$

$$k_E = \frac{264EI}{L^3}$$

c. First Trial - Actual Properties.

$$\text{Let } M_{Ps} = M_{Pm} = M_P$$

Assume D.L.F. = 2.0 and neglect dead load stresses (experience)

$$R_m = \text{D.L.F.}(B) = 2.0(138) = 276 \text{ kips (par. 6-11)}$$

$$M_P = 1.05Sf_{dy} = \frac{1.05(41.6)S}{12} = 3.64S \text{ kip-ft (S in in.}^3\text{) (eq 4.2a)}$$

$$R_m = \frac{22M_P}{L} = \frac{(22)3.64S}{18} = 276, \therefore S = 62 \text{ in.}^3$$

$$\text{Try 16 W40, } b/t_f = 7.0/0.503 = 13.9 < 14; \text{ OK, } S = 64.4 \text{ in.}^3$$

$$I = 515.5 \text{ in.}^4, \quad Z = 72.7 \text{ in.}^3, \quad 0.5(S + Z) = 68.5 \text{ in.}^3$$

$$M_P = 0.5f_{dy}(S + Z) = \frac{41.6(68.5)}{12} = 237 \text{ kip-ft}$$

$$R_m = \frac{22M_P}{L} = \frac{22(237)}{18} = 290 \text{ kips}$$

$$k = \frac{264EI}{L^3} = \frac{(264)30(10^3)515.5}{18^3(144)} = 4870 \text{ kips/ft}$$

$$\text{Weight} = \left\{ \left[ \frac{4.25(150)}{12} + 6.0 \right] 5.33 + 40 \right\} \frac{18}{1000} = 6.4 \text{ kips}$$

$$\text{Mass } m = \frac{6.4}{32.2} = 0.1985 \text{ kip-sec}^2/\text{ft}$$

d. First Trial - Equivalent Properties.

$$k_e = K_L k = 0.58(4870) = 2820 \text{ kips/ft (eq 6.6)}$$

$$m_e = K_M m = 0.45(0.1985) = 0.0894 \text{ kip-sec}^2/\text{ft} \text{ (eq 6.8)}$$

$$T_n = 2\pi\sqrt{m_e/k_e} = 6.28\sqrt{0.0894/2820} = 0.0353 \text{ sec (eq 6.14)}$$

e. First Trial - Available Resistance vs Required Resistance.

$$C_T = T/T_n = 0.38/0.0353 = 10.8$$

$$\text{D.L.F.} = 1.95 \text{ (fig. 5.20)}$$

$$t_m/T = 0.05 \text{ (fig. 5.20)}$$

$$t_m = 0.05(0.38) = 0.019 \text{ sec}$$

Idealized load-time curve is satisfactory up to  $t_m$  (fig. 7.40)

$$\text{Required } R_m = \text{D.L.F.}(B) = 1.95(138) = 269 \text{ kips}$$

The required  $R_m < \text{available } R_m$ ; therefore the selected proportions are satisfactory as a preliminary design.

f. Preliminary Design for Shear Stress.

$$\text{At } t_m = 0.019 \text{ sec, } P = \frac{9.5(5.33)18(144)}{1000} = 131 \text{ kips (fig. 7.40)}$$

$$V = 0.36R + 0.11P = 0.36(269) + 0.11(131) = 111.4 \text{ kips}$$

$$v = \frac{V}{dt_w} = \frac{111,400}{(16)(0.307)} = 22,700 \text{ psi (allowable } v = 21,000 \text{ psi)(par. 4-05c)}$$

Since  $V = 111.4$  kips is estimated it is desirable to continue with investigation of 16 W40.

g. Determination of Maximum Deflection and Dynamic Reactions by Numerical Integration.

$$M_P = 0.5f_{dy}(S + Z) = 237 \text{ kip-ft}$$

$$\text{Weight} = 6.4 \text{ kips}$$

$$\text{Mass } m = 0.1985 \text{ kip-sec}^2/\text{ft}$$

Elastic range:

$$R_{lm} = \frac{12M_P}{L} - \text{weight} = \frac{12(237)}{18} - 6.4 = 151.6 \text{ kips}$$

$$k_1 = \frac{384EI}{L^3} = \frac{(384)30(10)^3 515.5}{18^3(144)} = 7100 \text{ kips/ft}$$

$$y_e = \frac{R_{lm}}{k_1} = \frac{151.6}{7100} = 0.0214 \text{ ft}$$

$$k_e = K_L k_1 = 0.53(7100) = 3760 \text{ kips/ft}$$



Allowable  $v = 21,000$  psi; OK (par. 4-05c)

i. Check Proportions for Local Buckling. (par. 4-06d)

$$16 \text{ W } 40, \quad b = 7.0, \quad t_f = 0.503$$

$$a = 15.0, \quad t_w = 0.307$$

$$b/t_f = 7.0/0.503 = 13.9 < 14.0; \text{ OK}$$

$$a/t_w = 15.0/0.307 = 49.0 > 30; \text{ NG}$$

Longitudinal stiffeners are required

$$t_s = 3/8 > 0.307$$

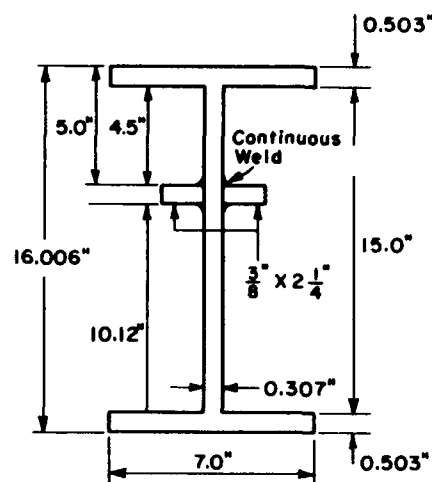
$$b_s/t_s = 6; \quad b_s = 6t_s = 6(3/8) = 2.25 \text{ in.}$$

Use 2 plates  $3/8$  in. by  $2-1/4$  in. full length welded continuously as described in paragraph 4-06d.

$$c/t_w = (4.5)/0.307 = 14.6 < 22; \text{ OK}$$

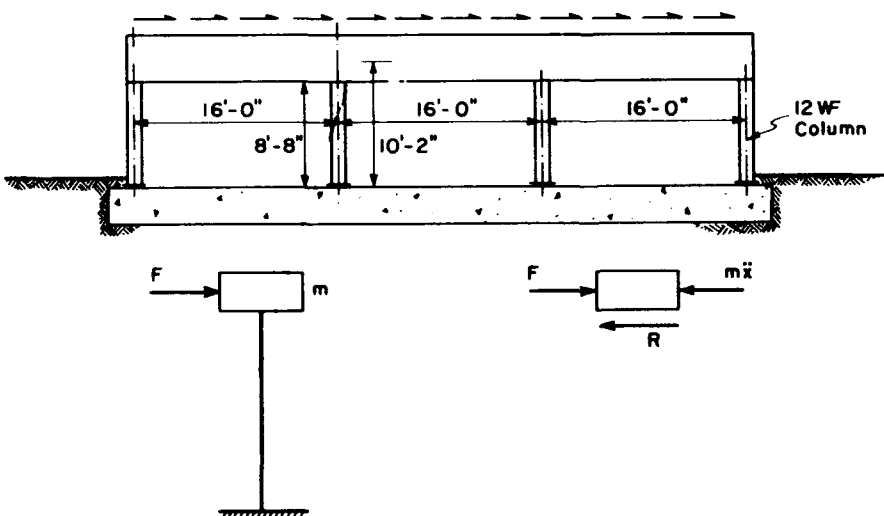
$$e/t_w = (10.12)/0.307 = 33 < 40; \text{ OK}$$

Weight of added stiffener plates = 5.7 lb/ft



7-34 PRELIMINARY DESIGN OF COLUMNS. A single-story frame subject to lateral load behaves essentially as a single-degree-of-freedom system with the column displaying the spring properties. It is therefore unnecessary to substitute an equivalent system for the original structure, and the mass and load factors that are necessary in the design of beams and slabs are not used in the design of single-story frames.

The preliminary elastic design procedure of paragraph 6-12 is used to determine the preliminary column size. The equations of paragraph 7-06



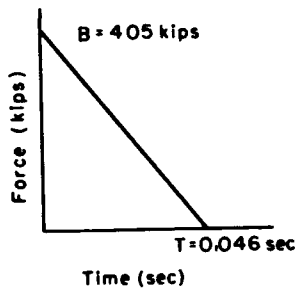
are used in the procedure of paragraph 6-12 to replace some of the factors used in determining equivalent single-degree-of-freedom systems.

For purposes of preliminary design the frame girders are assumed to be

infinitely rigid thus simplifying the determination of the column spring constant. For spring constant computation the effective column height is 10.17 ft based on an assumed girder depth of 3 ft and clear height of 8.67 ft. The clear height is used in determining the maximum resistance of the columns (par. 7-06).

In the preliminary design of steel columns it is desirable to be conservative to allow for the factors which are neglected in the preliminary design. These factors are: (1) the effect of direct stress on the plastic hinge moment and (2) the effect of girder flexibility.

a. Design Loading. The design lateral load on the frame is obtained from the dynamic reactions at the top of the front wall slab. However, for the preliminary design procedure it is satisfactory to use the net lateral overpressure curve (fig. 7.43). In this case the total concentrated lateral load on the frame is the product of  $(18)(6.18)\bar{P}_{net}$ . The effective wall height determining the frame load is obtained from the wall clear height and roof slab thickness



$$h' = \frac{11.67}{2} + 0.35 = 6.18$$

The design load as idealized from the computed loading shown by figure 7.43 is defined by

$$B = 25.3 \text{ psi} = \frac{25.3(144)18(6.18)}{1000} = 405 \text{ kips}$$

$$T = 0.046 \text{ sec}$$

b. Mass Computation.

$$\text{Walls } \frac{14(11.67)2(18)150}{12(1000)} = 73.5 \text{ kips}$$

$$\text{Roof slab } \left[ \frac{4.25(150)}{12} + 6 \right] \frac{18(54)}{1000} = 57.3 \text{ kips}$$

$$\text{Purlins } \frac{40(18)10}{1000} = 7.2 \text{ kips}$$

$$\text{Girders (estimated) } \frac{150(49)}{1000} = 7.4 \text{ kips}$$

$$\text{Columns (estimated) } \frac{150(8.67)4}{1000} = 5.2 \text{ kips}$$

$$\text{Connections} = 0.1(7.2 + 7.4) = 1.4 \text{ kips (10\% of girders and purlins)}$$



Single-degree-of-freedom-system mass = total roof + girder +

$$\begin{aligned} 1/3 \text{ (walls and columns)} &= \frac{57.3 + 7.2 + 7.4 + 1.4 + 0.33(73.5 + 5.2)}{32.2} \\ &= \frac{99.5}{32.2} = 3.09 \text{ kip-sec}^2/\text{ft} \end{aligned}$$

c. First Trial - Actual Properties.

Assume D.L.F. = 1.2 (experience)

$$R_m = \text{D.L.F.}(B) = 1.2(405) = 486 \text{ kips}$$

$$\text{Required } M_D = R_m h / 2n = 486(8.67) / 2(4) = 527 \text{ kip-ft (eq 7.14)}$$

The effect of axial column load is neglected

$$M_D = f_{dy} \frac{(S + Z)}{2} \text{ (eq 4.2)}$$

$$\frac{S + Z}{2} = \frac{M_D}{f_{dy}} = \frac{527(12)}{41.6} = 152 \text{ in.}^3$$

Smallest column that satisfies buckling criteria is 12 WF 120

$$S = 163.4 \text{ in.}^3, Z = 186.0 \text{ in.}^3, I = 1071.7 \text{ in.}^4, 0.5(S + Z) = 174.5 \text{ in.}^3$$

$$A = 35.31 \text{ in.}^2$$

$$\begin{aligned} a/t_w &= 10.91/0.71 = 15.4 < 22, \text{ OK; } b/t_f = 12.32/1.106 \\ &= 11.1 < 14.0, \text{ OK (par. 4-06d)} \end{aligned}$$

$$M_D = 0.5(S + Z)f_{dy} = (174.5)41.6/12 = 605 \text{ kip-ft}$$

$$R_m = \frac{2nM_D}{h} = \frac{2(4)605}{8.67} = 558 \text{ kips}$$

d. First Trial - Determination of D.L.F.

$$k = 12EI_n/h^3 = \frac{12(30)10^3(1071.7)^4}{144(10.17)^3} = 10,200 \text{ kips/ft (eq 7.10)}$$

$$T_n = 2\pi\sqrt{m/k} = 6.28\sqrt{3.09/10,200} = 0.1095 \text{ sec}$$

$$T/T_n = 0.046/0.1095 = 0.42$$

$$\text{D.L.F.} = 1.07, \quad t_m/T = 0.93 \text{ (fig. 5.20)}$$

$$t_m = 0.93(0.046) = 0.043 \text{ sec}$$

The original load-time curve should be revised to obtain a closer approximation to the total impulse up to time  $t_m$ . The impulse up to  $t = 0.05 \text{ sec}$  in figure 7.43 is:

$H = 0.82$  psi-sec (obtained by graphical integration)

$T = 2H/B = 2(0.82)/25.3 = 0.065$  sec

$T/T_n = 0.065/0.1095 = 0.593$

D.L.F. = 1.3,  $t_m/T = 0.70$  (fig. 5.20)

$t_m = 0.7(0.065) = 0.0455 < 0.05$

The revised idealized load-time curve is satisfactory

Required  $R_m = \text{D.L.F.}(B) = 1.3(405) = 527$  kips  $< 558$  kips; OK

There is no need for further trials. The required  $R_m$  of 527 kips is greater than original estimate of 486 kips but less than  $R$  available from 12 W 120 (558 kips). Therefore the 12 W 120 is accepted for preliminary design.

e. Shear Stress Check of 12 W 120.

Estimated  $V_{\max} = \frac{R_m}{4} = \frac{558}{4} = 139.5$  kips

$v = \frac{V}{t_w d} = \frac{139,500}{0.710(13.12)} = 15,000$  psi

Allowable  $v = 21,000$  psi; OK (par. 4-05c)

f. Slenderness Criterion for Beam - Columns. (par. 4-08) An approximate evaluation of the 12 W 120 column slenderness criterion is made before the column size is accepted for final analysis. The criterion is:

$$\left(\frac{M_D}{M_P}\right)\left(\frac{K' L_d}{100bt}\right) + \left(\frac{P_D}{P_P}\right)\left(\frac{K'' L}{15r}\right) < 1.0 \text{ (eq 4.10)}$$

$M_D = \frac{527}{558} (605) = 570$  kip-ft, rough estimate based on ratio of required

$R_m$  to available  $R_m$

$M_P = 605$  kip-ft

$P_P = f_{dy} A = 41.6(35.31) = 1470$  kips

$P_D = \frac{(8.4)144(54)18}{1000(3)} = 3920$  kips, at  $t_m = 0.0455$  sec (fig. 7.43)

$K' = 0.14$  (table 4.1)

$K'' = 0.50$  (table 4.2)

$L = 8.67$  ft (clear height)

$r = 3.13$  in.  $d = 13.12$  in.

$b = 12.32$  in.

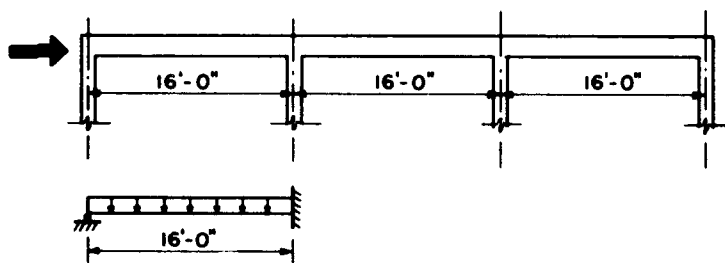
$t_f = 1.106$  in.

Substituting in equation (4.10) gives

$$\left(\frac{570}{605}\right) \left[ \frac{0.14(8.67)12(13.12)}{100(12.32)(1.106)} \right] + \left(\frac{392}{1470}\right) \left[ \frac{0.5(8.67)12}{15(3.13)} \right]$$

$$= 0.132 + 0.284 = 0.416 < 1; \text{ OK}$$

7-35 GIRDER DESIGN. The design procedure for the frame girder is identical to the girder design in paragraph 7-25. To provide continuity in the presentation of this building design example without excessive repetition, this girder design is limited to the computation of the resistance diagram and the numerical integration computation to check the adequacy of the girder resistance. The same preliminary computations performed in paragraph 7-25 have been performed for this girder but are omitted for simplicity of presentation. The girder is designed as a fixed-pinned beam. A 36 WF150 girder is selected by the preliminary computations.



The design load curve (fig. 7.51) is presented. The method of determining the load curve from purlin reactions is the same as described in paragraph 7-25. The dynamic reaction data are obtained from table 7.19.

a. Determination of Maximum Deflection by Numerical Integration.

Based on the preliminary computations which are omitted to reduce duplication the numerical integration analysis is performed for a 36 WF150 girder.

$$S = 503 \text{ in.}^3, \quad Z = 580 \text{ in.}^3, \quad I = 9012 \text{ in.}^4$$

$$\text{Girder } M_P = f_{dy} [(S + Z)/2] = 1875 \text{ kip-ft (eq 4.2)}$$

The static loads develop a moment at the fixed support

$$\text{Static load moment} = 40 \text{ kip-ft}$$

The moment in the girder to resist lateral frame motion is

$$\frac{2}{3} M_P \text{ of column} = \frac{2}{3} (605) = 403 \text{ kip-ft (pars. 7-34e and 7-11)}$$

$$R_{lm} = \frac{6M}{L} = \frac{6}{16} (1875 - 40 - 403) = 538 \text{ kips}$$

$$k = 132 \frac{EI}{L^3} = \frac{132(30)10^3(9012)}{16^3(144)} = 60,400 \text{ kips/ft}$$

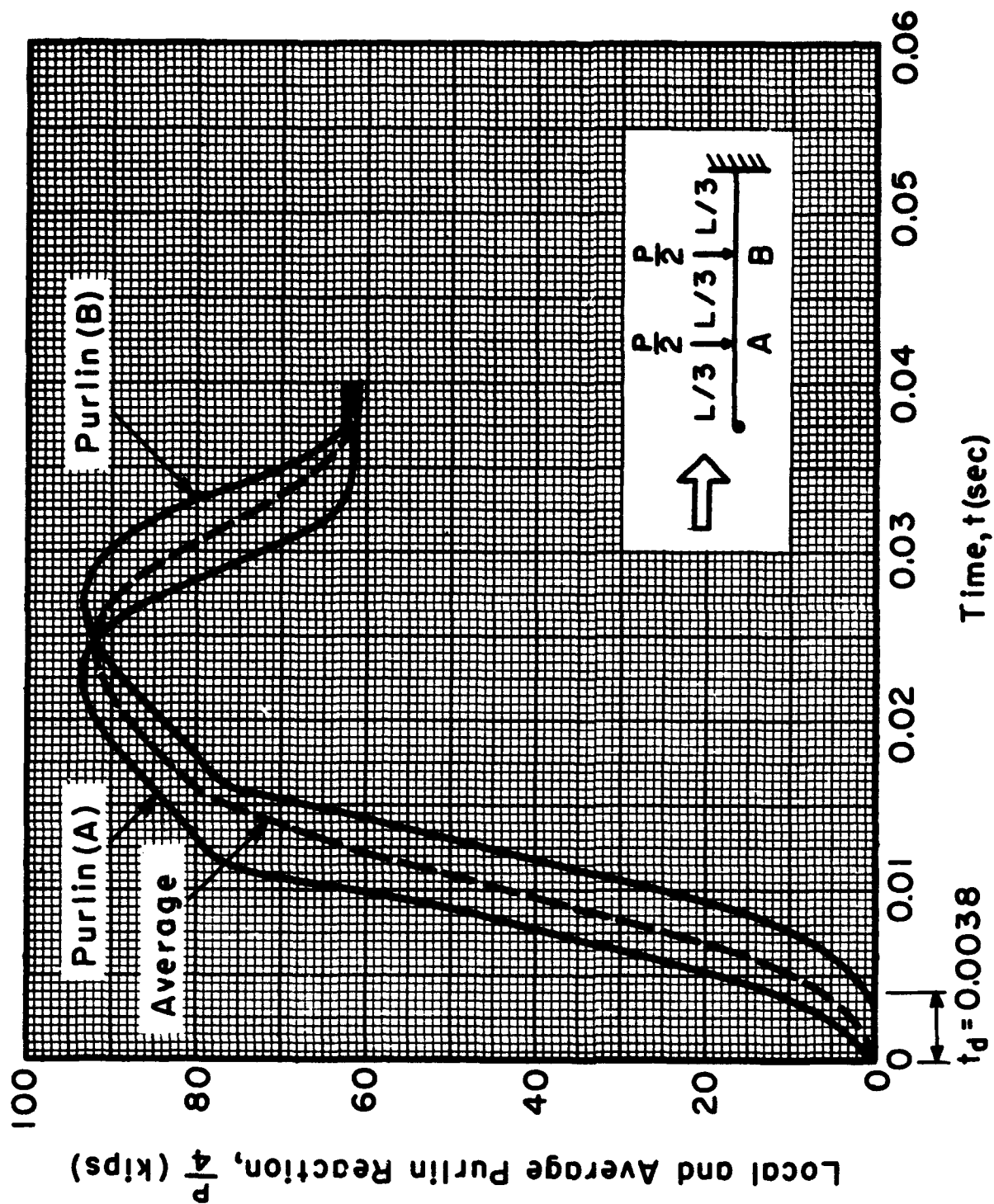


Figure 7.51. Girder design load

The resistance diagram is shown by figure 7.52.

Total concentrated mass = 0.40 kip-sec<sup>2</sup>/ft

Total uniform mass = 0.075 kip-sec<sup>2</sup>/ft

Concentrated load and mass,  $K_{IM} = 0.83$

(table 6.1)

Concentrated load and uniform mass,

$K_{IM} = 0.55$  (table 6.1)

$$T_n = 2\pi \left[ \frac{\Sigma K_{IM}(m)}{k} \right]^{1/2} = 0.0156 \text{ sec}$$

Use  $\Delta t = 0.0015 \text{ sec} < \frac{T_n}{10} = 0.00156$  (par. 5-08)

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(0.0015)^2}{0.40(0.83) + 0.55(0.075)} = 6.01(10^{-6})(P_n - R_n) \text{ ft}$$

The numerical integration in table 7.21 is based on the equation  $y_{n+1} = \ddot{y}_n(\Delta t)^2 + 2y_n - y_{n-1}$  (table 5.3). The dynamic reaction equations are:

$$V_1 = 0.17R + 0.17P \text{ (table 6.1)}$$

$$V_2 = 0.33R + 0.33P$$

The  $P_n$  values for the second column in table 7.21 are obtained from figure 7.51 multiplying by 4 to obtain the total concentrated load applied to the girder by the two purlins.

In table 7.21 the maximum  $R_n = 423 \text{ kips} < 538 \text{ kips}$ . The design is satisfactory.

b. Shear Stress Check.

$$V = 253 \text{ kips (table 7.21)}$$

Multiply by 1.75 in accordance with paragraph 6-09

$$v = \frac{V}{dt_w} = \frac{253,000(1.75)}{38.84(0.625)} = 18,200 \text{ psi}$$

Allowable  $v = 21,000 \text{ psi}$ ; OK (par. 4-05c)

c. Check Proportions of 36 W150 for Local Buckling.

$$b/t_f = 11.972/0.94 = 12.7 < 14.0; \text{ OK}$$

$$a/t_w = 33.96/0.625 = 54.3 > 30; \text{ NG}$$

Longitudinal stiffeners are required (par. 4-06d)

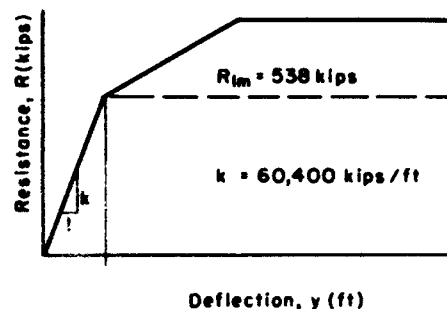
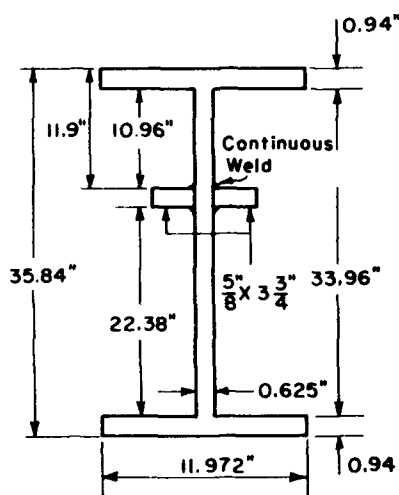


Figure 7.52. Resistance function for 36 W150 girder spanning 20 ft, fixed at one end and pinned at the other

Table 7.21. Determination of Maximum Deflection and Dynamic Reactions for Roof Girder

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> - R <sub>n</sub> (kips)	y <sub>n</sub> (Δt) <sup>2</sup> (ft)	y <sub>n</sub> (ft)	V <sub>n</sub> (kips)
0	0	0	0.6	0.000004	0	253 max
0.0015	4	0.2	3.8	0.000023	0.000004	
0.0030	12	1.0	10.1	0.000061	0.000031	
0.0045	26	7	19	0.000114	0.000119	
0.0060	54	19	35	0.000210	0.000321	
0.0075	84	44	40	0.000240	0.000733	
0.0090	136	84	52	0.000313	0.001385	
0.0105	180	142	38	0.000228	0.002350	
0.0120	232	214	18	0.000108	0.003543	
0.0135	268	293	-25	-0.000150	0.004844	
0.0150	294	362	-68	-0.000409	0.005995	
0.0165	328	407	-79	-0.000475	0.006737	
0.0180	340	423	-83	-0.000499	0.007004	
0.0195	352	414	-62	-0.000373	0.006862	
0.0210	360	383	-23	-0.000138	0.006347	
0.0225	366	344	+22	+0.000132	0.005694	
0.0240	372	312	60	0.000361	0.005173	
0.0255	370	303	67	0.000403	0.005013	
0.0270	366	317	49	0.000294	0.005256	
0.0285	352	350	2	0.000012	0.005793	
0.0300	330	383	-53	-0.000319	0.006342	
0.0315	312	397	-85	-0.000511	0.006572	
0.0330					0.006291	



$$t_s = 5/8 = 0.625$$

$$b_s/t_s = 6; b_s = 6t_s = 6(5/8) = 3.75 \text{ in.}$$

Use 2 longitudinal stiffeners, 2 plates  
5/8 in. by 3-3/4 in. - full length

$$\begin{aligned} c/t_w &= (11.9 - 0.94)/0.625 \\ &= 17.5 < 22; \text{OK} \end{aligned}$$

$$e/t_w = (23.32 - 0.94)/0.625 = 35.6 < 40; \text{OK}$$

Weight of added stiffener plates = 15.9 lb/ft

7-36 FINAL DESIGN OF COLUMN. The column design was begun in paragraph 7-34. The final steps in the column design are illustrated by this paragraph. The steps which follow all lead to the determination of the lateral deflection of the top of the columns by a numerical integration. In the preliminary design of the column (par. 7-34) some of the factors which affect the maximum deflection of the column are neglected to simplify the computations. These factors which are now considered are: the variation of plastic hinge moment with direct stress, the variation of column resistance with lateral deflection, the effect of girder flexibility on the stiffness of the columns, the difference between the load on the wall slab and the dynamic reactions from the wall which are used as the lateral design load for the frame columns.

For all pertinent dimensions refer to paragraph 7-34.

a. Mass Computation. (par. 7-34b)

Walls = 73.5 kips, roof slab = 57.3 kips, purlins = 7.2 kips

$$\text{Columns} = \frac{120(8.67)^4}{1000} = 4.2 \text{ kips}$$

$$\text{Girders} = \frac{160(49)}{1000} = 7.9 \text{ kips}$$

$$\text{Connections} = 0.1(7.2 + 7.9) = 1.5 \text{ kips}$$

Single-degree-of-freedom mass = total roof + 1/3 (columns + walls)

$$= \frac{57.3 + 7.2 + 7.9 + 1.5 + 0.33(73.5 + 4.2)}{32.2} = \frac{99.8}{32.2}$$

$$= 3.1 \text{ kip-sec}^2/\text{ft}$$

b. Column Properties.

$$12 \text{ W}120, I = 1071.7 \text{ in.}^4, S = 163.4 \text{ in.}^3, Z = 186.0 \text{ in.}^3,$$

$$0.5(S + Z) = 174.7 \text{ in.}^3, A = 35.31 \text{ in.}^2, b = 12.32 \text{ in.},$$

$$t_f = 1.106 \text{ in.}, a = 10.91 \text{ in.}, t_w = 0.71 \text{ in.}, d = 13.12 \text{ in.},$$

$$r = 3.13 \text{ in.}$$

c. Column Interaction Design Data. The plastic hinge moment, plastic axial load, and the values of  $M_1$  and  $P_1$  are computed below from the column properties.

$$M_P = f_{dy}(S + Z)0.5 = \frac{41.6}{12} (174.7) = 606 \text{ kip-ft (eq 4.2)}$$

$$P_P = f_{dy}A = 41.6(35.31) = 1470 \text{ kips (eq 4.7)}$$

$$P_1 = \frac{f_{dy}(2bt_f^2 + t_w d^2/2 - 2t_w t_f^2)}{d} \quad (\text{eq 4.12})$$

$$= \frac{41.6}{13.12} [2(12.32)(1.106)^2 + 0.5(0.71)13.12^2 - 2(0.71)(1.106)^2]$$

$$= 288 \text{ kips}$$

$$M_1 = \frac{f_{dy} \left[ 4t_w \left( \frac{d}{2} - t_f \right)^3 + bt_f(3d^2 - 6dt_f + 4t_f^2) \right]}{3d} \quad (\text{eq 4.11})$$

$$M_1 = \frac{41.6}{12(3)(13.12)} \left\{ 4(0.71)(5.46)^3 + 12.32(1.106)[3(13.12)^2 - 6(13.12)(1.106) + 4(1.106)^2] \right\} = 555 \text{ kip-ft}$$

For  $P_D > P_1$

$$M_D = \left( \frac{P_P - P_D}{P_P - P_1} \right) M_1 = \left( \frac{1470 - P_D}{1470 - 288} \right) 555 = 690 - 0.469P_D \quad (\text{eq 4.13})$$

For  $P_D < P_1$

$$M_D = M_P - \frac{P_D}{P_1} (M_P - M_1) = 606 - \frac{606 - 555}{288} P_D = 606 - 0.177P_D \quad (\text{eq 4.14})$$

d. Effect of Girder Flexibility. The relative flexibility of the girders reduces the spring constant  $k$  in the elastic range from the value obtained in paragraph 7-34 for the assumption of infinitely stiff girders (par. 7-08). (See par. 7-23d.) To obtain this revised value of  $k$  a simple sidesway analysis of the frame is performed. From the sidesway analysis the spring constant is the magnitude of the lateral force required to cause a unit displacement.

In figure 7.55 (page 145) the initial spring constant is applicable only up to the formation of the first hinge. However, to simplify the design procedure, this value is assumed to hold up to the plastic resistance  $R_m$ .

In the sidesway analysis (fig. 7.53) the frame dimensions are based on the centerline dimensions of the girders and columns. The lateral deflection for which the F.E.M. at top and bottom of each column is 1000 kip-ft is

$$x = (F.E.M.)h^2/6EI$$



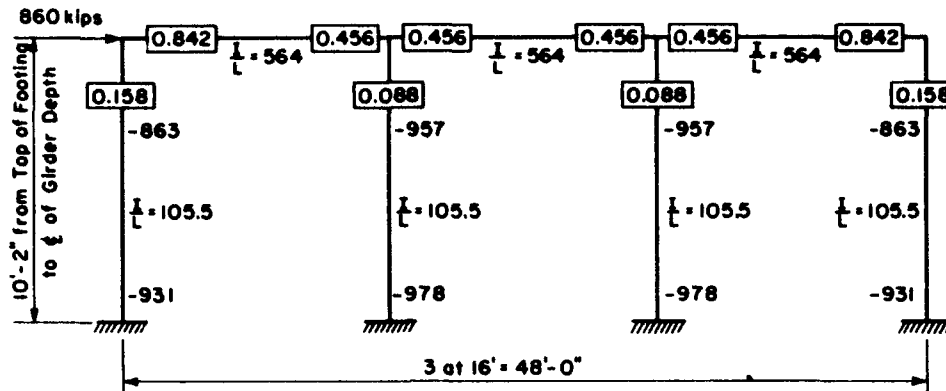


Figure 7.53. Sidesway frame analysis by moment distribution

$$x = \frac{(1000)(10.17)^2 144}{6(30)10^3(1071.7)} = 0.077 \text{ ft}$$

$$R = \frac{\Sigma M}{h} = \frac{2(863 + 957 + 931 + 978)}{10.17} = 734 \text{ kips}$$

$$k = \frac{R}{x} = \frac{734}{0.077} = 9540 \text{ kips/ft}$$

e. Loading. Both horizontal and vertical loads are considered. The lateral load for the numerical integration  $F_n$  (table 7.22, page 144) is obtained from the  $V_n$  dynamic reaction column of the numerical integration analysis of the front wall slabs in paragraph 7-31 (table 7.16). The dynamic reaction values for a 1-ft width are multiplied by 18, the width of one bay of the building. After the dynamic reactions of the front wall slab have decreased to the level of the applied load (at  $t = 0.018$  sec) the  $F_n$  values are taken from the net lateral overpressure curve (fig. 7.43). For  $\bar{P}_{net}$  in psi

$$F_n = \frac{144(18)6.18}{1000} \bar{P}_{net} = 16.0 \bar{P}_{net} \text{ kips}$$

These data are plotted in figure 7.54 to give the frame lateral design load for the second column of table 7.22. The dimension 6.18 is equal to one-half the clear height of the wall plus the roof slab thickness.

$$\frac{11.67}{2} + 0.35 = 6.18 \text{ ft}$$

The total vertical load  $P_n$  in table 7.22 is obtained by multiplying the average roof overpressure (fig. 7.44) by  $[54(18)144]/1000 = 140$  and

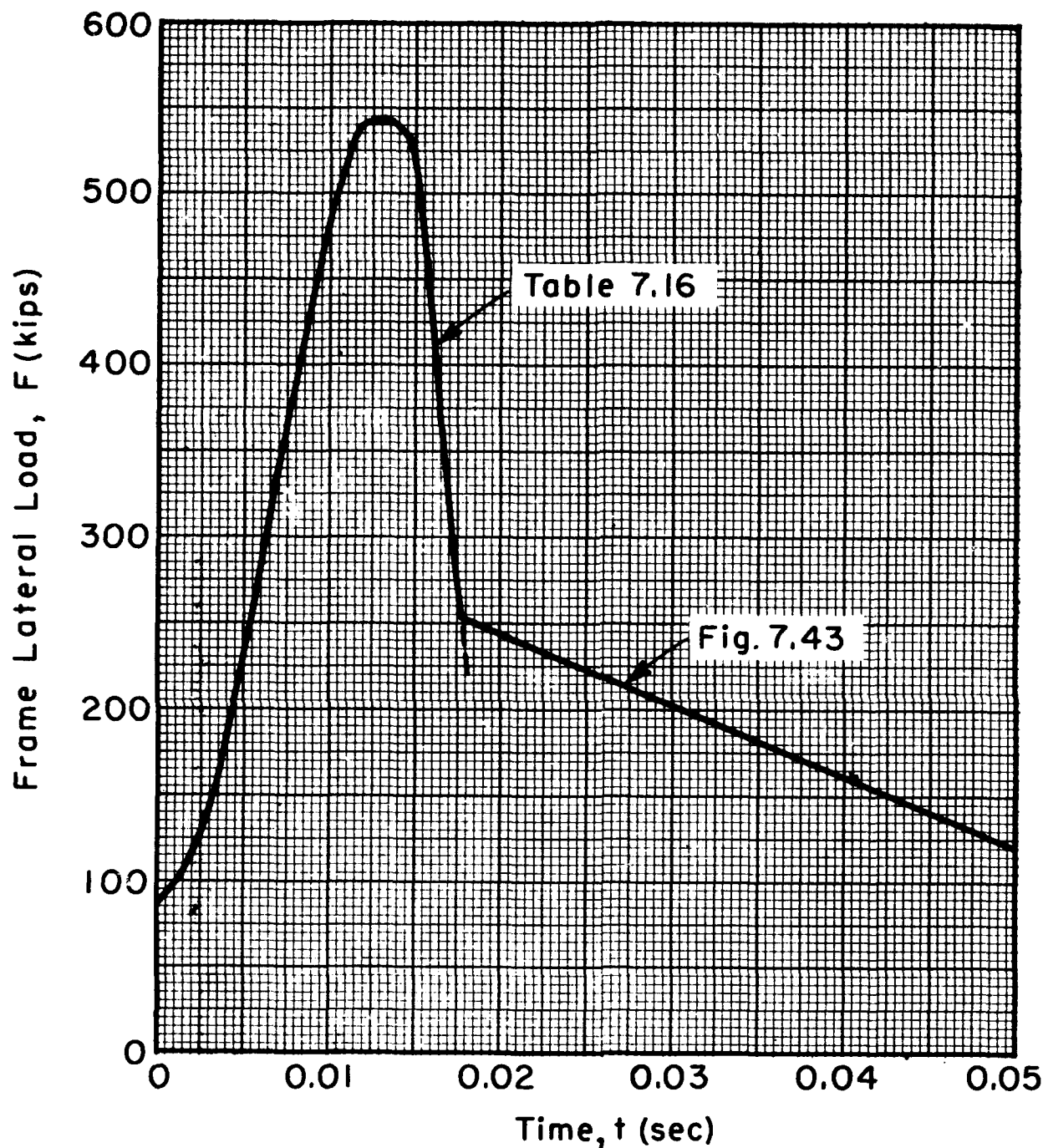


Figure 7.54. Frame design lateral load

adding to that the dead weight of the roof system  $57.3 + 7.2 + 7.9 + 1.5 = 73.9$  kips. The average roof overpressure is used because in the numerical analysis only the average column load is needed. This procedure neglects the dynamic effect of the roof slab, purlins, and girder on the vertical blast loads.

f. Computation of Deflection of Frame by Numerical Integration.

(See table 7.22.) The  $(P_D)_n$  values are the average axial column loads obtained by dividing the total vertical load  $P_n$  by the number of columns. The  $(P_D)_n$  values are used to determine the corresponding values of  $(M_D)_n$  by the relationships of paragraph 7-36c. From the value of  $(M_D)_n$  the maximum resistance is obtained by the relation

$$(R_m)_n = \frac{2n(M_D)_n}{h_c} = \frac{2(4)(M_D)_n}{8.67} = 0.922(M_D)_n$$

$R_n$  is equal to  $kx_n = 9540x_n$  in the elastic range. The expression  $\frac{P_n x_n}{h_c}$  indicates the decrease in resistance corresponding to the increase in bending moment resulting from the eccentric loading.

The basic equation for the numerical integration in table 7.22 is

$$x_{n+1} = \ddot{x}_n (\Delta t)^2 + 2x_n - x_{n-1} \quad (\text{table 5.3})$$

where

$$\begin{aligned} \ddot{x}_n (\Delta t)^2 &= \left[ F_n - R_n + \frac{P_n x_n}{h_c} \right] \frac{(\Delta t)^2}{m} = \left[ F_n - R_n + \frac{P_n x_n}{h_c} \right] \frac{(0.002)^2}{3.1} \\ &= 1.29(10^{-6}) \left[ F_n - R_n + \frac{P_n x_n}{h_c} \right] \text{ ft} \end{aligned}$$

To check the slenderness criteria (par. 4-08) the following equation is evaluated for each time interval

$$\frac{M_D}{M_P} \left[ \frac{K' L d}{100 b t_f} \right] + \frac{P_D}{P_P} \left[ \frac{K'' L}{15 r} \right] \leq 1 \quad (\text{eq 4.10})$$

$$K' = 0.14 \quad (\text{table 4.1}), \quad K'' = 0.50 \quad (\text{table 4.2})$$

$$L = 8.67 \text{ ft (clear height)}, \quad b = 12.32 \text{ in.}, \quad r = 3.13 \text{ in.}$$

$$t_f = 1.106 \text{ in.}, \quad d = 13.12 \text{ in.}$$

$$\frac{M_D}{M_P} \left[ \frac{(0.14)(8.67)(12)(13.12)}{100(12.32)(1.106)} \right] + \frac{P_D}{P_P} \left[ \frac{0.50(8.67)(12)}{15(3.13)} \right] \leq 1$$

$$(0.140) \frac{M_D}{M_P} + (1.11) \frac{P_D}{P_P} \leq 1$$

The time interval  $\Delta t = 0.002$  sec used in table 7.22 is based on the natural period  $T_n = 2\pi\sqrt{m/k} = 6.28\sqrt{3.1/9540} = 0.113$  sec.

$$\Delta t \approx \frac{T_n}{10} = \frac{0.113}{10} = 0.0113 \quad (\text{par. 5-08})$$

The analysis in table 7.22 shows that the maximum deflection

Table 7.22. Determination of Column Adequacy

t (sec)	P <sub>n</sub> (kips)	(P <sub>D</sub> ) <sub>n</sub> (kips)	M <sub>D</sub> (kip-ft)	R <sub>m</sub> (kips)	F <sub>n</sub> (kips)	R <sub>n</sub> (kips)	$\frac{\Sigma P_n}{h_c} x_n$ (kips)	$\frac{\Sigma P_n}{h_c}$ (kips)	$\Sigma$	$\bar{x}_n (\Delta t)^2$ (ft)	x <sub>n</sub> (ft)
0	74	18	603	556	85	0	0	8.5	42.5	0.000055	0
0.002	136	34	600	554	115	0.5	0	15.7	114.5	0.000148	0.000055
0.004	199	50	597	551	185	2.5	0	23.0	182.5	0.000235	0.000258
0.006	261	65	594	548	285	6.6	0	30.1	278.4	0.000359	0.000696
0.008	323	81	592	546	390	14.2	0.1	37.3	375.9	0.000485	0.001493
0.010	386	96	589	543	485	26.5	0.1	44.5	458.6	0.000591	0.002775
0.012	448	112	586	541	543	44.3	0.2	51.7	498.9	0.000644	0.004648
0.014	511	128	583	538	540	68.4	0.4	58.9	472.0	0.000609	0.007165
0.016	573	143	581	536	420	98.2	0.7	66.1	322.5	0.000416	0.010291
0.018	635	159	578	533	252	132.0	1.0	73.2	121.0	0.000156	0.013833
0.020	698	174	575	531	245	167.2	1.4	80.5	79.2	0.000102	0.017531
0.022	760	190	572	528	235	203.5	1.9	87.6	33.4	0.000043	0.021331
0.024	823	206	570	526	227	240.2	2.4	94.9	-10.8	-0.000014	0.025174
0.026	885	221	567	523	220	276.7	3.0	102.1	-53.7	-0.000069	0.029003
0.028	947	237	564	520	210	312.6	3.6	109.2	-99.0	-0.000128	0.032763
0.030	1010	252	561	518	202	347.2	4.2	116.5	-141.0	-0.000182	0.036395
0.032	1072	268	558	515	196	380.1	4.9	123.6	-179.2	-0.000231	0.039845
0.034	1134	283	556	513	188	410.8	5.6	130.8	-217.2	-0.000280	0.043064
0.036	1197	299	550	507	180	438.9	6.4	138.1	-252.5	-0.000326	0.046003
0.038	1259	315	542	500	172	463.8	7.0	145.2	-284.8	-0.000367	0.048616
0.040	1268	317	541	499	164	485.2	7.4	146.3	-313.8	-0.000405	0.050862
0.042	1261	315	542	500	156	500.0	7.7	145.4	-336.3	-0.000434	0.052703
0.044	1254	313	543	501	148	501.0	7.8	144.6	-345.2	-0.000445	0.054110
0.046	1244	311	544	502	140	502.0	7.9	143.4	-354.1	-0.000457	0.055072
0.048	1233	308	546	504	132	504.0	7.9	142.2	-364.1	-0.000470	0.055577
0.050	1222	306	547	505	124	505	7.8	140.9	-373.2	-0.000481	0.055612 *
* (x <sub>n</sub> ) <sub>max</sub> = 0.056 ft.											

$(x_n)_{\max} = 0.056$  exceeds the limiting elastic deflection  $x_e = \frac{R_m}{9540} = \frac{500}{9540} = 0.053$ . This small amount of plastic action is considered acceptable for purposes of this example. At  $t = 0.04$  sec the slenderness criteria equation

$$0.140 \frac{M_P}{M_P} + 1.11 \frac{P_P}{P_P} = 0.14 \frac{541}{606} + 1.11 \frac{317}{1470} \\ = 0.364 < 1.0, \therefore \text{OK}$$

g. Shear Stress Check.

$$(R_m)_{\max} = 505 \text{ kips (table 7.22)}$$

$$V_{\max} = 505/4 = 126.0 \text{ kips}$$

$$v = \frac{V}{dt_w} = \frac{126,000}{(13.12)(0.71)} = 13,400 \text{ psi}$$

Allowable  $v = 21,000 \text{ psi}$ ; OK (par. 4-05e)

NUMERICAL EXAMPLE, DESIGN OF A ONE-STORY REINFORCED-CONCRETE  
FRAME BUILDING, PLASTIC DEFORMATION PERMITTED

7-37 GENERAL. This numerical example presents the design of the important elements of one bay of a windowless one-story, reinforced-concrete frame building. Included are the design of the wall slab, roof slab, roof girder, and columns. The foundation design is not included because that design procedure is illustrated by paragraph 7-27.

In this example all elements with the exception of the girder are permitted to deform plastically. The reason for designing the girder in the elastic range is covered in paragraph 7-11. The limiting deflections are established on the basis of paragraph 6-26.

The structure consists of reinforced-concrete wall slabs spanning vertically with one-way reinforcement. The wall slab is supported at the bottom by the foundation and at the top by the roof slab. The roof slab is also a one-way slab spanning continuously over the reinforced concrete frames. The roof girders are tee beams formed by the roof slab and rectangular girder stems. The columns are rectangular tied columns symmetrically reinforced in the strong direction.

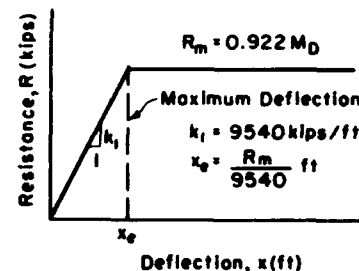


Figure 7.55. Frame  
resistance diagram

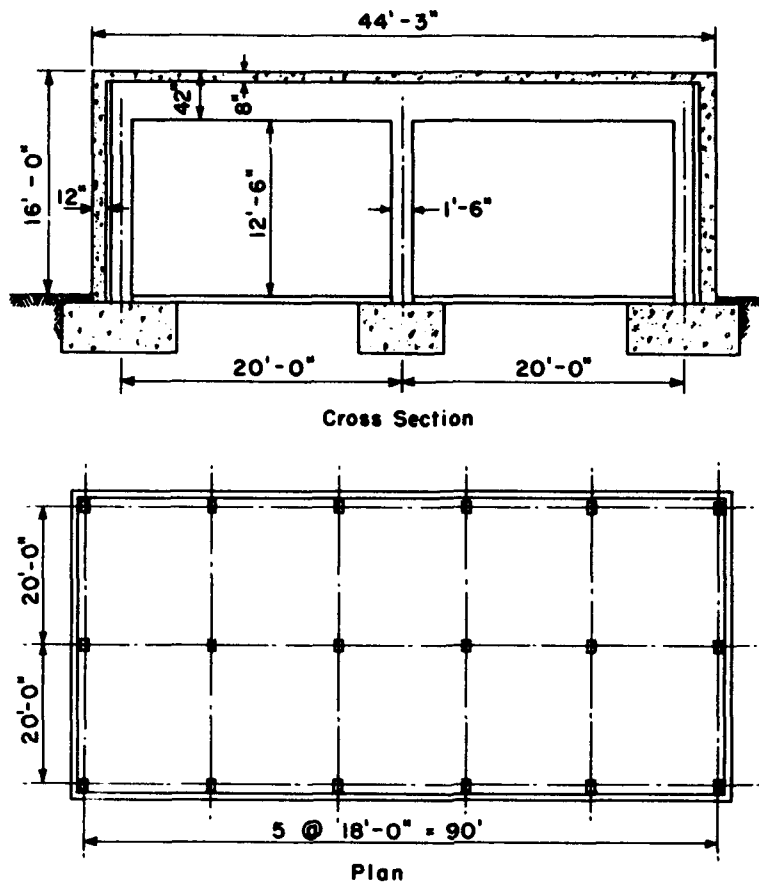


Figure 7.56. Plan and section of reinforced-concrete building

7-38 DESIGN PROCEDURE. The design procedure illustrated by this example is essentially the same as the procedure detailed in paragraph 7-18 for the plastic design of a steel frame building with reinforced-concrete walls and roof.

7-39 LOAD DETERMINATION. In this example only the completed overpressure curves are presented. The computation of the overpressure variation is explained in detail in EM 1110-345-413 and illustrated again in paragraph 7-19. The design overpressure of 10 psi is selected arbitrarily.

The overpressure vs time curves that are presented are:

- (1) Incident overpressure vs time (fig. 7.57)
- (2) Front face overpressure vs time (fig. 7.58)
- (3) Rear face overpressure vs time (fig. 7.59)
- (4) Net lateral overpressure vs time (fig. 7.60)
- (5) Average roof overpressure vs time (fig. 7.61)

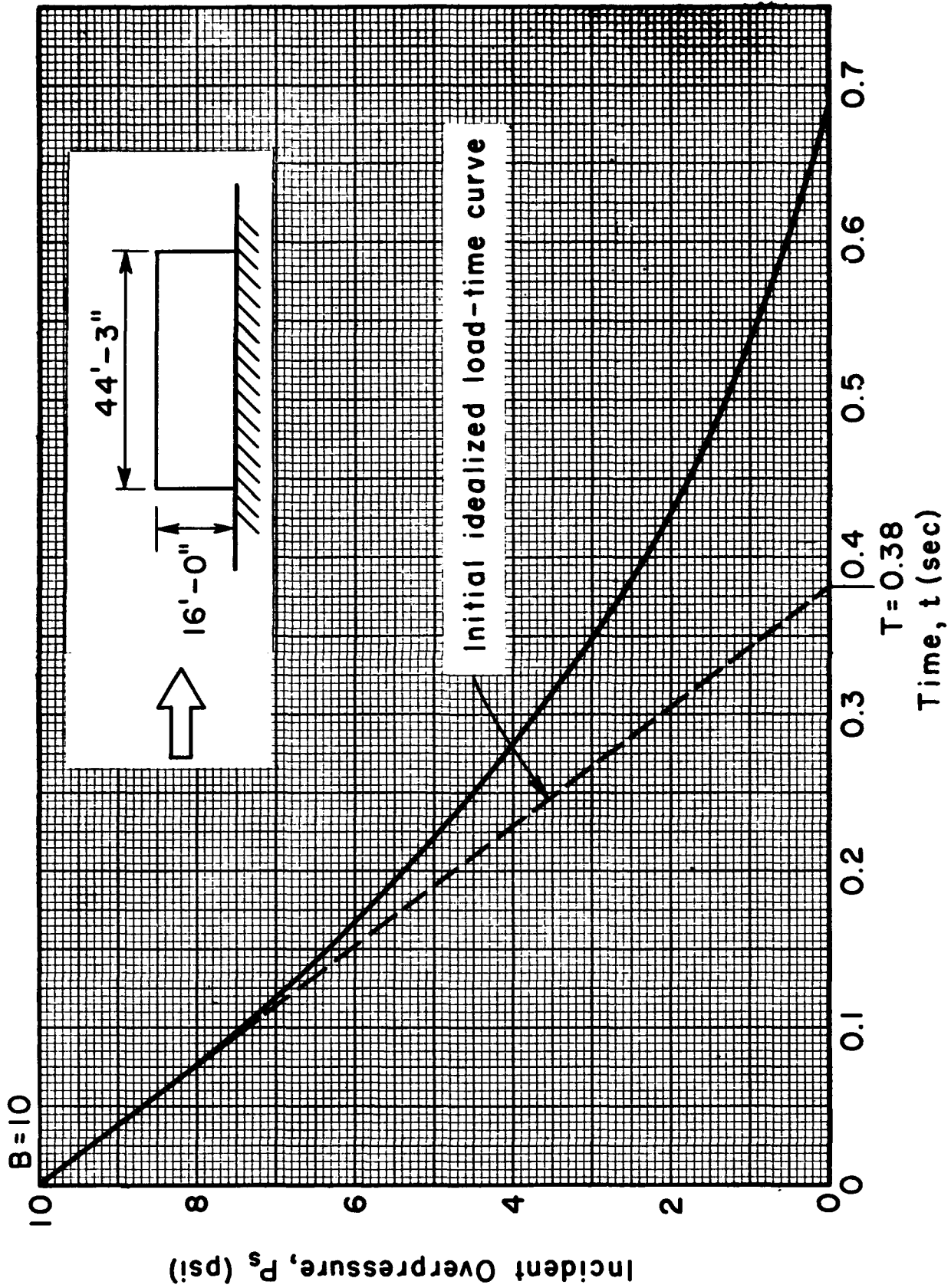


Figure 7.57. Incident overpressure vs time curve

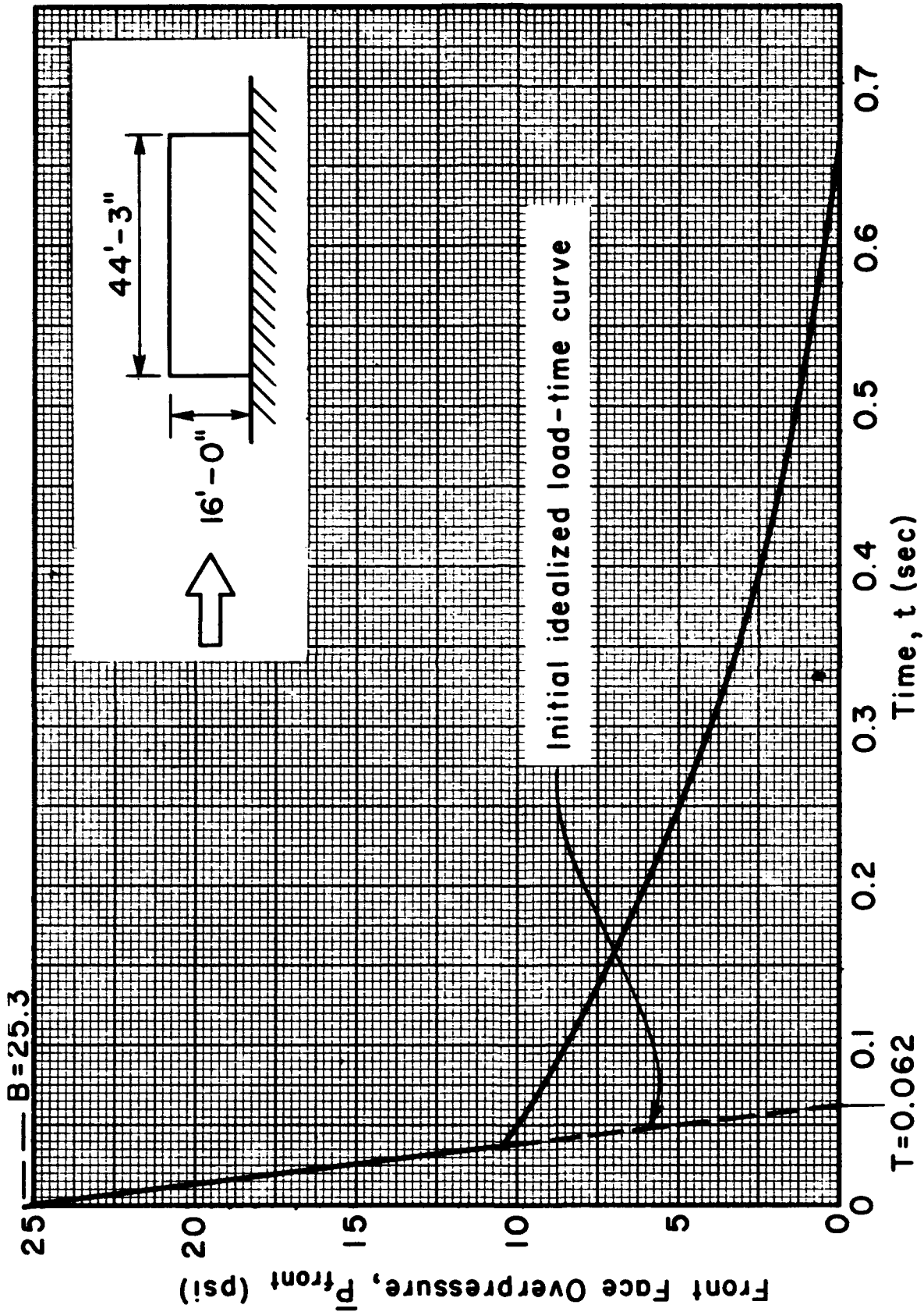


Figure 7.58. Front face overpressure vs time curve



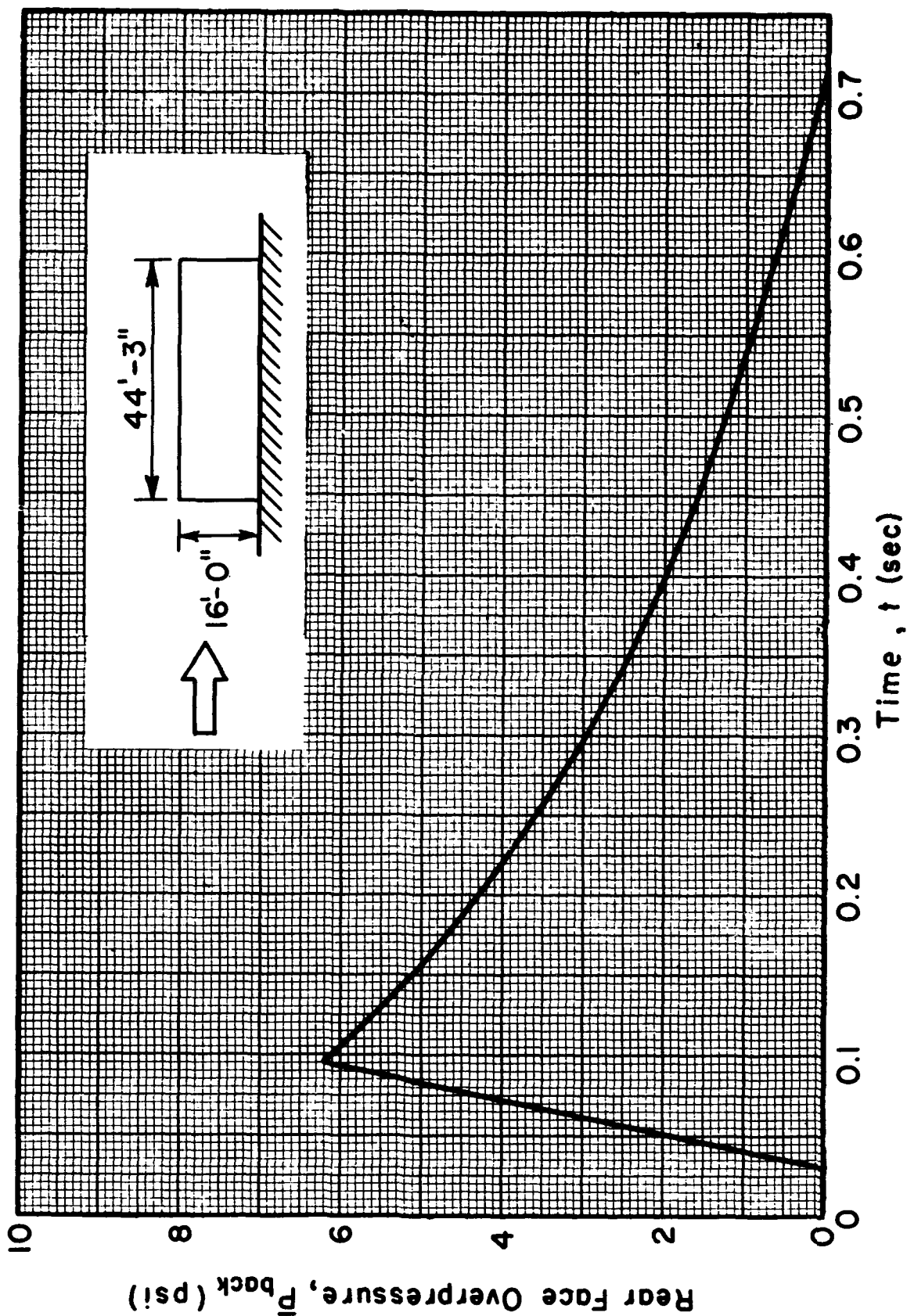


Figure 7.59. Rear face overpressure vs time curve

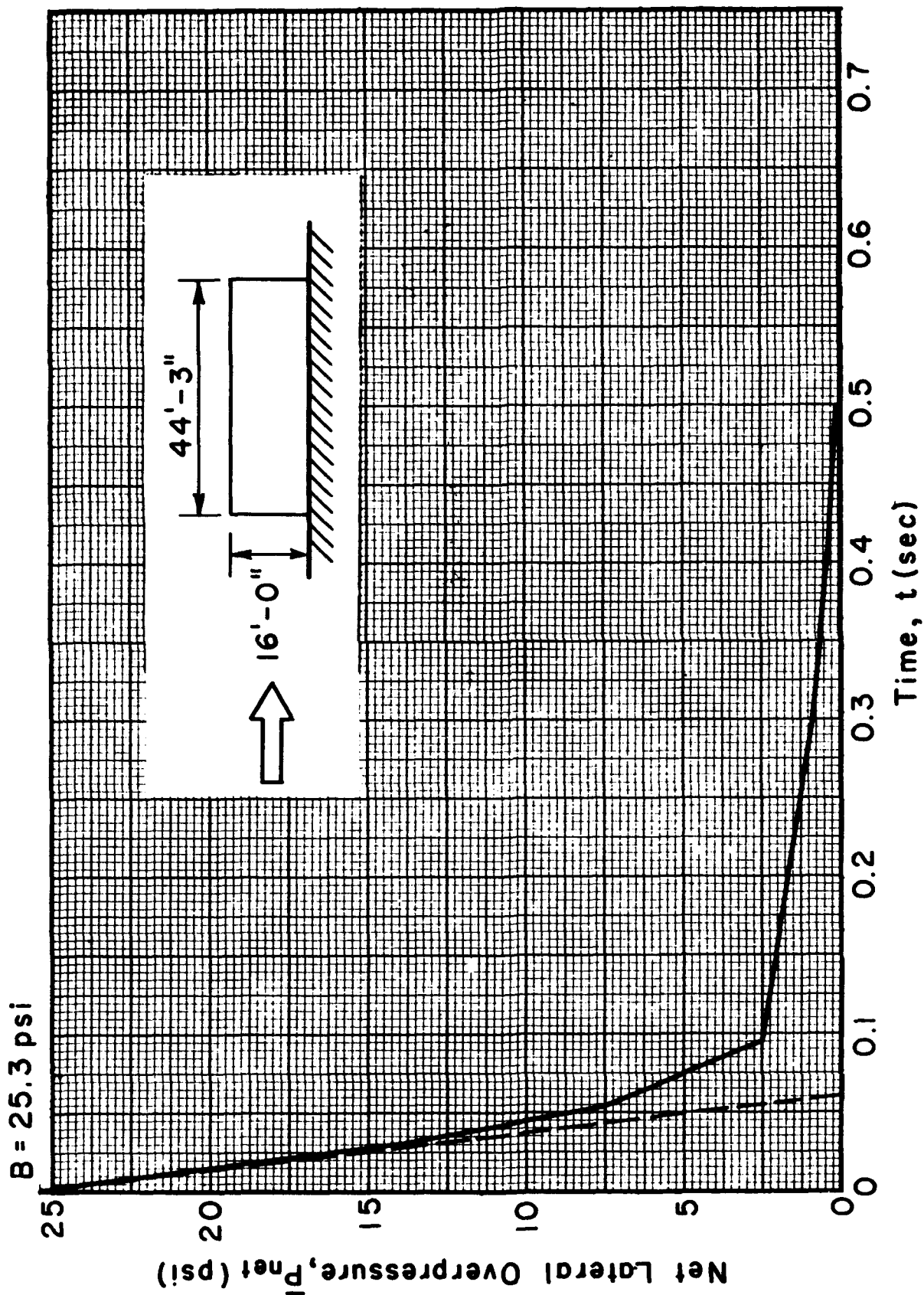


Figure 7.60. Net lateral overpressure vs time curve

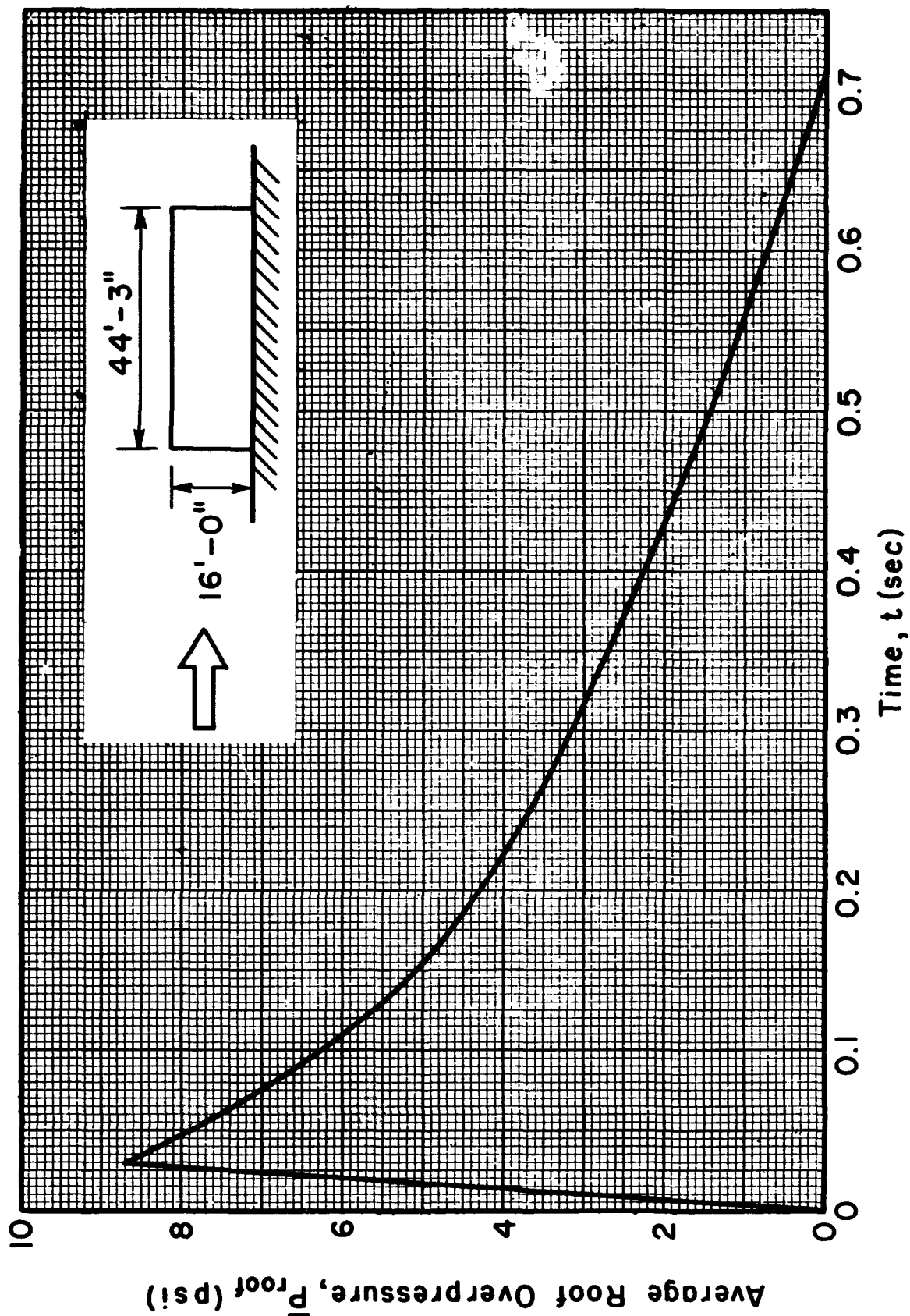
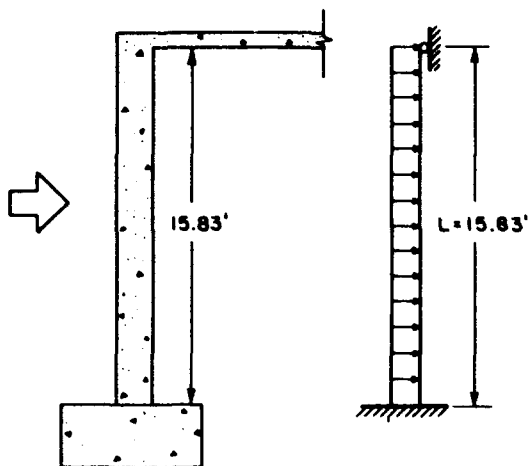


Figure 7.61. Average roof overpressure vs time curve

7-40 DESIGN OF WALL SLAB. The wall slab is designed as a reinforced concrete beam one foot in width fixed at the foundation and pinned at the roof



where it is framed into a thin slab. The wall slab is designed for plastic action so that a plastic hinge will be developed first at the foundation and then at mid-span when the slab is subjected to the design loading. The clear height of the slab is considered the design span length.

The preliminary design procedure for plastic design is described and illustrated by an example in paragraph 6-11.

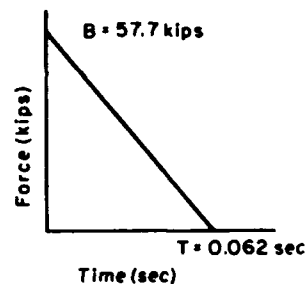
The dead load stresses are neglected for a vertical wall slab.

a. Design Loading. The design load as idealized from the computed loading shown by figure 7.58 is defined by:

$$B = \frac{25.3(144)15.83}{1000} = 57.7 \text{ kips}$$

$$T = 0.062 \text{ sec}$$

$$H = \frac{BT}{2} = \frac{(57.7)0.062}{2} = 1.79 \text{ kip-sec (par. 6-11)}$$



b. Dynamic Design Factors. (Refer to table 6.1.)

Elastic range:

$$K_L = 0.58,$$

$$K_M = 0.45,$$

$$K_{LM} = 0.78$$

$$R_{lm} = \frac{8M_{Ps}}{L},$$

$$k_1 = \frac{185EI}{L^3}$$

$$V_1 = 0.26R + 0.12P,$$

$$V_2 = 0.43R + 0.19P$$

Elasto-plastic range:

$$K_L = 0.64,$$

$$K_M = 0.50,$$

$$K_{LM} = 0.78$$

$$R_m = \frac{4}{L} (M_{Ps} + M_{Pm}),$$

$$k_{ep} = \frac{384EI}{5L^3}$$

$$V = 0.39R + 0.11P$$

Plastic range:

$$K_L = 0.50,$$

$$K_M = 0.33,$$

$$K_{LM} = 0.66$$

$$R_m = \frac{4}{L} (M_{Ps} + 2M_{Pm})$$

$$V = 0.38R + 0.12P$$

Average values:

$$K_L = \frac{(0.64 + 0.50)}{2} = 0.57$$

$$K_M = \frac{(0.50 + 0.33)}{2} = 0.42$$

$$R_m = \frac{4}{L} (M_{Ps} + 2M_{Pm})$$

$$k_E = \frac{160EI}{L^3} \quad (\text{if } M_{Ps} = M_{Pm})$$

c. First Trial - Actual Properties.

$$\text{Let } M_{Ps} = M_{Pm} = M_P$$

$$\text{Assume } p = 0.015 \text{ (par. 4-10)}$$

$$\text{Let } \alpha\beta = 5 \text{ (par. 6-26)}$$

$$\text{Assume } C_R = 0.75 \text{ (experience)}$$

$$R_m = C_R B = 0.75(57.7) = 43.3 \text{ kips}$$

(eq 4.16)

$$M_P = pf_{dy} bd^2 \left( 1 - \frac{pf_{dy}}{1.7f'_{dc}} \right)$$

$$= 0.015(52)(1)d^2 \left[ 1 - \frac{0.015(52)}{1.7(3.9)} \right] = 0.688d^2 \text{ kip-ft (d in inches)}$$

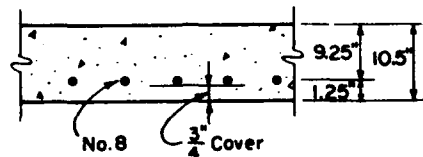
$$R_m = \frac{12M_P}{L} = \frac{(12)0.688d^2}{15.83} = 43.3, \therefore d = 9.11 \text{ in.}$$

$$\text{Try } h = 10.5 \text{ in., } d = 9.25 \text{ in., } p = 0.015, \text{ np} = 0.15$$

$$M_P = 0.688(9.25)^2 = 58.9 \text{ kip-ft}$$

$$R_m = \frac{12M_P}{L} = \frac{12(58.9)}{15.83} = 44.6 \text{ kips}$$

$$I_g = bh^3/12 = (10.5)^3 = 1158 \text{ in.}^4$$



$$I_t = bd^3 \left[ \frac{k^3}{3} + np(1 - k)^2 \right] = 12(d)^3 \left[ \frac{(0.42)^3}{3} + 0.15(1 - 0.42)^2 \right]$$

$$= 0.905d^3 = 0.905(9.25)^3 = 716 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 0.5(1158 + 716) = 937 \text{ in.}^4$$

$$k_E = \frac{160EI}{L^3} = \frac{(160)(3)(10)^3 937}{144(15.83)^3} = 787 \text{ kips/ft}$$

$$y_E = \frac{R_m}{k_E} = \frac{44.6}{787} = 0.0567 \text{ ft}$$

$$y_m = \alpha \beta y_E = 5(0.0567) = 0.2835 \text{ ft (par. 6-26)}$$

$$\text{Weight} = \frac{10.5(150)15.83}{(12)1000} = 2.078 \text{ kips}$$

$$\text{Mass } m = \frac{2.078}{32.2} = 0.0645 \text{ kip-sec}^2/\text{ft}$$

d. First Trial - Equivalent System Properties.

$$R_{me} = K_L R_m = 0.57(44.6) = 25.4 \text{ kips (eq 6.12)}$$

$$H_e = K_L H = 0.57(1.79) = 1.02 \text{ kip-sec (eq 6.8)}$$

$$m_e = K_M m = 0.42(0.0645) = 0.0271 \text{ kip-sec}^2/\text{ft (eq 6.2)}$$

$$W_P = \frac{(H_e)^2}{2m_e} = \frac{(1.02)^2}{2(0.0271)} = 19.20 \text{ ft-kips (eq 6.10)}$$

$$T_n = 2\pi \sqrt{\frac{K_{IM} m}{k_E}} = 6.28 \sqrt{0.78(0.0645)/787} = 0.050 \text{ sec (eq 6.14)}$$

e. First Trial - Work Done vs Energy Absorption Capacity.

$$C_T = T/T_n = 0.062/0.050 = 1.24$$

$$C_R = R_m/B = 44.6/57.7 = 0.77 \text{ (eqs 6.15, 6.16)}$$

$$t_m/T = 0.71 \text{ (fig. 5.29)}$$

$$t_m = (0.71)0.062 = 0.044 \text{ sec}$$

The idealized load-time variation is considered to be a satisfactory approximation to the actual front face overpressure curve (fig. 7.58) up to  $t_m = 0.044 \text{ sec}$  (par. 5-13).

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$$C_W = 0.31 \text{ (fig. 5.27)}$$

$$W_m = C_W W_P = 0.31(19.20) = 5.95 \text{ ft-kips (eq 6.17)}$$

$$E = R_{me}(y_m - 0.5y_E) = 25.4 [0.2835 - 0.5(0.0567)] \\ = 6.48 \text{ ft-kips (eq 6.18)}$$

$E > W$ , therefore the selected proportions are satisfactory as a preliminary design.

A 10-in. slab was tried and found to be unsatisfactory.

Use 10.5-in. slab.

f. Preliminary Design for Bond Stress. It is now necessary to select the reinforcing steel for the critical cross sections. At the fixed end of the wall the cover requirement results in a smaller value of  $d = 8.0$  in. than at midspan,  $d = 9.25$  in. To achieve approximately the same  $p$  at both critical cross sections several values of  $p$  are investigated to obtain the value of  $p$  for which

$$\frac{8M_{Pm} + 4M_{Ps}}{12} = M_P = 58.9 \text{ kip-ft}$$

A plot of equation (4.16) simplifies this computation. From such a plot  $p \approx 0.017$ .

At the fixed end

$$\text{Estimated } V_{\max} = 0.5R_m = 0.5(44.6) = 22.3 \text{ kips}$$

$$\text{Allowable } u = 0.15f'_c = 0.15(3000) = 450 \text{ psi}$$

$$\Sigma o = \frac{V}{u_j d} = \frac{8(22,300)}{7(450)8.0} = 7.1 \text{ in.}$$

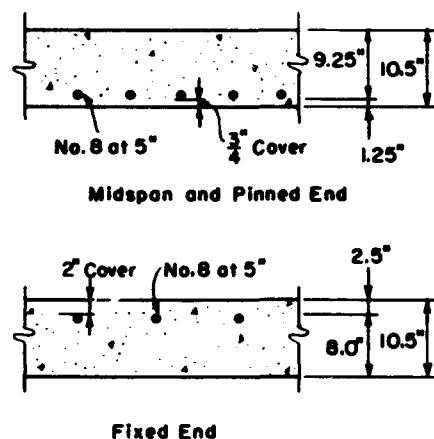
$$A_s = pbd = 0.017(12)8.0 = 1.63 \text{ in.}^2$$

$$\text{Try \#8 at 5 in., } A_s = 1.90 \text{ in.}^2, \Sigma o = 7.5 \text{ in.}$$

$$p = \frac{1.90}{12(8)} = 0.0198$$

At the pinned end

$$\text{Estimated } V_{\max} = 1/3 R_m = \frac{44.6}{3} = 14.9 \text{ kips}$$



$$\Sigma o = \frac{V}{u_j d} = \frac{8(14,900)}{450(7)9.25} = 4.1 \text{ in.}$$

$$A_s = pbd = 0.017(12)9.25 = 1.89 \text{ in.}^2$$

$$\text{Try \#8 at 5 in., } A_s = 1.9 \text{ in.}^2, \Sigma o = 7.5 \text{ in.}$$

$$p = \frac{1.9}{12(9.25)} = 0.0171$$

g. Determination of Maximum Deflection and Dynamic Reactions by Numerical Integration.

$$M_{Pm} = pf_{dy} bd^2 \left[ 1 - \frac{pf_{dy}}{1.7f_{dc}} \right] = 0.0171(52)(1)(9.25)^2 \left[ 1 - \frac{0.0171(52)}{1.7(3.9)} \right]$$

$$= 66.0 \text{ kip-ft (eq 4.16)}$$

$$M_{Ps} = 0.0198(52)(1)(8.0)^2 \left[ 1 - \frac{0.0198(52)}{1.7(3.9)} \right] = 56.0 \text{ kip-ft}$$

$$I_g = bh^3/12 = (10.5)^3 = 1158 \text{ in.}^4$$

$$I_t = bd^3 \left[ \frac{k^3}{3} + np(1 - k)^2 \right] = 12(9.25)^3 \left[ \frac{(0.44)^3}{3} + 0.171(1 - 0.44)^2 \right]$$

$$= 775 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 0.5(1158 + 775) = 967 \text{ in.}^4$$

$$\text{Weight} = \frac{10.5(150)15.83}{12(1000)} = 2.078 \text{ kips}$$

$$\text{Mass } m = \frac{2.078}{32.2} = 0.0645 \text{ kip-sec}^2/\text{ft}$$

Elastic range:

$$R_{lm} = \frac{8M_{Ps}}{L} = \frac{8(56.0)}{15.83} = 28.2 \text{ kips}$$

$$k_1 = \frac{185EI}{L^3} = \frac{(185)3(10)^3 967}{144(15.83)^3} = 930 \text{ kips/ft}$$

$$y_e = \frac{R_{lm}}{k_1} = \frac{28.2}{930} = 0.0304 \text{ ft}$$

Elasto-plastic range:

$$R_m = \frac{4(M_{Ps} + 2M_{Pm})}{L} = \frac{4[56.0 + 2(66.0)]}{15.83} = 47.5 \text{ kips}$$

$$k_{ep} = \frac{384EI}{5L^3} = \frac{384}{5(185)} k_1 = 386 \text{ kips/ft}$$



$$y_{ep} = y_e + \frac{R_m - R_{lm}}{k_{ep}} = 0.0304 + \frac{47.5 - 28.2}{386} = 0.080 \text{ ft}$$

Plastic range:

$$R_m = 47.5 \text{ kips}$$

By providing equal areas under the "effective resistance" line and the computed elasto-plastic resistance line,  $y_E = 0.0641 \text{ ft}$  (fig. 7.62).

$$k_E = R_m / y_E = 47.5 / 0.0641 = 740 \text{ kips/ft}$$

$$T_n = 2\pi \sqrt{K_{LM}(m) / k_E}$$

$$= 6.28 \sqrt{\frac{0.78(0.0645)}{740}} = 0.0517 \text{ sec}$$

$$y_m = \alpha \beta y_E = 5(0.0641) = 0.3205 \text{ ft}$$

The basic equation for the numerical integration in table 7.23 is  $y_{n+1} = \ddot{y}_n(\Delta t)^2 + 2y_n - y_{n-1}$  (table 5.3) where

$$\ddot{y}_n(\Delta t)^2 = \frac{P_n - R_n(\Delta t)^2}{K_{LM}(m)}$$

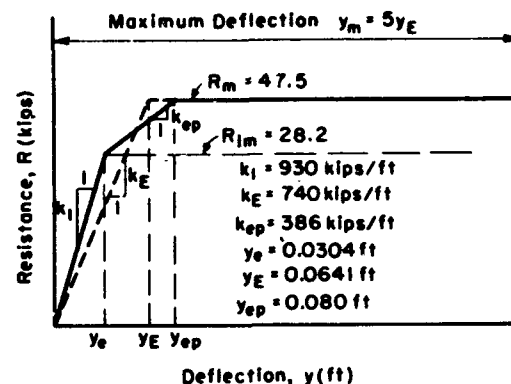


Figure 7.62. Resistance function for 10-1/2-in. slab spanning 15.83 ft, fixed at one end and pinned at the other

Table 7.23. Determination of Maximum Deflection and Dynamic Reactions for Front Wall Slab

t (sec)	$P_n$ (kips)	$R_n$ (kips)	$P_n - R_n$ (kips)	$\ddot{y}_n(\Delta t)^2$ (ft)	$y_n$ (ft)	$V_{1n}$ (kips)	$V_{2n}$ (kips)
0	57.7	0	28.8	0.0143	0	6.9	11.0
0.005	53.0	13.3	39.7	0.0197	0.0143	9.8	15.8
0.010	48.3	35.1	13.2	0.00656	0.0483	19.0	19.0
0.015	43.7	47.5	-3.8	-0.00223	0.0888	23.2	23.2
0.020	39.1	47.5	-8.4	-0.00493	0.1271	22.7	22.7
0.025	34.3	47.5	-13.2	-0.00775	0.1605	22.1	22.1
0.030	29.7	47.5	-17.8	-0.01045	0.1862	21.6	21.6
0.035	26.4	47.5	-21.1	-0.01239	0.2015	21.2	21.2
0.040	25.3	47.5	-22.2	-0.01303	0.2044*	21.2	21.2
0.045	24.2	38.1			0.1943	12.8	18.5

\* $(y_n)_{max} = 0.20 \text{ ft.}$

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)25(10)^{-6}}{0.78(0.0645)} = 4.97(10)^{-4}(P_n - R_n) \text{ ft, elastic range}$$

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)25(10)^{-6}}{0.78(0.0645)} = 4.97(10)^{-4}(P_n - R_n) \text{ ft, elasto-plastic range}$$

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)25(10)^{-6}}{0.66(0.0645)} = 5.87(10)^{-4}(P_n - R_n) \text{ ft, plastic range}$$

The time interval  $\Delta t = 0.005$  sec is approximately  $T_n/10 = 0.00517$  sec (par. 5-08).

The dynamic reaction equations are listed in paragraph 7-40b. The  $P_n$  values for the second column are obtained from figure 7.58, multiplying by  $144(15.83)/1000 = 2.28$ .

The maximum deflection  $(y_n)_{\max}$ , computed in table 7.23, is 0.204 ft, which is less than the allowable  $y_m$  of 0.3205 ft. Thus

$$\alpha\beta = \frac{0.204}{0.3205} (5) = 3.2; \text{ OK}$$

h. Shear and Bond Strength. For bottom of wall (fixed end of idealized slab):

$$V_{\max} = 23.0 \text{ kips (table 7.23)}$$

For no shear reinforcement, allowable  $v_y = 0.04f'_c + 5000p$  (eq 4.24)

$$v_y = 0.04(3000) + 5000(0.0171) = 120 + 85 = 205 \text{ psi}$$

$$v = \frac{8V}{7bd} = \frac{8(23,200)}{7(12)8.0} = 274 \text{ psi}$$

Shear reinforcement required for  $274 - 205 = 69$  psi

Contribution of shear reinforcement to allowable shear stress =  $rf_y$

$$r = \frac{69}{40,000} = 0.0017$$

$$\text{Try 1 \#3, } A_s = 0.11 \text{ in.}^2$$

$$r = \frac{A_s}{bs} = \frac{0.11}{10(s)} = 0.0017; \therefore s = 6.5 \text{ in., use } s = 6 \text{ in.}$$

For top of wall (pinned end of idealized slab)

$$V_{\max} = 23.2 \text{ kips (table 7.23)}$$

$$v_y = 205 \text{ psi}$$

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$$v = \frac{8V}{7bd} = \frac{8(23,200)}{7(12)9.25} = 240 \text{ psi}$$

Shear reinforcement required  
for  $240 - 205 = 35 \text{ psi}$

$$r = 35/40,000 = 0.00088$$

$$\text{Try 1 \#2, } A_s = 0.05 \text{ in.}^2$$

$$r = \frac{A_s}{bs} = \frac{0.05}{10(s)} = 0.00088;$$

$$\therefore s = 5.7 \text{ in., use } s = 5 \text{ in.}$$

Bond:

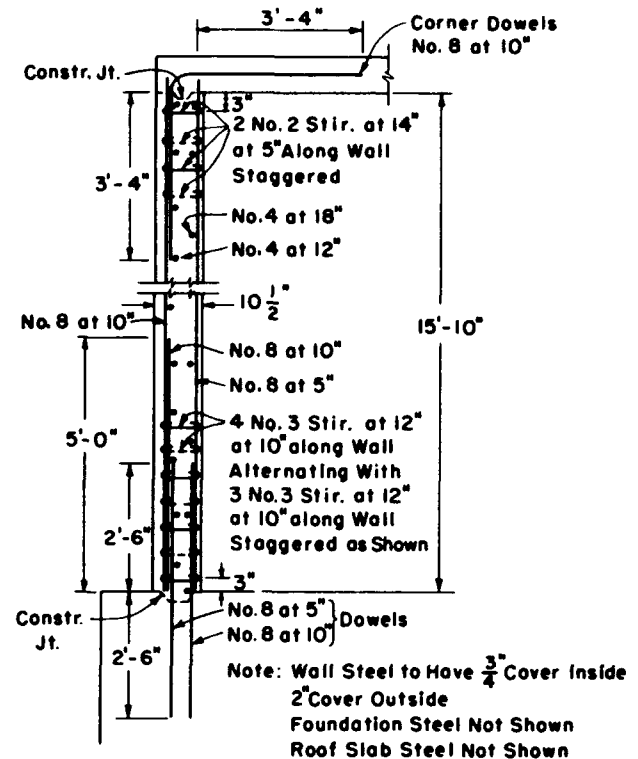
$$\text{Allowable } u = 0.15f'_c = 0.15(3000) = 450 \text{ psi}$$

$$\Sigma o = \frac{V}{u_j d} = \frac{8(23,000)}{7(450)(8.0)} = 7.3 \text{ in.}$$

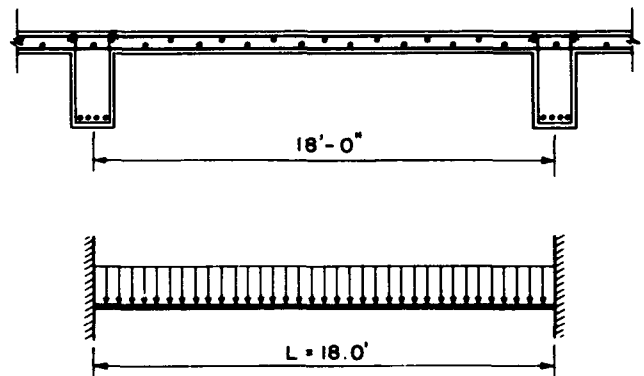
Use #8 at 5 in.,

$$\Sigma o = 7.5 \text{ in., } A_s = 1.9 \text{ in.}^2$$

1. Summary. 10-1/2-in. slab (see sketch above)



7-41 DESIGN OF ROOF SLAB. The roof is formed by a one-way slab spanning continuously over the frame bents spaced at 18 ft. This manual is limited to consideration of single-span elements. To account for the continuity of the slab it is designed as a fixed-end beam of 18-ft span. Since plastic behavior is desired the design procedure of paragraph 6-11 is followed. The permissible deflection is established in accordance with paragraph 6-26 and the static loads are allowed for by reducing the resistance.



a. Design Loading. The roof slab spanning between frames in a long building has different design conditions from the roof framing over purlins in a similar building (par. 7-22).

For the blast wave moving parallel to the long axis of the building the overpressure variation with time at all points is the same. However, for any slab element the overpressure along the span varies with time as a result of the lag time

$$t_{lag} = \frac{\text{length of slab}}{\text{velocity of blast wave}} = \frac{18}{1403} = 0.0128 \text{ sec}$$

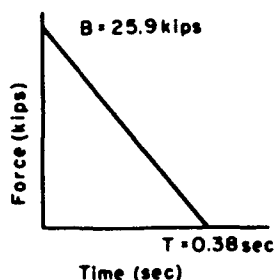
For this case the average slab loading is determined by introducing the rise time equal to  $t_{lag}$  as indicated in figure 7.63 to the incident overpressure.

For the blast wave moving perpendicular to the long axis of the building the overpressure variation with time at all points is different as a result of vortex action. However, at any slab element the overpressure-time variation may be treated as constant across the span. If the response is rapid, i.e., before the vortex action takes place, the slab element is subjected to the incident overpressure uniformly distributed.

In this example the preliminary design loading for the roof slab is the incident overpressure. For checking the preliminary design both directions of loading are considered and two numerical integrations are presented (tables 7.24 and 7.25, page 167).

The design load as idealized from the computed loading shown by

figure 7.57 is defined by:



$$B = 10 \text{ psi} = \frac{10(144)(18)}{1000} = 25.9 \text{ kips}$$

$$T = 0.38 \text{ sec}$$

$$H = \frac{BT}{2} = \frac{(25.9)(0.38)}{2} = 4.92 \text{ kip-sec (par. 6-11)}$$

b. Dynamic Design Factors. (Refer to table 6.1.)

Elastic range:

$$K_L = 0.53,$$

$$K_M = 0.41,$$

$$K_{IM} = 0.77$$

$$R_{1m} = 12M_{Ps}/L,$$

$$k_1 = \frac{384EI}{L^3}$$

$$V = 0.36R + 0.14P$$

Elasto-plastic range:

$$K_L = 0.64,$$

$$K_M = 0.50,$$

$$K_{IM} = 0.78$$

$$R_m = \frac{8}{L} (M_{Ps} + M_{Pm}),$$

$$k_{ep} = \frac{384EI}{5L^3}$$

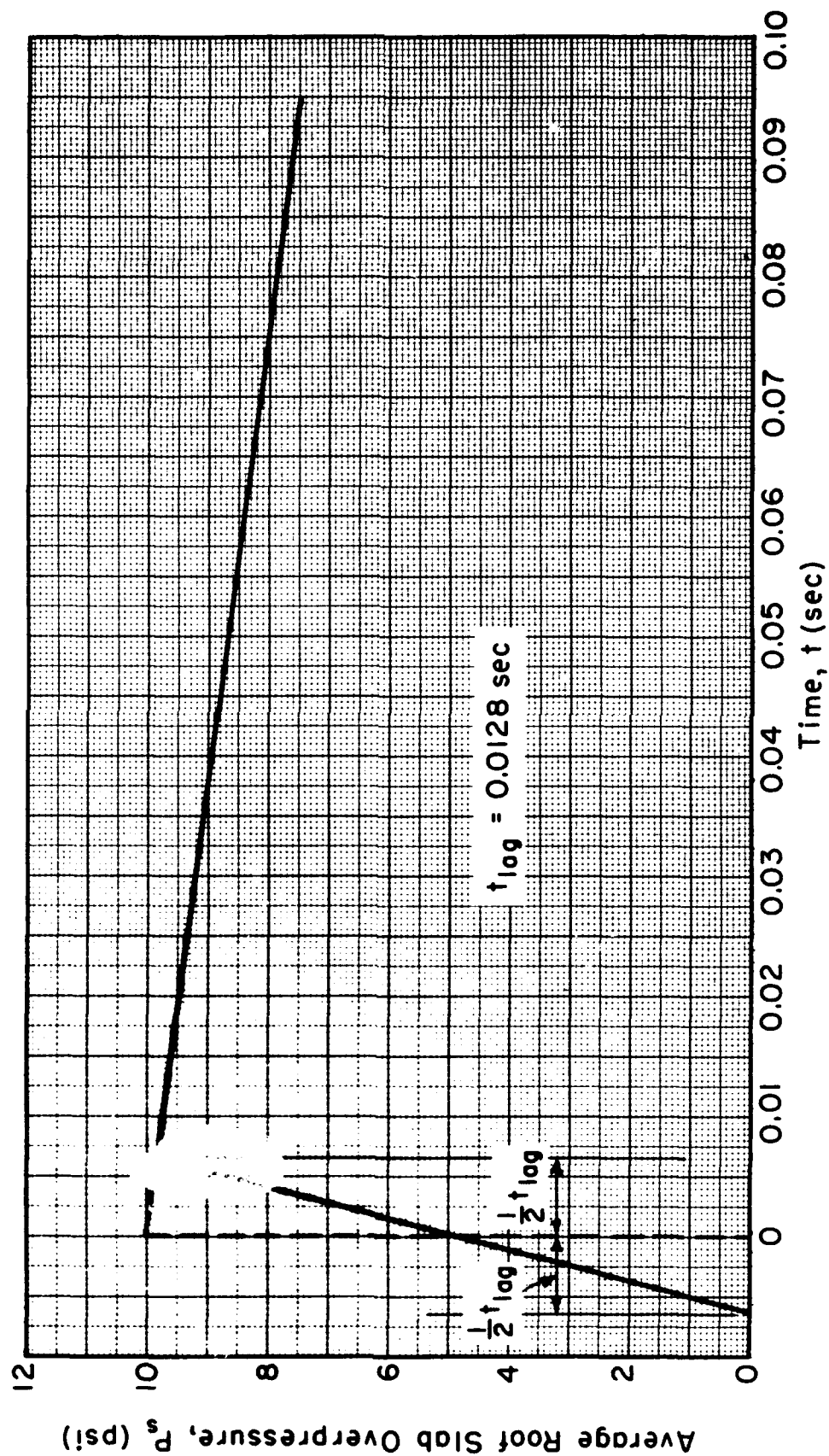


Figure 7.63. Average roof slab overpressure, blast wave parallel to slab span

Plastic range:

$$K_L = 0.50,$$

$$K_M = 0.33,$$

$$K_{LM} = 0.66$$

$$R_m = \frac{8}{L} (M_{Ps} + M_{Pm})$$

$$V = 0.38R + 0.12P$$

Average values:

$$K_L = 0.5(0.64 + 0.50) = 0.57$$

$$K_M = 0.5(0.50 + 0.33) = 0.42$$

$$K_{LM} = 0.5(0.78 + 0.77) = 0.77$$

$$R_m = \frac{8}{L} (M_{Ps} + M_{Pm})$$

$$k_E = 307EI/L^3$$

c. First Trial - Actual Properties.

$$\text{Let } M_{Ps} = M_{Pm} = M_P$$

$$\text{Assume } p = 0.015 \text{ (par. 4-10)}$$

$$\text{Let } \alpha\beta = 5 \text{ (par. 6-26)}$$

$$\text{Assume } C_R = 1.0 \text{ (experience)}$$

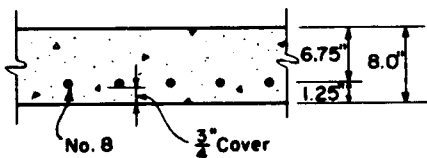
$$R_m = C_R B = 1.0(25.9) = 25.9 \text{ kips}$$

$$M_P = pf_{dy} bd^2 \left( 1 - \frac{pf_{dy}}{1.7f'_{dc}} \right) \quad (\text{eq 4.16})$$

$$= 0.015(52)(1)d^2 \left[ 1 - \frac{0.015(52)}{1.7(3.9)} \right] = 0.688d^2 \text{ kip-ft (d in inches)}$$

$$R_m = \frac{16M_P}{L} = \frac{(16)0.688d^2}{18} = 25.9, \therefore d = 6.5 \text{ in.}$$

$$\text{Try } h = 8 \text{ in., } d = 6.75 \text{ in., } p = 0.015$$



$$M_P = 0.688(6.75)^2 = 31.4 \text{ kip-ft}$$

$$R_m = \frac{16M_P}{L} = \frac{16(31.4)}{18} = 27.9 \text{ kips}$$

$$I_g = bh^3/12 = (8.0)^3 = 511 \text{ in.}^4$$

$$I_t = bd^3 \left[ \frac{k^3}{3} + np(1 - k)^2 \right] = 12(d)^3 \left[ \frac{(0.42)^3}{3} + 0.15(1 - 0.42)^2 \right]$$

$$= 0.905d^3 = 0.905(6.75)^3 = 278 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 0.5(511 + 278) = 394 \text{ in.}^4$$

$$k_E = \frac{307EI}{L^3} = \frac{(307)3(10)^3 394}{(18)^3 144} = 432 \text{ kips/ft}$$

$$y_E = \frac{R_m}{k_E} = \frac{27.9}{432} = 0.0645 \text{ ft}$$

$$y_m = \alpha y_E = 5(0.0645) = 0.3225 \text{ ft (par. 6-26)}$$

$$\text{Weight} = \left[ \frac{8(150)}{(12)} + 6 \right] \frac{18}{1000} = 1.91 \text{ kips}$$

$$\text{Mass } m = \frac{1.91}{32.2} = 0.0591 \text{ kip-sec}^2/\text{ft}$$

d. First Trial - Equivalent System Properties.

$$R_{me} = K_L R_m = 0.57(27.9) = 15.9 \text{ kips (eq 6.12)}$$

$$H_e = K_L H = 0.57(4.92) = 2.8 \text{ kip-sec (eq 6.2)}$$

$$m_e = K_M m = 0.42(0.0591) = 0.0248 \text{ kip-sec}^2/\text{ft (eq 6.2)}$$

$$W_P = \frac{(H_e)^2}{2m_e} = \frac{(2.8)^2}{2(0.0248)} = 158 \text{ ft-kips (eq 6.10)}$$

$$T_n = 2\pi \sqrt{K_{LM}/k_E} = 6.28 \sqrt{0.77(0.0591)/432} = 0.0645 \text{ sec}$$

e. Work Done vs Energy Absorption Capacity.

$$C_T = T/T_n = 0.38/0.0645 = 5.9$$

$$C_R = R_m/B = 27.9/25.9 = 1.08 \text{ (eqs 6.15, 6.16)}$$

$$t_m/T = 0.17 \text{ (fig. 5.29)}$$

$$t_m = (0.17)0.38 = 0.0646 \text{ sec}$$

Idealized load-time curve is satisfactory at  $t = 0.0646 \text{ sec}$  (par. 5-13)

$$C_W = 0.02 \text{ (fig. 5.27)}$$

$$W_m = C_W W_P = 0.02(158) = 3.16 \text{ ft-kips (eq 6.17)}$$

$$E = R_{me}(y_m - 0.5y_E) = 15.9 [0.3225 - 0.5(0.0645)] \\ = 4.62 \text{ ft-kips (eq 6.18)}$$

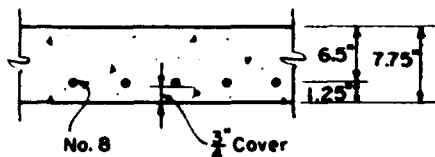
$E > W$ , therefore the selected proportions are satisfactory as a preliminary design.

Try to reduce slab to bring E closer to W.

f. Second Trial - Actual Properties.

$$R_m = \frac{0.25W_m + 0.75E}{K_L(y_m - 0.5y_E)} = \frac{0.25(3.16) + 4.82(0.75)}{(0.57)[0.3225 - 0.5(0.0645)]} = 26.6 \text{ kips (eq 6.19)}$$

$$R_m = \frac{16M_P}{L} = \frac{(16)0.688d^2}{18} = 26.6, \therefore d = 6.6 \text{ in.}$$



Try  $h = 7.75 \text{ in.}$ ,  $d = 6.50 \text{ in.}$ ,  $p = 0.015$

$$M_P = 0.688d^2 = 0.688(6.5)^2 = 29.1 \text{ kip-ft}$$

$$R_m = \frac{16M_P}{L} = \frac{16(29.1)}{18} = 25.9 \text{ kips}$$

$$I_g = \frac{bh^3}{12} = (7.75)^3 = 465 \text{ in.}^4$$

$$I_t = 0.905d^3 = 0.905(6.50)^3 = 248 \text{ in.}^4 \quad (k = 0.42)$$

$$I_a = 0.5(I_g + I_t) = 0.5(465 + 248) = 356 \text{ in.}^4$$

$$k_E = \frac{307EI}{L^3} = \frac{(307)3(10)^3 356}{(18)^3 144} = 391 \text{ kips/ft}$$

$$y_E = \frac{R_m}{k_E} = \frac{25.9}{391} = 0.066 \text{ ft}$$

$$y_m = \alpha \beta y_E = 5(0.066) = 0.330 \text{ ft (par. 6-26)}$$

$$\text{Weight} = \left[ \frac{7.75(150)}{(12)} + 6.0 \right] \frac{18}{1000} = 1.86 \text{ kips}$$

$$\text{Mass } m = \frac{1.86}{32.2} = 0.0575 \text{ kip-sec}^2/\text{ft}$$

g. Second Trial - Equivalent System Properties.

$$R_{me} = K_L R_m = 0.57(25.9) = 14.75 \text{ kips (eq 6.12)}$$

$$H_e = K_L H = 0.57(4.92) = 2.8 \text{ kip-sec (eq 6.2)}$$

$$m_e = K_M m = 0.42(0.0575) = 0.0242 \text{ kip-sec}^2/\text{ft (eq 6.2)}$$

$$W_P = \frac{(H_e)^2}{2m_e} = \frac{(2.8)^2}{2(0.0242)} = 162 \text{ ft-kips (eq 6.10)}$$

$$T_n = 2\pi \sqrt{K_{LM}/k_E} = 6.28 \sqrt{0.77(0.0575)/391} = 0.0667 \text{ sec}$$



h. Work Done vs Energy Absorption Capacity.

$$C_T = T/T_n = 0.38/0.0667 = 5.7$$

$$C_R = R_m/B = 25.9/25.9 = 1.0 \text{ (eqs 6.15, 6.16)}$$

$$t_m/T = 0.24 \text{ (fig. 5.29)}$$

$$t_m = (0.24)0.38 = 0.091 \text{ sec}$$

Idealized load-time curve is satisfactory at time  $t = 0.091$

(par. 5-13)

$$C_W = 0.028 \text{ (fig. 5.27)}$$

$$W_m = C_W W_P = 0.028(162) = 4.55 \text{ ft-kips (eq 6.17)}$$

$$E = R_{me}(y_m - 0.5y_E) = 14.75 [0.330 - 0.5(0.066)] \\ = 4.38 \text{ ft-kips (eq 6.18)}$$

$E \approx W$ , therefore the selected proportions are satisfactory as a preliminary design.

i. Preliminary Design for Bond Stress.

$$\text{Estimated } V_{\max} = 0.5R_m = 0.5(25.9) = 12.9 \text{ kips}$$

$$\text{Allowable } u = 0.15f'_c = 0.15(3000) = 450 \text{ psi (par. 4-09)}$$

$$x_c = \frac{V}{u_j d} = \frac{8(12,900)}{450(7)6.5} = 5.04 \text{ in.}$$

$$\text{Try \#7 at 6 in., } A_s = 1.2 \text{ in.}^2, \Sigma o = 5.5 \text{ in., } p = \frac{A_s}{bd} = \frac{1.2}{12(6.56)} = 0.0152$$

$$np = 10(0.0152) = 0.152, h = 7.75 \text{ in., } d = 6.56 \text{ in.}$$

j. Determination of Maximum Deflection and Dynamic Reactions by Numerical Integration.

$$M_P = pf_{dy} bd^2 \left[ 1 - \frac{pf_{dy}}{1.7f'_{dc}} \right] = 0.0152(52)(1)(6.56)^2 \left[ 1 - \frac{(0.0152)52}{1.7(3.9)} \right] \\ = 30.4 \text{ kip-ft (eq 4.16)}$$

$$I_g = bh^3/12 = (7.75)^3 = 465 \text{ in.}^4$$

$$I_t = bd^3 \left[ \frac{k^3}{3} + np(1 - k)^2 \right] = 0.905(d^3) = 0.905(6.56)^3 = 256 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 0.5(465 + 256) = 360 \text{ in.}^4$$

$$\text{Weight} = \left[ \frac{7.75(150)}{12} + 6 \right] \frac{18}{1000} = 1.86 \text{ kips}$$

$$\text{Mass } m = \frac{1.86}{32.2} = 0.0575 \text{ kip-sec}^2/\text{ft}$$

Elastic range:

$$R_{lm} = \frac{12M_P}{L} - \text{weight} = \frac{12(30.4)}{18} - 1.86 = 18.3 \text{ kips}$$

$$k_1 = \frac{384EI}{L^3} = \frac{(384)3(10)^3 360}{18^3(144)} = 480 \text{ kips/ft}$$

$$y_e = \frac{R_{lm}}{k_1} = \frac{18.3}{480} = 0.0381 \text{ ft}$$

Elasto-plastic range:

$$R_m = \frac{16M_P}{L} - \text{weight} = \frac{16(30.4)}{18} - 1.86 = 25.1 \text{ kips}$$

$$k_{ep} = \frac{384EI}{5L^3} = \frac{1}{5} k_1 = 96 \text{ kips/ft}$$

$$y_{ep} = y_e + \frac{R_m - R_{lm}}{k_{ep}} = 0.0381 + \frac{25.1 - 18.3}{96} = 0.0381 + 0.071 = 0.109 \text{ ft}$$

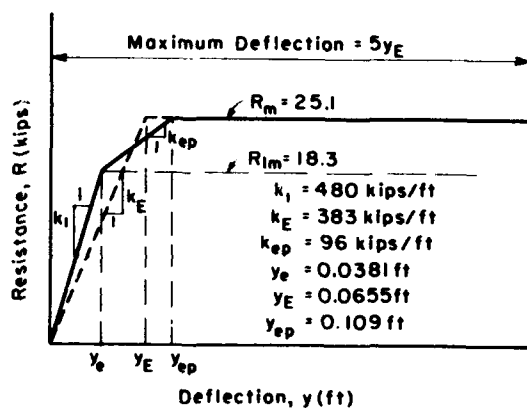


Figure 7.64. Resistance function for 7-3/4-in. slab spanning 18 ft, fixed at both ends

Plastic range:

$$R_m = \frac{16M_P}{L} - \text{weight} = 25.1 \text{ kips}$$

$$k_E = \frac{307EI}{L^3} = \frac{307}{384} k_1 = 383 \text{ kips/ft}$$

$$y_E = \frac{R_m}{k_E} = \frac{25.1}{383} = 0.0655 \text{ ft}$$

$$y_m = \alpha \beta y_e = 5(0.0655) = 0.3275 \text{ ft}$$

$$\begin{aligned} T_n &= 2\pi \sqrt{K_{LM} m / k_E} \\ &= 6.28 \sqrt{0.77(0.0575) / 383} \\ &= 0.0674 \text{ sec} \end{aligned}$$

The basic equation for the numerical integrations in tables 7.24 and 7.25 is  $y_{n+1} = \ddot{y}_n (\Delta t)^2 + 2y_n - y_{n-1}$  (table 5.3) where

**Table 7.24. Determination of Maximum Deflection and Dynamic Reactions for Roof Slab,  
Incident Overpressure (Modified for Rise Time)**

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> - R <sub>n</sub> (kips)	y <sub>n</sub> (Δt) <sup>2</sup> (ft)	y <sub>n</sub> (ft)	V <sub>n</sub> (kips)
0	0	0	2.0	0.0022	0	0
0.007	13.0	1.0	12.0	0.0132	0.0022	2.2
0.014	24.5	8.4	16.1	0.0178	0.0176	6.5
0.021	24.0	19.5	4.5	0.0049	0.0508	10.3
0.028	23.6	23.2	0.4	0.0004	0.0889	11.6
0.035	23.1	25.1	-2.0	-0.0026	0.1274	12.3
0.042	22.6	25.1	-2.5	-0.0032	0.1633	12.3
0.049	22.2	25.1	-2.9	-0.0037	0.1960	12.2
0.056	21.7	25.1	-3.4	-0.0044	0.2250	12.1
0.063	21.3	25.1	-3.8	-0.0049	0.2496	12.1
0.070	20.8	25.1	-4.3	-0.0056	0.2693	12.0
0.077	20.4	25.1	-4.7	-0.0061	0.2834	12.0
0.084	19.9	25.1	-5.2	-0.0067	0.2914	11.9
0.091	19.5	25.1	-5.6	-0.0072	0.2927*	11.9
					0.2868	
*(y <sub>n</sub> ) <sub>max</sub> = 0.29.						

**Table 7.25. Determination of Maximum Deflection and Dynamic Reactions for Roof Slab,  
Incident Overpressure (No Rise Time)**

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> - R <sub>n</sub> (kips)	y <sub>n</sub> (Δt) <sup>2</sup> (ft)	y <sub>n</sub> (ft)	V <sub>n</sub> (kips)
0	25.9	0	12.95	0.01373	0	3.6
0.00685	25.4	6.6	18.8	0.01993	0.01373	4.9
0.01370	24.9	19.2	5.7	0.00596	0.04739	10.2
0.02055	24.4	23.0	1.4	0.00146	0.08701	11.6
0.02740	24.0	25.1	-1.1	-0.00136	0.12809	12.4
0.03425	23.4	25.1	-1.7	-0.00211	0.16781	12.4
0.04110	22.8		-2.3	-0.00285	0.20542	12.3
0.04795	22.4		-2.7	-0.00335	0.24018	12.2
0.05480	22.0		-3.1	-0.00384	0.27159	12.2
0.06165	21.6		-3.5	-0.00434	0.29916	12.1
0.06800	21.1		-4.0	-0.00496	0.32239	12.1
0.07535	20.7		-4.4	-0.00545	0.34066	12.0
0.08220	20.7		-4.4	-0.00545	0.35348	12.0
0.08905	20.2		-4.9	-0.00607	0.36085	12.0
0.09590	19.9		-5.2	-0.00644	0.36215*	11.9
0.10275	18.9	22.6			0.35701	10.8
*(y <sub>n</sub> ) <sub>max</sub> = 0.362.						

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(\Delta t)^2}{K_{IM}(m)} = \frac{(P_n - R_n)(0.007)^2}{K_{IM}(m)}$$

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(49)10^{-6}}{0.77(0.0575)} = 1.107(10^{-3})(P_n - R_n) \text{ ft, elastic range}$$

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(49)10^{-6}}{0.78(0.0575)} = 1.093(10^{-3})(P_n - R_n) \text{ ft, elasto-plastic range}$$

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(49)10^{-6}}{0.66(0.0575)} = 1.291(10^{-3})(P_n - R_n) \text{ ft, plastic range}$$

The time interval  $\Delta t = 0.007$  sec is approximately  $T_n/10 = 0.00674$  sec (par. 5-08). The dynamic reaction equations are listed in paragraph 7-41b. In table 7.24 the slab analysis considers the loading resulting from the shock wave moving along the long axis of the building (incident overpressure modified for rise time). The  $P_n$  values for the second column are obtained from figures 7.57 and 7.63, multiplying by  $144(18)/1000 = 2.59$ . The slab is not critical for this condition  $(y_n)_{\max} = 0.2927 < 0.3275$ .

In table 7.25 the slab analysis considers the loading resulting from the shock wave moving along the short axis of the building (local roof overpressure with no time of rise). The  $P_n$  values for the second column are obtained from figure 7.57, multiplying by 2.59. The slab is critical for this case and in fact the maximum deflection  $(y_n)_{\max} = 0.3621 > 0.3275$ . This is accepted as a satisfactory design since

$$\alpha\beta = \frac{0.3621}{0.0655} = 5.5$$

is close enough to value of 5 established above.

k. Shear Strength and Bond Stress.

$$V_{\max} = 12.4 \text{ kips (table 7.25)}$$

For no shear reinforcement

$$\text{Allowable } v_y = 0.04f'_c + 5000p \text{ (eq 4.24)}$$

$$v_y = 0.04(3000) + 5000(0.0152) = 120 + 76 = 196 \text{ psi}$$

$$v = \frac{8V}{7bd} = \frac{8(12,400)}{7(12)(6.56)} = 178 \text{ psi; OK, no shear reinforcement required}$$

$$u = \frac{8V}{7\Sigma o d} = \frac{8(12,400)}{7(5.5)6.56} = 390 \text{ psi}$$

Allowable  $u = 0.15f'_c = 0.15(3000) = 450$  psi; OK

1. Summary.

7-3/4-in. slab

$p = 0.0152$

$\Sigma o = 5.5$  in.

No shear reinforcement

7-42 PRELIMINARY COLUMN DESIGN. It has been found desirable to make a preliminary design of the column before designing the girders because the column resisting moment is a factor in design of the girder (par. 7-11).

A single-story frame subject to lateral load behaves essentially as a single-degree-of-freedom system. In determining the requirements for the columns which are the springs of this system it is not necessary to use the equivalent system technique used in designing all elements.

Using the principles of paragraph 6-11 and equations from paragraph 7-06 provides a procedure for obtaining preliminary column sizes.

In the preliminary design the girders are assumed to be infinitely rigid to simplify the analysis. In determining the spring constant, the column height is 14.75 ft from the centerline of the girder to the top of the footing. The resistance computation is based on the clear height  $h_c = 13.0$  ft.

If the first trial section is overstrength or understrength it is desirable to make a second trial. In this plastic column design, since the adequacy of the selected section is based on a comparison of  $E$ , the energy absorption capacity, with  $W_m$ , the work done on the frame, successive trials are obtained by estimating a design energy level somewhere between the computed values of  $E$  and  $W$  for use in the equation

$$R_m = \frac{\text{energy}}{x_m}$$

for determining the new trial  $R$ . If  $W_m > E$  the energy level should be set at  $W_m$  so that

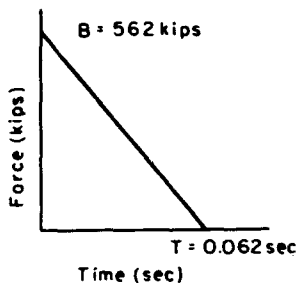
$$R_m = \frac{W_m}{x_m}$$

However, if  $E > W_m$ , as in the following example, the intermediate energy level may be obtained as illustrated on the following page.

a. Design Loading. The design lateral load on the frame is obtained from the dynamic reactions at the top of the front wall slab. However, for a preliminary design it is satisfactory to use the net lateral overpressure curve (fig. 7.60).

In computing the total concentrated lateral load on the frame it is assumed that the wall slab transmits the blast loads equally to the roof slab and foundation. This is generally conservative for the frame because, in the elastic phase, the dynamic reaction at the roof is less than that at the foundation (table 6.1).

The design load as idealized from the computed loading shown by figure 7.60 is defined by:



$$B = 25.3 \text{ psi} = \frac{25.3(144)18 \left[ \frac{15.83}{2} + 0.65 \right]}{1000}$$

$$= 562 \text{ kips}$$

$$T = 0.062 \text{ sec}$$

$$H = \frac{BT}{2} = \frac{(562)(0.062)}{2} = 17.4 \text{ kip-sec (par. 6-11)}$$

b. Mass Computation.

$$\text{Roof slab} \left[ \frac{7.75(150)}{12} + 6.0 \right] \frac{18(44.25)}{1000} = 82.0 \text{ kips}$$

$$\text{Girder (assumed)} \frac{18(32)42(150)}{12(12)1000} = 25.2 \text{ kips}$$

$$3 \text{ columns (assumed)} \frac{12(24)13(150)3}{12(12)1000} = 11.7 \text{ kips}$$

$$2 \text{ wall slabs} \frac{10.5(18)15.83(150)2}{12(1000)} = 74.8 \text{ kips}$$

Mass of single-degree-of-freedom system = total (roof + girder) +

$$1/3(\text{columns} + \text{wall}) = \frac{82.0 + 25.2 + 0.33(11.7 + 74.0)}{32.2} = 4.22 \text{ kip-sec}^2/\text{ft}$$

c. First Trial - Actual Properties.

Assume  $p = 0.015$  (par. 4-10)

Let  $\alpha\beta = 6$  (par. 6-26)

Assume  $C_R = 0.50$  (experience)

$$R_m = C_R B = 0.50(562) = 281 \text{ kips}$$

$$\text{Required } M_D = \frac{R_m h}{2n} = \frac{281(13.0)}{2(3)} = 610 \text{ kip-ft}$$

Approximate average roof pressure = 6.5 psi (estimated in fig. 7.61)

$$\text{Average blast load per column} = \frac{44.25(18)6.5(144)}{3(1000)} = 248 \text{ kips}$$

$$\text{Dead load per column} = 1/3(82.0 + 25.2) = 36.0 \text{ kips}$$

$$\text{Average column design load } P_D = 248 + 36 = 284 \text{ kips}$$

$$M_D = A_s f_{dy} d' + P_D \left( 0.5t - \frac{P_D}{1.76 f'_{dc}} \right) \quad (\text{eq 4.32})$$

$$\text{Let } p = p' = 0.015, d' = (t - 4.5) \text{ in.}, d'' = 2.25 \text{ in.}$$

$$b = 12 \text{ in.}, A_s = pbt$$

Substituting into previous  $M_D$  equation

$$610(12) = 0.015(12)t(52)(t - 4.5) + 284 \left[ 0.5t - \frac{284}{1.7(12)3.9} \right]$$

$$9.35t^2 - 42t + 142t - 1015 - 7320 = 0$$

$$\text{Solving } t = 25.0 \text{ in.}$$

$$\text{Try } b = 12 \text{ in.}, t = 26 \text{ in.}$$

$$M_D = \frac{0.015(12)26(52)(26 - 4.5)}{12} + \frac{284}{12} \left[ 0.5(26) - \frac{284}{1.7(12)3.9} \right]$$

$$M_D = 436 + 223 = 659 \text{ kip-ft}$$

$$d''/d = 2.25/23.75 = 0.0948$$

$$\text{For } p = p' = 0.015 \text{ and } n = 10$$

$$m = np + (n - 1)p' = 0.15 + 9(0.015) = 0.285$$

$$q = np + (n - 1)p' \frac{d''}{d} = 0.15 + 9(0.015)0.0948 = 0.1628$$

$$k = 0.35 \text{ (table 11 RCDH of ACI)}$$

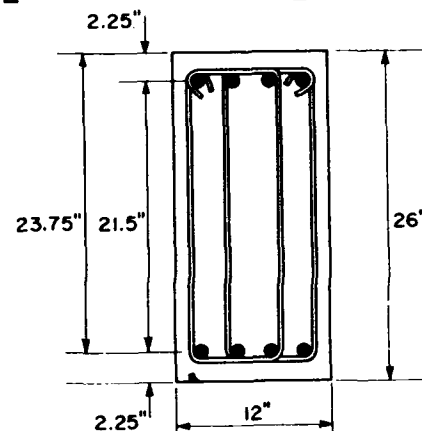
$$I_t = bd^3 \left[ \frac{k^3}{3} + (n - 1)p' \left\{ k^2 - 2k \left( \frac{d''}{d} \right) + \left( \frac{d''}{d} \right)^2 \right\} + np(1 - k)^2 \right]$$

$$= 12(23.75)^3 \left[ \frac{(0.35)^3}{3} + 9(0.015) \left\{ (0.35)^2 - 2(0.35)(0.0948) + (0.0948)^2 \right\} + 0.15(1 - 0.35)^2 \right] = 13,800 \text{ in.}^4$$

$$I_g = bt^3/12 = 26^3/12 = 17,576 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 15,700 \text{ in.}^4$$

$$k = \frac{12EI_a}{h^3} = \frac{12(3)10^3(15,700)3}{(14.75)^3 144} = 3670 \text{ kips/ft}$$



$$R_m = \frac{2n}{h_c} M_D = \frac{2(3)659}{13.0} = 304 \text{ kips}$$

$$x_e = R_m/k = 304/3670 = 0.083 \text{ ft}$$

$$x_m = \alpha \beta x_e = 6x_e = 6(0.083) = 0.498 \text{ ft (par. 6-26)}$$

$$T_n = 2\pi\sqrt{m/k} = 6.28\sqrt{4.22/3670} = 0.213 \text{ sec}$$

d. First Trial - Work Done vs Energy Absorption Capacity.

$$T/T_n = 0.062/0.213 = 0.291$$

$$C_R = R_m/B = 304/562 = 0.54$$

$$t_m/T = 1.35 \text{ (fig. 5.29)}, t_m = 1.35(0.062) = 0.084 \text{ sec}$$

The original load-time curve should be revised to obtain a closer approximation to the total impulse up to time  $t_m$ . The impulse up to  $t = 0.10$  sec in figure 7.60 is  $H = 1.167$  psi-sec (obtained by graphical integration).

$$T = 2H/B = 2(1.167)/25.3 = 0.0923 \text{ sec}$$

$$T/T_n = 0.0923/0.213 = 0.437$$

$$t_m/T = 1.2 \text{ (fig. 5.29)}, t_m = 1.2(0.0923) = 0.111 \text{ sec}$$

Try again for impulse up to  $t = 0.12$  sec

$$H = 1.214 \text{ psi-sec}, T = 2(1.214)/25.3 = 0.096 \text{ sec}, T/T_n = 0.096/0.213 = 0.45$$

$$t_m/T = 1.2 \text{ (fig. 5.29)}, t_m = 1.2(0.096) = 0.115 \text{ sec; OK}$$

$$C_W = 0.71 \text{ (fig. 5.27)}$$

$$W_P = \frac{H^2}{2m} = \frac{(BT/2)^2}{2m} = \frac{(562)^2(0.096)^2}{8(4.22)} = 86.5 \text{ ft-kips}$$

$$W_m = C_W W_P = 0.71(86.5) = 61.5 \text{ ft-kips}$$

$$E = R_m(x_m - 0.5x_e) = 304 [0.498 - 0.5(0.083)] = 139 \text{ ft-kips}$$

Since  $E \gg W_m$  a new trial should be made.

e. Second Trial - Actual Properties. Since  $E \gg W_m$ , the intermediate value of energy for use in determining the second trial  $R_m$  is obtained as follows:

$$R_m = \frac{0.5(W_m + E)}{x_m} = \frac{0.5(61.5 + 139.0)}{0.498} = 201 \text{ kips}$$

$$\text{Required } M_D = \frac{R_m h_c}{2n} = \frac{201(13.0)}{2(3)} = 436 \text{ kip-ft}$$



Use same column constants as above and substitute into equation (4-32)

$$436(12) = 0.015(12)t(52)(t - 4.5) + 284 \left[ 0.5t - \frac{284}{1.7(12)3.9} \right]$$

$$9.35t^2 - 42t + 142t - 1015 - 5240 = 0$$

$$t = 21 \text{ in.}, \text{ try } b = 12 \text{ in.}, t = 20 \text{ in.}$$

$$M_D = \frac{0.015(12)20(52)(20 - 4.5)}{12} + \frac{284}{12} \left[ 10 - \frac{284}{1.7(12)3.9} \right]$$

$$= 242 + 152 = 394 \text{ kip-ft}$$

$$d''/d = 2.25/(20 - 2.25) = 0.127$$

$$\text{For } p = p' = 0.015 \text{ and } n = 10$$

$$m = np + (n - 1)p' = 0.15 + 9(0.015) = 0.285$$

$$q = np + (n - 1)p' d''/d = 0.15 + 9(0.015)0.127 = 0.167$$

$$k = 0.36 \text{ (table 11 RCDH of ACI)}$$

$$I_t = bd^3 \left[ \frac{k^3}{3} + (n - 1)p' \left\{ k^2 - 2k \left( \frac{d''}{d} \right) + \left( \frac{d''}{d} \right)^2 \right\} + np(1 - k)^2 \right]$$

$$= 12(17.75)^3 \left[ \frac{(0.36)^3}{3} + 9(0.015) \left\{ (0.36)^2 - 2(0.36)(0.127) + (0.127)^2 \right\} + 0.15(1 - 0.36)^2 \right]$$

$$I_t = 67,200(0.0845) = 5680 \text{ in.}^4$$

$$I_g = bt^3/12 = (20^3)/12 = 8000 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 6840 \text{ in.}^4$$

$$k = \frac{12EI_n}{h^3} = \frac{12(3)10^3(6840)3}{(14.75)^3 144} = 1590 \text{ kips/ft}$$

$$R_m = \frac{2nM_D}{h_c} = \frac{2(3)394}{13.0} = 182 \text{ kips}$$

$$x_e = R_m/k = 182/1590 = 0.114 \text{ ft}$$

$$x_m = \alpha_B x_e = 6x_e = 6(0.114) = 0.684 \text{ ft (par. 6-26)}$$

$$T_n = 2\pi\sqrt{m/k} = 6.28\sqrt{4.22/1590} = 0.322 \text{ sec}$$

f. Second Trial - Work Done vs Energy Absorption Capacity.

$$T/T_n = 0.096/0.322 = 0.298$$

$$C_R = R_m/B = 182/562 = 0.324$$

$$t_m/T = 1.8 \text{ (fig. 5.29)}, t_m = 1.8(0.096) = 0.173 \text{ sec}$$

The value of  $t_m$  exceeds the value of  $t = 0.12$  sec for which the value of  $T = 0.096$  was obtained. The idealized load-time curve should be revised to approximate more closely the total impulse up to  $t_m$ . The total impulse up to  $t = 0.20$  sec (fig. 7.60) is  $H = 1.374$  psi-sec (obtained by graphical integration).

$$T = 2H/B = 2(1.374)/25.3 = 0.109 \text{ sec}$$

$$T/T_n = 0.109/0.322 = 0.338$$

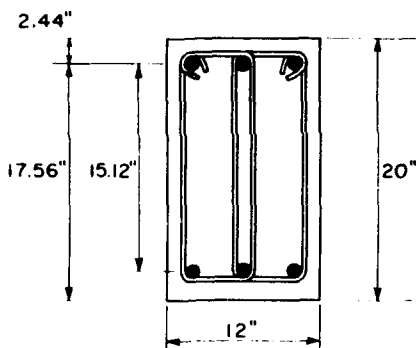
$$t_m/T = 1.75 \text{ (fig. 5.29)}, t_m = 1.75(0.109) = 0.191 \text{ sec; OK}$$

$$C_W = 0.83 \text{ (fig. 5.27)}$$

$$W_P = \frac{H^2}{2m} = \frac{(BT/2)^2}{2m} = \frac{(562)^2(0.109)^2}{8(4.22)} = 111.0 \text{ ft-kips}$$

$$W_m = C_W W_P = 0.83(111.0) = 92.1 \text{ ft-kips}$$

$$E = R_m(x_m - 0.5x_e) = 182 [0.684 - 0.5(0.114)] = 114 \text{ ft-kips}$$



$E$  is slightly greater than  $W$  and another trial might be made. It is desirable, however, to be conservative in the preliminary design because simplifying assumptions are used (par. 7-44). Therefore the 12 by 20 column is selected as the preliminary design and an actual column section is selected to establish the column

plastic bending moment for use in the girder design that follows in paragraph 7-43.

$$d'' = 2.44 \text{ in.}, d = 17.56 \text{ in.}, d' = 15.12 \text{ in.}, 3 \text{ \#9 bars,}$$

$$A_s = A'_s = 3 \text{ in.}^2$$

$$M_D = A_s f_{dy} d' + P_D \left[ 0.5t - \frac{P_D}{1.7(b)f'_{dc}} \right]$$

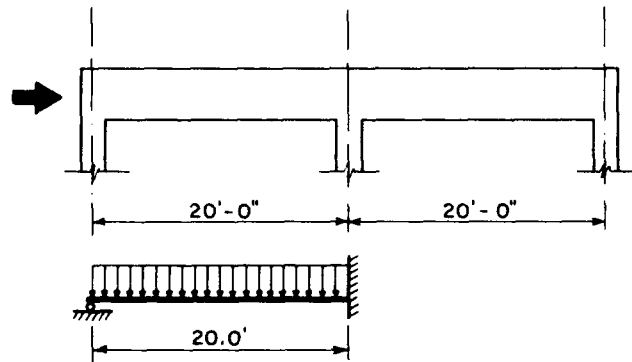
$$= \frac{3(52)15.12}{12} + \frac{284}{12} \left[ 10 - \frac{284}{1.7(12)3.9} \right] = 197 + 237 - 84.5 = 350 \text{ kip-ft}$$

7-43 DESIGN OF ROOF GIRDER. The frame of this building consists of three rectangular columns supporting a rectangular girder which forms a tee beam with the roof slab. The roof girder is designed to resist the combined vertical loads on the roof and the lateral loads on the frame as explained

in paragraph 7-11. Since this manual provides a technique for handling only single-span elements, the continuous girder is designed to withstand vertical loads as a beam fixed at the interior support and pinned at the exterior support.

Although the other structural elements of this building are permitted to deflect plastically, the girder should be designed to behave elastically so that proper restraint is maintained for the column throughout the deflection history of the frame.

The design bending moment of the girder is the sum of the moments at the fixed support due to (1) the roof slab dynamic reactions, (2) the static loads, and (3) the frame action. The moment due to frame action in a two-bay frame is equal to one-half the column plastic moment (par. 7-11).



The girder is designed as a tee beam in the region of positive moment. In the region of negative moment near the interior support the girder is of rectangular section. This results in a girder of variable moment of inertia for which there are special expressions for  $R_m$ ,  $y_m$ , and  $k_1$  (par. 6-21a and table 6.4).

a. Loading. The critical frame girder loading results from the blast wave traveling parallel to the girder. Since the girder is parallel to the short side of the building the appropriate roof loading is from the Zone 3 condition. For Zone 3 loading the time variation of the local roof overpressure varies continuously from front to back of the building.

To obtain the true variation of slab dynamic reaction on the girder would be a tedious task as well as unjustified in the light of all the possible inaccuracies. In this example a conservative loading based on the incident overpressure curve (fig. 7.57) is used as the basis for the slab load in determining the girder load. This is warranted because in general the slab reaches its maximum displacement before the vortex action in Zone 3 has an opportunity to reduce the incident overpressure to the local

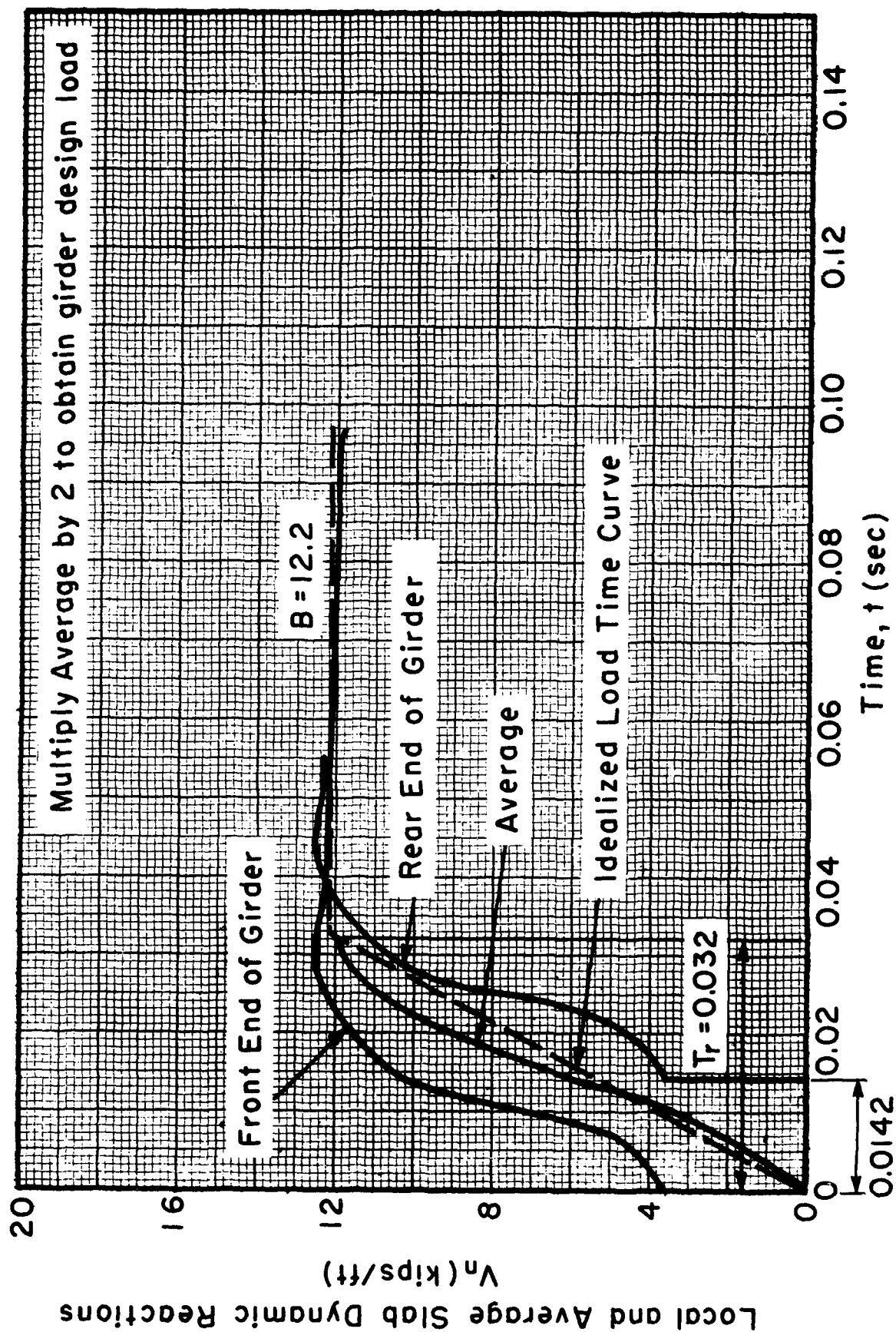
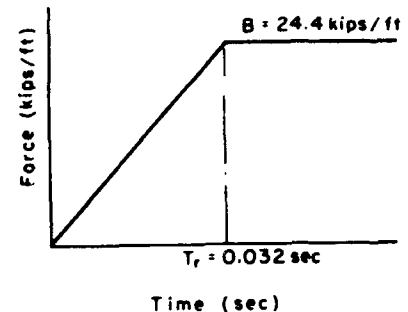


Figure 7.65. Local and average slab dynamic reactions for incident overpressure, blast wave parallel to girder

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roof overpressure (see pars. 7-23 and 7-25 and table 7.5).

The girder load is determined from the slab dynamic reactions for incident overpressure with the blast wave traveling parallel to the girder. Plotting from table 7.25 the same variation of dynamic reaction at the front and rear ends of the girder and averaging the total load on the girder gives the variation of average girder load from one slab in figure 7.65. The preliminary design load is obtained by idealizing the load-time curve in figure 7.65 and multiplying by two to account for the two slabs loading the girder. The idealized load is



$$B = 12.2(2) = 24.4 \text{ kips/ft}$$

$$T_r = 0.032 \text{ sec}$$

b. Elastic Range Dynamic Design Factors. (Refer to table 6.4.)

$$K_L = 0.58, \quad K_M = 0.45, \quad K_{IM} = 0.78$$

$$k = f_3 EI_1 / L^3 \text{ (fig. 6.29)}$$

$$R_m = f_1 M_{Ps} / L \text{ (fig. 6.27)}$$

$$M_{pos} = f_2 M_n \text{ (fig. 6.28)}$$

$$M_n = R_1 L / f_1 \text{ (fig. 6.27)}$$

$$V_1 = 0.26R + 0.12P$$

$$V_2 = 0.43R + 0.19P$$

c. Mass Computation.

$$\text{Slab and roofing} = \left[ \frac{7.75(150)}{12} + 6.0 \right] \frac{18(44.25)}{1000(2)} = 41.0 \text{ kips}$$

$$\text{Girder (estimate)} = \frac{18(36)20(150)}{(12)12(1000)} = 13.5 \text{ kips}$$

$$\text{Total mass} = \frac{41.0 + 13.5}{32.2} = 1.69 \text{ kip-sec}^2/\text{ft}$$

d. First Trial - Actual Properties.

Estimate tee beam action for first trial,

$$f_1 = 9, \quad f_2 = 0.67, \quad f_3 = 240$$

Assume D.L.F. = 1.5 (experience)

$$R_m = \text{D.L.F.}(B) = 1.5(24.4)(20) = 732 \text{ kips}$$

Moment in girder at interior support (fixed support) due to static loads

$$M = WL/8 = (41.0 + 13.5)20/8 = 136 \text{ kip-ft}$$

Moment in girder at midspan due to static load

$$M = 9WL/128 = 9(54.5)20/128 = 77 \text{ kip-ft}$$

Moment from column

$$M = 0.5M_p = 0.5(350) = 175 \text{ kip-ft (par. 7-42f)}$$

Moment resistance required for vertical blast loads

$$M = R_m L/9 = 732(20)/9 = 1630 \text{ kip-ft}$$

The total moment resistance required

$$1630 + 175 + 136 = 1941 \text{ kip-ft}$$

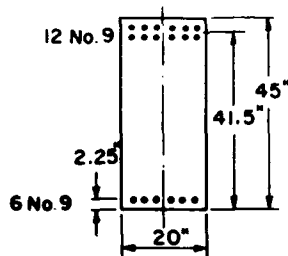
$$M_p = p f_{dy} b d^2 \left( 1 - \frac{p f_{dy}}{1.7 f'_{dc}} \right) \text{ (eq 4.16)}$$

Assume  $p = 0.015$

$$M_p = 0.015(52) b d^2 \left[ 1 - \frac{0.015(52)}{1.7(3.9)} \right] = 0.688 b d^2 = 1941(12) \\ = 23,300 \text{ kip-in.}$$

If  $b = 20 \text{ in.}$ ,  $d = 41.2 \text{ in.}$ ; try  $h = 45 \text{ in.}$ ,  $d = 41.5 \text{ in.}$

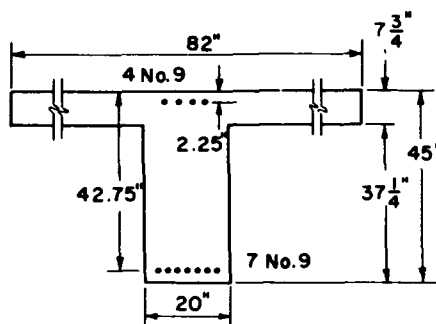
From figure 6.28 estimate  $f_2 = 0.67$  (from experience)



Effective Section at Interior Support

$$\text{Midspan moment} = 0.67(1630) + 0 + 77 \\ = 1090 + 77 = 1167 \text{ kip-ft}$$

Ratio of midspan tension reinforcement to interior support tension reinforcement is  $\frac{1167}{1941} = 0.60$ .



Effective Section at Midspan

Rectangular section at fixed support

$$np = 10(12)/20(41.5) = 0.1445$$

$$np' = 10(6)/20(41.5) = 0.0722$$

$$I_g = 20(45)^3/12 = 152,000 \text{ in.}^4$$

$$m = np + (n - 1)p'$$

$$= 0.1445 + \frac{9}{10} (0.0722) = 0.2095$$

$$q = np + (n - 1)p' \frac{d'}{d}$$

$$= 0.1445 + \frac{9}{10} (0.0722) \frac{2.25}{41.5} = 0.148$$

$$k = 0.38 \text{ (table 11 RCDH of ACI)}$$

$$kd = 0.38(41.5) = 15.7$$

$$I_t = 20(15.7)^3/3 + 9(6)(13.45)^2 + 12.0(10)(25.8)^2 \\ = 19,350 + 9750 + 80,000 = 109,000 \text{ in.}^4$$

$$I_1 = (I_g + I_t)0.5 = 130,000 \text{ in.}^4$$

Tee section at midspan

$$np = 10(7)/82(42.75) = 0.0200$$

$$np' = 10(4)/82(42.75) = 0.0114$$

$$\bar{y} = \frac{[-82(7.75)^2]/2 + [20(37.25)^2]/2}{82(7.75) + 20(37.25)} = \frac{11,640}{1380} = 8.45 \text{ in.}$$

$$I_g = \frac{82(7.75)^3}{3} + \frac{20(37.25)^3}{3} - 1380(8.45)^2 \\ = 12,700 + 345,000 - 98,500 = 250,200 \text{ in.}^4$$

$$m = np + (n - 1)p' = 0.0200 + 0.9(0.0114) = 0.0303$$

$$q = np + (n - 1)p' \frac{d'}{d} = 0.0200 + 0.9(0.0114) \frac{2.25}{42.75} = 0.0205$$

$$k = 0.18 \text{ (table 11 RCDH of ACI)}, kd = (0.18)(42.75) = 7.7 \text{ in.}$$

$$I_t = \frac{82(7.7)^3}{3} + 9(4)(7.7 - 2.25)^2 + 10(7)(42.75 - 7.7)^2 \\ = 12,500 + 1070 + 86,000 = 99,570 \text{ in.}^4$$

$$I_2 = 0.5(I_g + I_t) = 179,400 \text{ in.}^4$$

$$I_1/I_2 = 130,000/179,400 = 0.725$$

$$f_1 = 8.85 \text{ (fig. 6.27)}$$

$$f_2 = 0.66 \text{ (fig. 6.28)}$$

$$f_3 = 235 \text{ (fig. 6.29)}$$

$$k_1 = \frac{f_3 EI_1}{L^3} = \frac{235(3)10^3(130,000)}{(20)^3 144} = 79,500 \text{ kips/ft}$$

At the interior support

$$M_P = pf_{dy} bd^2 \left( 1 - \frac{pf_{dy}}{1.7f'_{dc}} \right) \text{ (eq 4.16)}$$

$$= \frac{0.01445(52)20(41.5)^2}{12} \left[ 1 - \frac{0.01445(52)}{1.7(3.9)} \right] = 1910 \text{ kip-ft}$$

$$T_n = 2\pi \sqrt{\frac{K_{1M}^m}{k_1}} = 6.28 \sqrt{\frac{0.78(1.69)}{79,500}} = 0.0256 \text{ sec}$$

$$T_r/T_n = 0.032/0.0256 = 1.25$$

$$\left. \begin{array}{l} \text{D.L.F.} = 1.18 \\ t_m/T_r = 1.3 \end{array} \right\} \text{ (fig. 5.21)}$$

$$t_m = 1.3(0.032) = 0.0417 \text{ sec}$$

The idealized loading is a satisfactory approximation for the loading in figure 7.65.

$$\text{Required } R = 1.18(24.4)20 = 575 \text{ kips}$$

The available resisting moment at the interior support for vertical blast loads is

$$M = 1910 - 136 - 197 = 1577 \text{ kip-ft}$$

The available resistance is

$$R_{1m} = f_1 M_{Ps}/L = 8.9(1577)/20 = 700 \text{ kip-ft}$$

The available  $R_{1m}$  is greater than the required  $R_{1m}$ . Another trial will be made by reducing the amount of reinforcing steel. Reducing the steel required by the ratio of the resistance calculated above

$$\text{Use } 10 \text{ \#9 tension bars; } A_s = (575/700)(12) = 9.85 \text{ in.}^2$$

$$p = (10)/20(41.5) = 0.01205$$

$$M_P = \frac{0.01205(52)20(41.5)^2}{12} \left[ 1 - \frac{0.01205(52)}{1.7(3.9)} \right] = 1635 \text{ kip-ft}$$

Assuming no substantial change in  $I_1/I_2$  and  $T_n$  the available resistance is  $R_{1m} = 8.9(1635 - 197 - 136)/20 = 580 \text{ kips} \approx 575 \text{ kips}$

e. Second Trial - Actual Properties.

Rectangular section at fixed support

$$np = 10(10)/20(41.5) = 0.1205$$

$$np' = 10(6)/20(41.5) = 0.0702$$

$$I_g = 152,000 \text{ in.}^4$$

$$m = np + (n - 1)p' = 0.1205 + \frac{9}{10} (0.0702) = 0.184$$

$$q = np + (n - 1)p' \frac{d'}{d} = 0.1205 + \frac{9}{10} (0.0702) \frac{2.25}{41.5} = 0.124$$

$$k = 0.35 \text{ (table 11 RCDH of ACI)}$$

$$kd = 0.35(41.5) = 14.5 \text{ in.}$$



$$I_t = \frac{20(14.5)^3}{3} + 9(6)(12.25)^2 + 10(10)(27.0)^2$$

$$= 20,300 + 8100 + 73,000 = 101,400 \text{ in.}^4$$

$$I_1 = 0.5(I_g + I_t) = 126,000 \text{ in.}^4$$

The section at midspan

The tension steel at midspan =  $0.6(10) = 6$  bars (par. 7-43d)

$$np = 10(6)/82 (42.75) = 0.0171$$

$$np' = 10(3)/82 (42.75) = 0.00855$$

$$m = np + (n - 1)p' = 0.0171 + 0.9(0.00855) = 0.0248$$

$$q = np + (n - 1)p' \frac{d'}{d} = 0.0171 + 0.9(0.00855) \frac{2.25}{42.75} = 0.0175$$

$$k = 0.165 \text{ (table 11 RCDH of ACI)}$$

$$kd = 0.165(42.75) = 7.06 \text{ in.}$$

$$I_t = 81,300 \text{ in.}^4, I_g = 259,200 \text{ in.}^4, I_a = (I_g + I_t)0.5 = 170,200 \text{ in.}^4$$

$$I_1/I_2 = 126,700/170,200 = 0.74$$

$$f_1 = 8.8 \text{ (fig. 6.27)}$$

$$f_2 = 0.655 \text{ (fig. 6.28)}$$

$$f_3 = 232 \text{ (fig. 6.29)}$$

$$k_1 = \frac{f_3 EI_1}{L^3} = \frac{(232)3(10)^3 126,700}{(20)^3 144} = 79,000 \text{ kips/ft}$$

$$T_n = 2\pi \sqrt{\frac{K_{IM}^m}{k_1}} = 6.28 \sqrt{\frac{0.78(1.69)}{79,000}} = 0.0257 \text{ sec}$$

$$T_r/T_n = 0.032/0.0257 = 1.25$$

$$\text{D.L.F.} = 1.18 \text{ (fig. 5.21)}$$

$$\text{Required } R_{lm} = 1.18(24.4)(20) = 575 \text{ kips}$$

f. Preliminary Design for Bond Stress.

$$\text{Estimated } V_{\max} = 0.62R_m = 0.62(575) = 356 \text{ kips}$$

$$\text{Allowable } u = 0.15f'_c = 0.15(3000) = 450 \text{ psi (par. 4-09)}$$

$$\Sigma o = \frac{V}{u_j d} = \frac{8(356,000)}{450(7)41.5} = 22.0 \text{ in.}$$

$$A_s = 10 \text{ \#9 bars in two rows, } A_s = 10 \text{ in.}^2$$

$$\Sigma o = 35.4 \text{ in., } p = A_s/bd = \frac{10}{20(41.5)} = 0.01205$$

g. Determination of Maximum Deflection and Dynamic Reactions by Numerical Integration. Since the trial size may be used directly without modification reference is made to the previous computations for pertinent data.

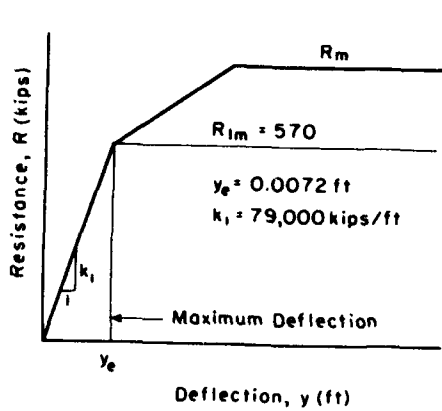


Figure 7.66. Resistance function for girder spanning 20 ft

$$\text{Revised girder weight} = \frac{20(37.25)(20)150}{12(12)1000}$$

$$= 15.6 \text{ kips}$$

$$\text{Total mass} = \frac{41.0 + 15.6}{32.2} = 1.76 \text{ kip-sec}^2/\text{ft}$$

$$\text{Static load girder moment} = \frac{WL}{8}$$

$$= \frac{(41.0 + 15.6)20}{8} = 142 \text{ kip-ft}$$

Maximum girder resistance in elastic

$$\text{range } R_{lm} = \frac{8.8(1635 - 142 - 197)}{20}$$

$$= 570 \text{ kips}$$

$$y_e = \frac{R_{lm}}{k_1} = \frac{570}{79,000} = 0.0072 \text{ ft}$$

The basic equation for the numerical integration in table 7.20 is  $y_{n+1} = \ddot{y}_n(\Delta t)^2 + 2y_n - y_{n-1}$  (table 5.3) where

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(\Delta t)^2}{K_{IM}(m)} = \frac{(P_n - R_n)(0.0025)^2}{K_{IM}(m)}$$

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(6.25)(10^{-6})}{0.78(1.76)} = 4.55(10^{-6})(P_n - R_n) \text{ ft, elastic range}$$

The time interval  $\Delta t = 0.0025$  sec is approximately  $T_n/10 = 0.00257$  sec (par. 5-08).

The dynamic reaction equations are listed in paragraph 7-43b. The  $P_n$  values for the second column are obtained from figure 7.65, multiplying by  $2 \times 20$  to account for the 20-ft span and the two slabs loading the girder.

The maximum deflection computed in table 7.26 is  $(y_n)_{\max} = 0.0069$  ft. This is less than, but close to, the specified maximum deflection,  $y_e = 0.0072$  ft. The design is satisfactory.

Table 7.26. Determination of Maximum Deflection and Dynamic Reactions for Roof Girder

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> - R <sub>n</sub> (kips)	y <sub>n</sub> (Δt) <sup>2</sup> (ft)	y <sub>n</sub> (ft)	V <sub>1n</sub> (kips)	V <sub>2n</sub> (kips)
0	0	0	4.4	0.000020	0	0	0
0.0025	28.0	1.6	26.4	0.000120	0.000020	4	6
0.0050	56.0	12.6	43.4	0.000197	0.000160	10	16
0.0075	96.0	39.3	56.7	0.000258	0.000497	22	35
0.0100	136.0	86.3	49.7	0.000226	0.001092	39	63
0.0125	192.0	151.1	40.9	0.000186	0.001913	62	102
0.0150	256.0	230.7	25.3	0.000115	0.002920	91	148
0.0175	312.0	319.3	-7.3	-0.000033	0.004042	120	197
0.0200	360.0	402.2	-42.2	-0.000192	0.005091	148	241
0.0225	408.0	469.9	-61.9	-0.000282	0.005948	171	280
0.0250	436.0	515.3	-79.3	-0.000361	0.006523	186	304
0.0275	460.0	532.2	-72.2	-0.000328	0.006737	194	316
0.0300	472.0	523.2	-51.2	-0.000233	0.006623	193	315
0.0325	480.0	495.8	-15.8	-0.000072	0.006276	186	304
0.0350	484.0	402.7	21.3	0.000097	0.005857	178	291
0.0375	488.0	437.3	50.7	0.000231	0.005535	172	281
0.0400	488.0	430.1	57.9	0.000263	0.005444	170	278
0.0425	488.0	443.7	44.3	0.000202	0.005616	174	284
0.0450	488.0	473.2	14.8	0.000067	0.005990	182	296
0.0475	488.0	508.0	-20.0	-0.000091	0.006431	191	311
0.0500	488.0	535.7	-47.7	-0.000217	0.006781	198	323
0.0525	488.0	546.2	-58.2	-0.000265	0.006914	201	328
					0.006782		
* (y <sub>n</sub> ) <sub>max</sub> = 0.0069 ft.							

h. Shear and Bond Strength.

For interior support end of girder (fixed end of idealized girder)

$$V_{\max} = 328 \text{ kips (table 7.26)}$$

For no shear reinforcement, allowable  $v_y = 0.04f'_c + 5000p$  (eq 4.24)

$$v_y = 0.04(3000) + 5000(0.01205) = 120 + 61 = 181 \text{ psi}$$

$$v = \frac{8V}{7bd} = \frac{8(328,000)}{7(20)41.5} = 452 \text{ psi}$$

Shear reinforcement required for  $452 - 181 = 271 \text{ psi}$ Contribution of shear reinforcement to allowable shear stress =  $rf_y$

$$r = \frac{271}{40,000} = 0.00677$$

$$\text{Try 4 \#4, } A_s = 0.80 \text{ in.}^2$$

$$r = \frac{A_s}{bs} = \frac{0.80}{20(s)} = 0.00677, \therefore s = 5.9 \text{ in., use } s = 6 \text{ in.}$$

For exterior support end of girder (pinned end of idealized girder)

$$V_{\max} = 201 \text{ kips (table 7.26)}$$

$$v_y = 0.04(3000) + 5000(0.00171) = 120 + 9 = 129 \text{ psi}$$

$$v = \frac{8V}{7bd} = \frac{8(201,000)}{7(20)(42.75)} = 268 \text{ psi}$$

Shear reinforcement required for  $268 - 129 = 139 \text{ psi}$

$$r = \frac{139}{40,000} = 0.00347$$

$$\text{Try 4 \#4, } A_s = 0.80 \text{ in.}^2$$

$$r = \frac{A_s}{bs} = \frac{0.80}{20(s)} = 0.00347; \therefore s = 11.5 \text{ in., use } s = 11 \text{ in.}$$

Bond

$$u = \frac{8V}{7\Sigma o d} = \frac{8(201,000)}{7(21.3)(42.75)} = 252 \text{ psi}$$

Allowable  $u = 0.15f'_c = 0.15(3000) = 450 \text{ psi; OK}$

i. Summary.

Girder 20-in.  $\times$  45-in. tee beam

7-44 FINAL DESIGN OF COLUMN. This column design for plastic behavior was begun in paragraph 7-42. The calculations which follow illustrate the steps which are needed to determine the adequacy of the preliminary design. The primary objective is the calculation of the lateral displacement of the top of the column as a function of time by a numerical integration.

In the preliminary design of the column some of the factors which affect the maximum deflection of the column are neglected to simplify the computations. These factors which are now considered are: the variation of plastic hinge moment with direct stress, the variation of column resistance with lateral deflection, the effect of girder flexibility on the stiffness of the frame, the difference between the load on the wall slab, and the dynamic reactions from the wall which are used as the lateral design load for the frame columns.

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Reference should be made to the preliminary design in paragraph 7-42.

a. Mass Computation.

Roof slab = 82.0 kips (par. 7-42b)

$$\text{Girder stem} = \frac{34.25(20)(41.67)150}{12(12)1000} = 29.8 \text{ kips}$$

$$\text{Columns} = \frac{12(20)13(150)3}{12(12)1000} = 9.75 \text{ kips}$$

Walls = 74.8 kips (par. 7-42b)

Mass of single-degree-of-freedom system = total roof +

$$\frac{1}{3} (\text{columns} + \text{walls}) = \frac{82.0 + 29.8 + 0.33(9.75 + 74.8)}{32.2}$$

$$= 4.35 \text{ kip-sec}^2/\text{ft}$$

b. Column Properties. (See par. 7-42h.)

b = 12 in., t = 20 in., d = 17.56 in.

d' = 15.12 in., d'' = 2.44 in.,  $A_s = A'_s = 3 \text{ in.}^2$ , p = 0.0142

$$M_D = A_s f_{dy} d' + P_D \left[ 0.5t - \frac{P_D}{1.7bf'_{dc}} \right] \quad (\text{eq 4.32})$$

$$= \frac{3(52)(15.12)}{12} + \frac{P_D}{12} \left[ 10 - \frac{P_D}{1.7(12)3.9} \right]$$

$$M_D = 196.6 + 0.83P_D - 0.00105P_D^2$$

c. Effect of Girder Flexibility. Consideration of the relative flexibility of the girders generally results in a value of the frame elastic spring constant k which is less than the value obtained for infinitely stiff girders in the preliminary design (par. 7-08). To obtain this revised value a simple sidesway analysis of the frame is made. From the sidesway analysis k is the magnitude of the lateral load required to cause unit displacement.

The elastic sidesway analysis (the results of which are shown in figure 7.67) is performed for initial column moments of -1000 kip-ft at top and bottom of each column. This is equivalent to a lateral displacement of the top of the column.

$$x = \frac{(F.E.M.)h^2}{6EI} = \frac{1000(14.63)^2 144}{6(3)10^3(6890)} = 0.249 \text{ ft}$$

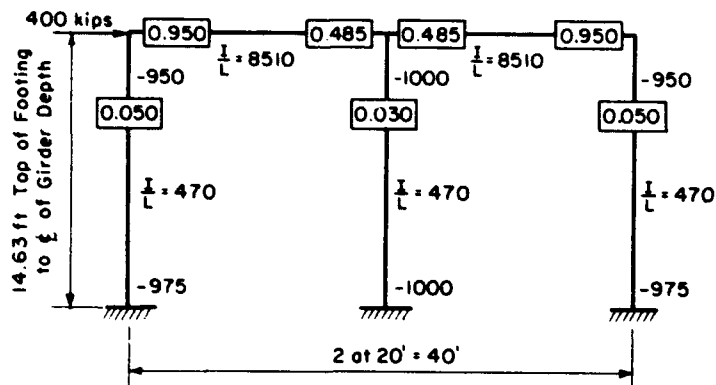


Figure 7.67. Sidesway analysis by moment distribution

spring constants (par. 7-42e).

d. Loading. The  $F_n$  column of the numerical integration analysis (table 7.27) is obtained from figure 7.68. The first part of figure 7.68

From figure 7.67

$$R = \frac{\Sigma M}{h} = \frac{2(950 + 975 + 1000)}{14.63}$$

$$= 400 \text{ kips}$$

$$k = \frac{R}{x} = \frac{400}{0.249}$$

$$= 1600 \text{ kips/ft}$$

In this example the girders are relatively stiff and there is no significant reduction in

Table 7.27. Determination of Column Adequacy

t (sec)	$\bar{P}_{\text{roof}}$ (psi)	$\frac{P_n}{3} = (P_D)_n$ (kips)	$P_n/h_c$ (kips/ft)	$M_D$ (kips/ft)	$R_m$ (kips)	$F_n$ (kips)	$R_n$ (kips)	$\frac{P_n}{h_c} x_n$ (kips)	$F_n - R_n + \frac{P_n}{h_c} x_n$ (kips)	$x_n(\Delta t)^2$ (ft)	$x_n$ (ft)
0	0	37.3	8.8			125	0	0	62.5	0.00144	0
0.01	2.90	148.1	34.9			355	2.3	0.1	352.8	0.00811	0.00144
0.02	5.80	259.0	60.9			386	17.6	0.7	369.1	0.00849	0.01099
0.03	8.70	369.9	87.0			366	46.4	2.5	322.1	0.00740	0.02903
0.04	8.30	354.6	83.4	358.9	168.9	355	87.2	4.5	272.3	0.00626	0.05447
0.05	7.90	339.3	79.8	357.3	168.1	282	137.9	6.9	151.0	0.00347	0.08617
0.06	7.51	324.4	76.3	355.4	167.2	145	167.2	9.3	-12.9	-0.00030	0.12134
0.07	7.15	310.6	73.1	353.1	166.2	100	166.2	11.4	-54.8	-0.00126	0.15621
0.08	6.84	298.8	70.3	350.9	165.1	55	165.1	13.3	-96.8	-0.00222	0.18982
0.09	6.53	286.9	67.5	348.3	163.9	54	163.9	14.9	-95.0	-0.00218	0.22121
0.10	6.26	276.6	65.1	345.9	162.8	52	162.8	16.3	-94.5	-0.00217	0.25042
0.11	6.00	266.7	62.7	343.3	161.6	51	161.6	17.4	-93.2	-0.00214	0.27746
0.12	5.76	257.5	60.6	340.7	160.3	50	160.3	18.3	-92.0	-0.00211	0.30236
0.13	5.52	248.3	58.4	338.0	159.1	49	159.1	19.0	-91.1	-0.00209	0.32515
0.14	5.30	239.9	56.4	335.3	157.8	47	157.8	19.5	-91.3	-0.00210	0.34585
0.15	5.11	232.6	54.7	332.9	156.7	46	156.7	19.9	-90.8	-0.00209	0.36445
0.16	4.92	225.4	53.0	330.4	155.5	45	155.5	20.2	-90.3	-0.00208	0.38096
0.17	4.74	218.5	51.4	327.9	154.3	44	154.3	20.3	-90.0	-0.00207	0.39539
0.18	4.58	212.4	50.0	325.5	153.2	43	153.2	20.4	-89.8	-0.00206	0.40775
0.19	4.44	207.0	48.7	323.4	152.2	41	152.2	20.4	-90.8	-0.00209	0.41805
0.20	4.31	202.1	47.5	321.4	151.2	40	151.2	20.2	-91.0	-0.00209	0.42626
0.21	4.18	197.1	46.4	319.4	150.3	38	150.3	20.1	-92.2	-0.00212	0.43238
0.22	4.06	192.5	45.3	317.5	149.4	36	149.4	19.8	-93.6	-0.00215	0.43638
0.23	3.94	187.9	44.2	315.5	148.5	34	148.5	19.4	-95.1	-0.00219	0.43823*
0.24	3.81	182.9	43.0	313.3	147.4	32	147.4				0.43789
0.25	3.70	178.7	42.0	311.4	146.5	30	146.5				

\*  $x_{\text{max}} = 0.44 \text{ ft.}$

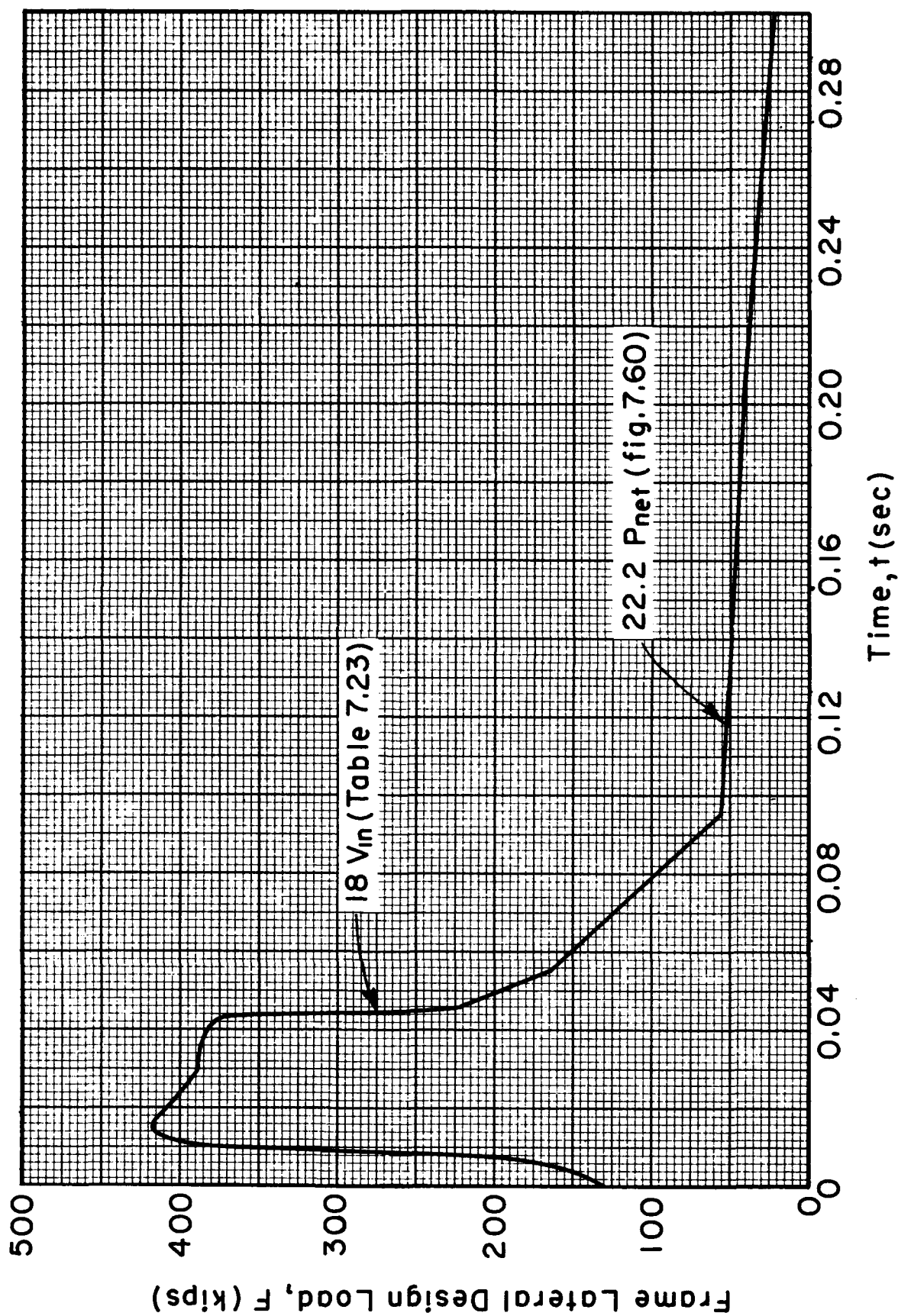


Figure 7.68. Frame lateral design load

is based on  $V_{ln}$  dynamic reaction column of the wall slab analysis (table 7.23). The dynamic reactions for a one-foot width of wall are multiplied by 18, the width of one frame bay.

The portion of the curve after  $t = 0.045$  sec is based on the net lateral overpressure curve (fig. 7.60). Here the values of  $F_n$  are obtained from

$$F_n = \frac{144(18)8.56}{1000} \bar{P}_{net} = 22.2\bar{P}_{net} \text{ kips}$$

where  $\bar{P}_{net}$  is in psi. The dimension 8.56 is equal to one-half the front wall clear span plus the roof slab thickness,  $1/2 (15.82) + 0.65 = 8.56$  ft.

e. Numerical Integration Computation to Determine Column Adequacy.

The total vertical load  $P_n$  is obtained by multiplying the average roof overpressure (fig. 7.61) by

$$\frac{(44.25)(18)144}{1000} = 115$$

and adding the dead weight of the roof system (112 kips). The  $(P_D)_n$  values are the average axial column loads and are obtained by dividing the total vertical load by 3, the number of columns. The  $(P_D)_n$  column is used in the formula of paragraph 7-44b to obtain the value of  $(M_D)_n$ . The value of  $(M_D)_n$  is used in turn to obtain the maximum resistance at any time from the relation

$$(R_m)_n = \frac{2n(M_D)_n}{h_c} = \frac{2(3)(M_D)_n}{12.75} = 0.47(M_D)_n$$

$R_n$  is equal to  $kx_n = 1600x_n$  in the elastic range. In the plastic range the limiting value of  $R$  is  $R_m$ . The expression  $(P_n x_n)/h_c$  indicates the decrease in resistance corresponding to the increase in moment resulting from the eccentric loading.

The basic equation for the numerical integration analysis in table 7.21 is

$$x_{n+1} = \ddot{x}_n (\Delta t)^2 + 2x_n - x_{n-1} \text{ (table 5.3)}$$

where



$$\ddot{x}_n(\Delta t)^2 = \frac{\left[ F_n - R_n + \frac{P_n x_n}{h_c} \right]}{m} (\Delta t)^2 = \frac{\left[ F_n - R_n + \frac{P_n x_n}{h_c} \right]}{4.35} (0.01)^2$$

$$= 2.3(10^{-5}) \left[ F_n - R_n + \frac{P_n}{h_c} (x_n) \right]$$

The time interval  $\Delta t = 0.01$  sec used in table 7.27 is less than one-tenth the natural period,  $T_n = 2\pi\sqrt{m/k} = 6.28\sqrt{4.35/1600} = 0.327$  sec.

$\Delta t = 0.01 < \frac{0.327}{10} = 0.0327$  (par. 5-08) in order to provide a more faithful representation of the load curve (fig. 7.68). The allowable maximum displacement =  $\frac{\alpha\beta R_m}{k} = \frac{6(167.2)}{1600} = 0.627$  ft (par. 7-42c and table 7.21). The computed maximum displacement = 0.44. Thus the design  $\alpha\beta = 6 \left( \frac{0.44}{0.627} \right) = 4.2$ . The design is satisfactory.

f. Shear and Bond Stress.

$$V_{\max} = \frac{167.2}{3} = 55.7 \text{ kips (table 7.27)}$$

For no shear reinforcement

$$\text{Allowable } v_y = 0.04f'_c + 5000p \text{ (eq 4.24)}$$

$$v_y = 0.04(3000) + 5000(0.0142) = 120 + 71 = 191 \text{ psi}$$

$$v = \frac{8V}{7bd} = \frac{8(55,700)}{7(12)(17.56)} = 303 \text{ psi}$$

Shear reinforcement required for  $303 - 191 = 112$  psi

$$\text{Try 1 \#4, } A_s = 0.20 \text{ in.}^2$$

$$r = \frac{112}{40,000} = 0.0028$$

$$r = \frac{A_s}{bs} = 0.0028 = \frac{0.20}{12(s)} ; \therefore s = 5.95 \text{ in., use } s = 6 \text{ in.}$$

$$\Sigma o = 8.9 \text{ in.}$$

$$u = \frac{8(55,700)}{7(8.9)(17.56)} = 407 \text{ psi}$$

$$\text{Allowable } u = 0.15f'_c = 0.15(3000) = 450 \text{ psi} > 407 \text{ psi; OK}$$

**NUMERICAL EXAMPLE, DESIGN OF A ONE-STORY REINFORCED-CONCRETE FRAME BUILDING--ELASTIC AND ELASTO-PLASTIC BEHAVIOR**

7-45 GENERAL. This numerical example presents the design of a typical bay of a windowless, one-story, reinforced-concrete rigid-frame building in

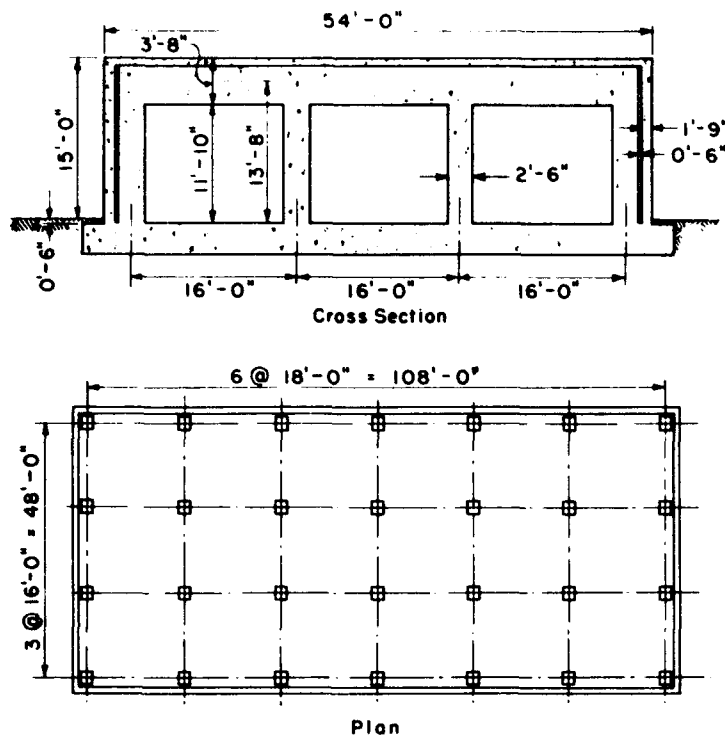


Figure 7.69. Plan and section of building

is also a one-way slab spanning continuously over the reinforced-concrete frames. The roof girders are tee beams formed by the roof slab and a rectangular girder stem. The columns are rectangular tied columns symmetrically reinforced in the strong direction.

7-46 DESIGN PROCEDURE. The over-all design procedure illustrated by this example is essentially the same as the procedure detailed in paragraph 7-29 for the elastic design of a steel rigid-frame building with reinforced concrete walls and roof.

7-47 LOAD DETERMINATION. The computation of loads is explained in EM 1110-345-413 and illustrated again in paragraph 7-19 for a one-story building. In this example the necessary load curves are presented without explanation or computation. The design overpressure of 10 psi is selected arbitrarily for this example.

The overpressure vs time curves that are presented are:

- (1) Incident overpressure vs time (fig. 7.70)
- (2) Front face overpressure vs time (fig. 7.71)
- (3) Rear face overpressure vs time (fig. 7.72)
- (4) Net lateral overpressure vs time (fig. 7.73)
- (5) Average roof overpressure vs time (fig. 7.74)

which the primary elements are permitted to deflect elastically and the others to deflect into the elasto-plastic region. The example includes the design of a typical wall slab, roof slab, girder, and column.

The building consists of reinforced-concrete rigid frames and reinforced-concrete wall and roof slabs. The wall slab is a one-way slab spanning vertically, supported at the bottom by the wall footing which is integral with the foundation, and at the top by the roof slab. The roof slab

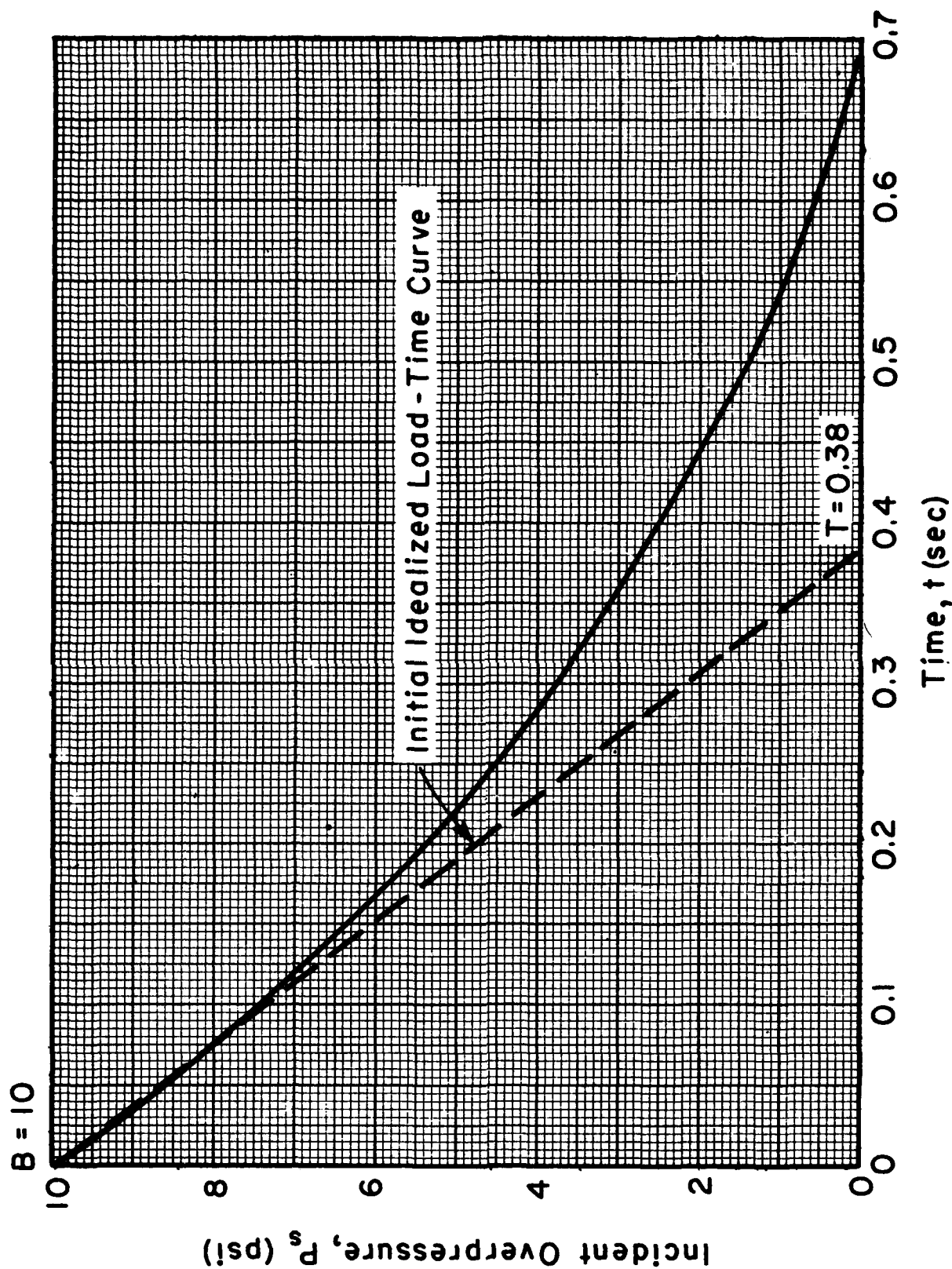


Figure 7.70. Incident overpressure vs time curve

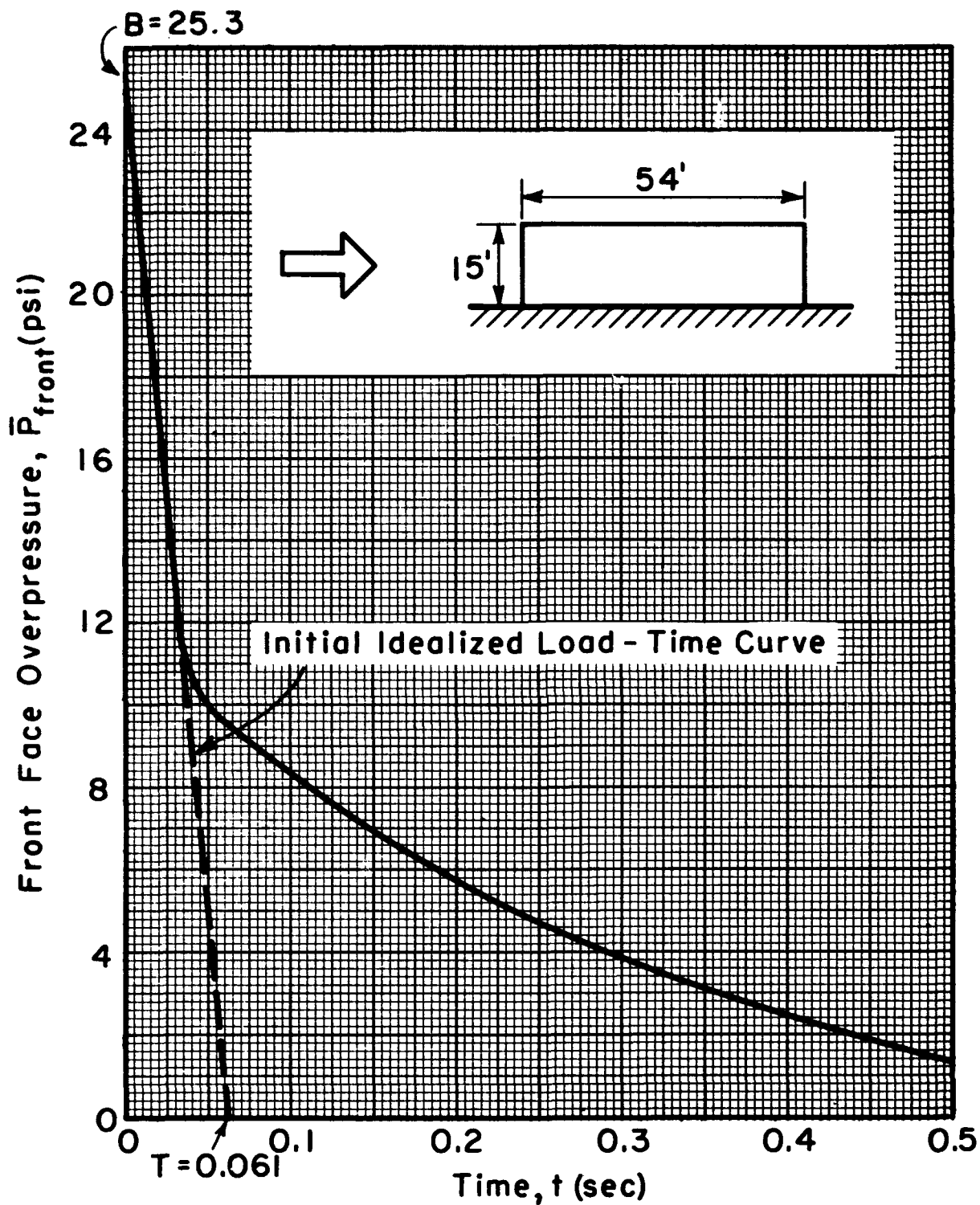


Figure 7.7L Front face overpressure vs time curve

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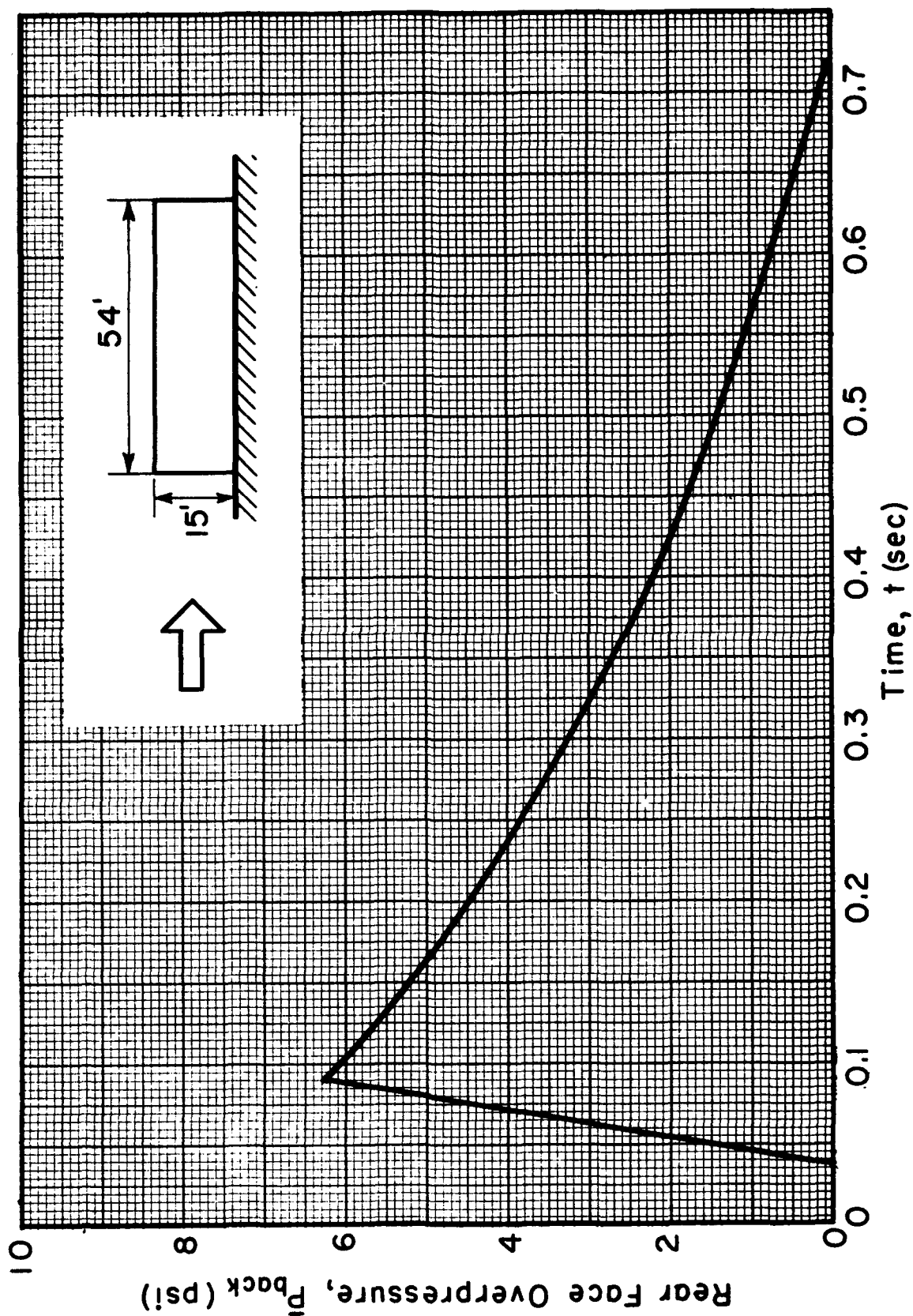


Figure 7.72. Rear face overpressure vs time curve

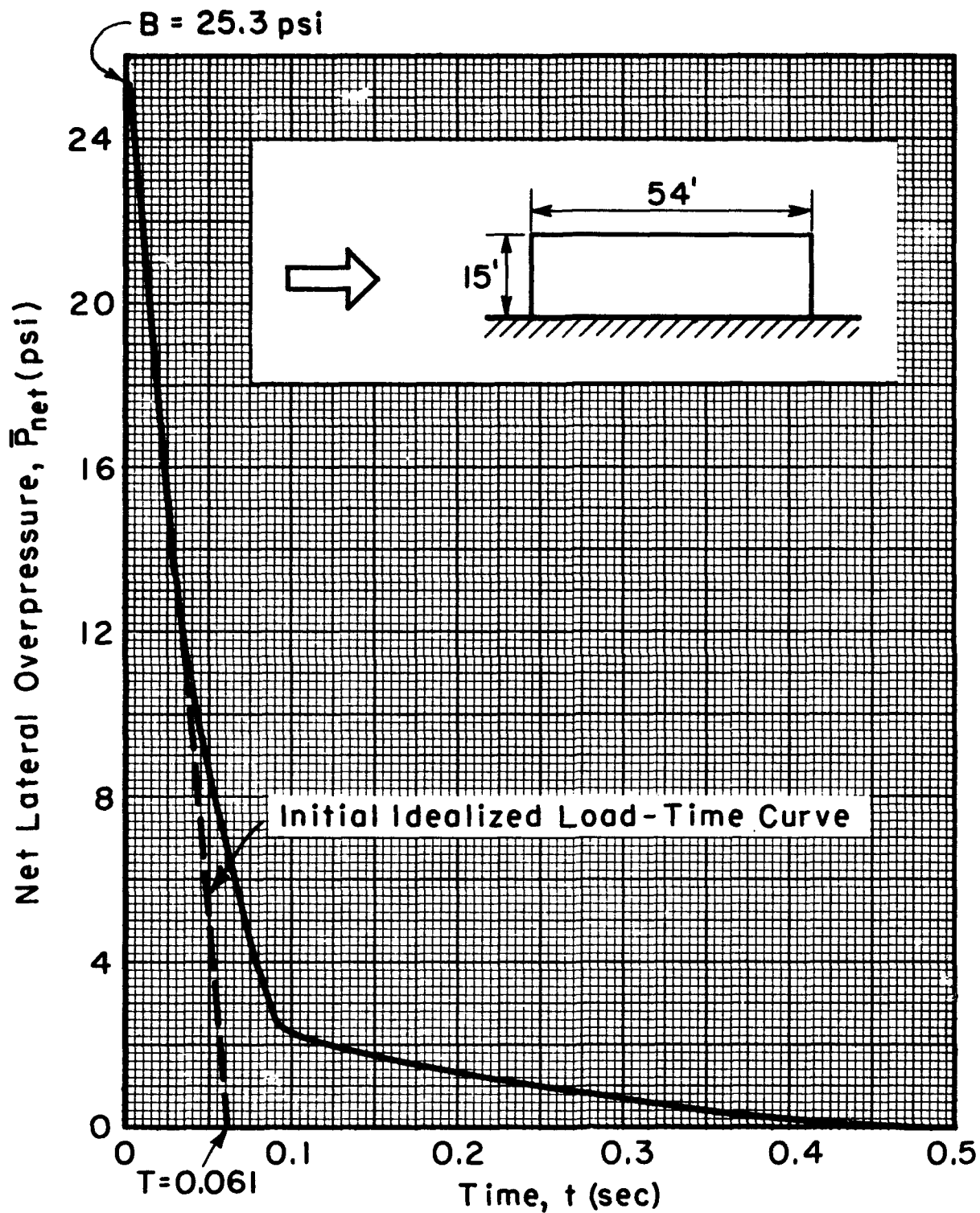


Figure 7.73. Net lateral overpressure vs time curve

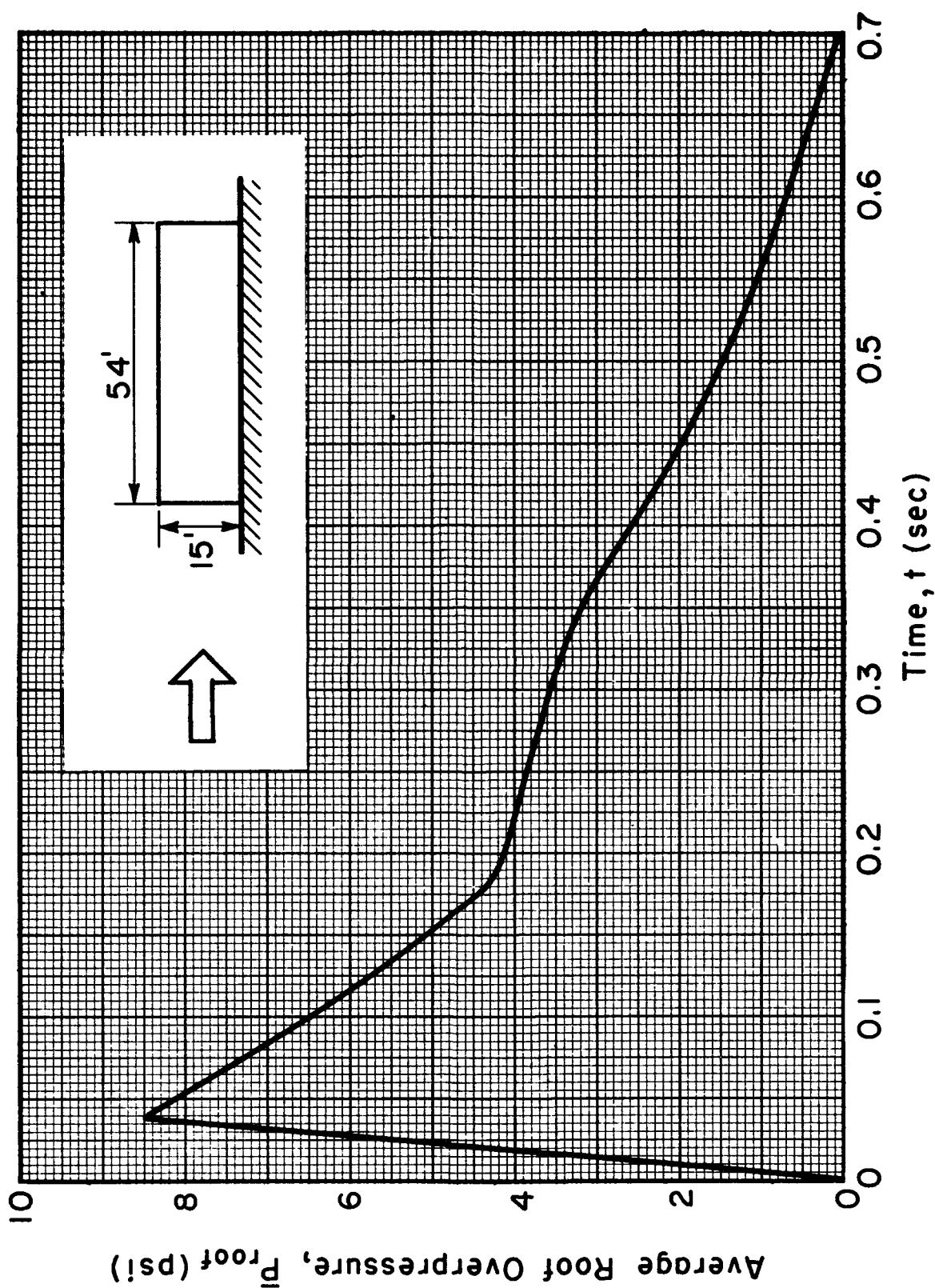
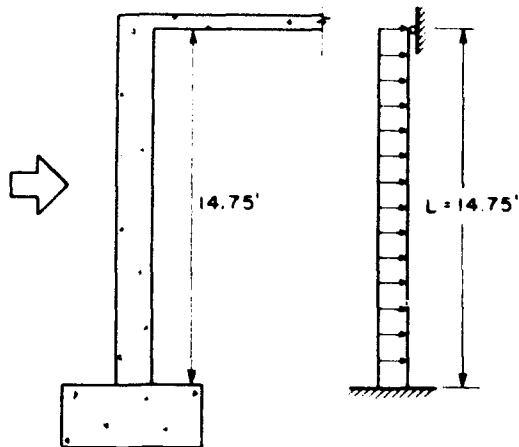


Figure 7.74. Average roof overpressure vs time curve

7-48 DESIGN OF WALL SLAB. The wall slab is designed by considering an element one foot wide, spanning vertically between a fixed support at the foundation and a pinned support at the roof slab. The wall slab is designed to deflect through the elasto-plastic range up to the beginning of

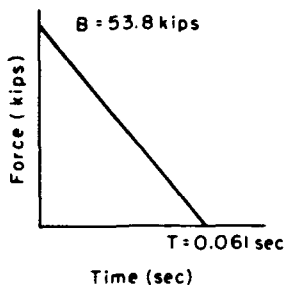
the plastic range. This means that the design load is expected to cause development of plastic hinges first at the fixed support and then near midspan.



This is an example of a design for which the elasto-plastic design procedure is used. Another method of design which could be used utilizes the elastic design procedure and an artificial maximum resistance (par. 6-14e).

The design span length is equal to the clear distance between the foundation and the roof slab. The dead load stresses are not considered in a vertical wall.

a. Design Loading. The design load as idealized from the computed loading shown by figure 7.71 is defined by:



$$B = 25.3 \text{ psi} = \frac{25.3(144)14.75}{1000} = 53.8 \text{ kips}$$

$$T = 0.061 \text{ sec}$$

$$H = \frac{BT}{2} = \frac{(53.8)0.061}{2} = 1.64 \text{ kip-sec (par. 6-11)}$$

b. Dynamic Design Factors. (Refer to table 6.1.)

Elastic range:

$$K_L = 0.58,$$

$$K_M = 0.45,$$

$$K_{LM} = 0.78$$

$$R_{lm} = \frac{8M_{Ps}}{L},$$

$$k_1 = \frac{185EI}{L^3}$$

$$V_1 = 0.26R + 0.12P,$$

$$V_2 = 0.43R + 0.19P$$

Elasto-plastic range:

$$K_L = 0.64,$$

$$K_M = 0.50,$$

$$K_{LM} = 0.78$$



$$R_{lm} = \frac{4}{L} (M_{Ps} + 2M_{Pm}), \quad k_{ep} = \frac{384EI}{5L^3}$$

$$V = 0.39R + 0.11P$$

Plastic range:

$$K_{IM} = 0.66$$

Average values:

$$K_L = 0.5(0.58 + 0.64) = 0.61$$

$$K_M = 0.5(0.45 + 0.50) = 0.47$$

$$R_m = \frac{4}{L} (M_{Ps} + 2M_{Pm})$$

$$k_E = \frac{160EI}{L^3} \text{ (plastic design, } M_{Ps} = M_{Pm} \text{)}$$

c. First Trial - Actual Properties.

$$\text{Let } M_{Ps} = M_{Pm} = M_P$$

Assume  $p = 0.015$  at midspan cross section (par. 4-10)

Assume  $C_R = 1.7$  (experience)

$$R_m = C_R B = 1.7(53.8) = 91.5 \text{ kips}$$

$$M_P = pf_{dy} bd^2 \left( 1 - \frac{pf_{dy}}{1.7f'_{dc}} \right) \text{ (eq 4.16)}$$

$$= 0.015(52)(1)d^2 \left[ 1 - \frac{0.015(52)}{1.7(3.9)} \right] = 0.688d^2 \text{ kip-ft (d in inches)}$$

$$R_m = \frac{12M_P}{L} = \frac{(12)0.688d^2}{14.75} = 91.5, \therefore d = 12.75 \text{ in.}$$

Try  $h = 14 \text{ in.}$ ,  $d = 12.5 \text{ in.}$ ,  $p = 0.015$ ,  $np = 0.15$

$$M_P = 0.688(12.5)^2 = 107.5 \text{ kip-ft}$$

$$R_m = \frac{12M_P}{L} = \frac{12(107.5)}{14.75} = 87.5 \text{ kips}$$

$$I_g = bh^3/12 = (14.0)^3 = 2740 \text{ in.}^4$$

$$I_t = bd^3 \left[ \frac{k^3}{3} + np(1 - k)^2 \right] = 12(d)^3 \left[ \frac{(0.42)^3}{3} + 0.15(1 - 0.42)^2 \right]$$

$$= 0.905d^3 = 0.905(12.5)^3 = 1770 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 0.5(2740 + 1770) = 2255 \text{ in.}^4$$

$$k_E = \frac{160EI}{L^3} = \frac{(160)3(10)^3 2255}{(14.75)^3 144} = 2340 \text{ kips/ft}$$

$$y_E = \frac{R_m}{k_E} = \frac{87.5}{2340} = 0.0374 \text{ ft}$$

$$\text{Weight} = \frac{14(150)(14.75)}{(12)1000} = 2.58 \text{ kips}$$

$$\text{Mass } m = \frac{2.58}{32.2} = 0.08 \text{ kip-sec}^2/\text{ft}$$

$$k_1 = \frac{185}{160} k_E = 1.155(2340) = 2700 \text{ kips/ft}$$

$$k_{ep} = \frac{384}{5(160)} k_E = 0.48(2340) = 1120 \text{ kips/ft}$$

$$R_{lm} = \frac{8M_P}{L} = \frac{8(107.5)}{14.75} = 58.3 \text{ kips}$$

$$y_e = \frac{R_{lm}}{k_1} = \frac{58.3}{2700} = 0.0216 \text{ ft}$$

$$y_m = y_{ep} = y_e + \frac{R_m - R_{lm}}{k_{ep}} = 0.0216 + \frac{87.5 - 58.3}{1120} = 0.0476 \text{ ft}$$

d. First Trial - Equivalent System Properties.

$$R_{me} = K_L R_m = 0.61(87.5) = 53.4 \text{ kips (eq 6.12)}$$

$$H_e = K_L H = 0.61(1.64) = 1.0 \text{ kip-sec (eq 6.8)}$$

$$m_e = K_M m = 0.47(0.08) = 0.0376 \text{ kip-sec}^2/\text{ft (eq 6.2)}$$

$$T_n = 2\pi \sqrt{\frac{K_{LM} m}{k_E}} = 6.28 \sqrt{\frac{0.78(0.08)}{2340}} = 0.0324 \text{ sec}$$

$$W_P = \frac{(H_e)^2}{2m_e} = \frac{(1.0)^2}{2(0.0376)} = 13.3 \text{ ft-kips (eq 6.10)}$$

e. First Trial-Work Done vs Energy Absorption Capacity.

$$C_T = T/T_n = 0.061/0.0324 = 1.88$$

$$C_R = R_m/B = 87.5/53.8 = 1.63 \text{ (eqs 6.15, 6.16)}$$

$$t_m/T = 0.26 \text{ (fig. 5.29)}$$

$$t_m = (0.26)0.061 = 0.016 \text{ sec}$$

Idealized load-time curve is satisfactory (fig. 7.71)(par. 5-13)

$$C_W = 0.091 \text{ (fig. 5.27)}$$

$$W_m = C_W W_P = 0.091(13.3) = 1.21 \text{ ft-kips (eq 6.17)}$$

$$E = R_{me}(y_m - 0.5y_e) = 53.4 [0.0476 - 0.5(0.0216)]$$

$$= 1.96 \text{ ft-kips (eq 6.18)}$$

$E \gg W$ , therefore the selected proportions are satisfactory as a preliminary design.

f. Second Trial - Actual Properties.

$$R_m = \frac{0.5(W_m + E)}{K_L(y_m - 0.5y_e)} = \frac{0.5(1.96 + 1.2)}{(0.61) [0.0476 - 0.5(0.0216)]} = 70.5 \text{ kips (eq 6.19)}$$

$$R_m = \frac{12M_P}{L} = \frac{(12)0.688d^2}{14.75} = 70.5, \therefore d = 11.2 \text{ in.}$$

$$\text{Try } h = 12.5 \text{ in., } d = 11.0 \text{ in., } p = 0.015$$

$$M_P = 0.688d^2 = 0.688(11)^2 = 83.3 \text{ kip-ft}$$

$$R_m = \frac{12M_P}{L} = \frac{12(83.3)}{14.75} = 68 \text{ kips}$$

$$I_g = \frac{bh^3}{12} = (12.5)^3 = 1950 \text{ in.}^4$$

$$I_t = 0.905d^3 = 0.905(11.0)^3 = 1210 \text{ in.}^4 \quad (k = 0.42)$$

$$I_a = 0.5(I_g + I_t) = 0.5(1950 + 1210) = 1580 \text{ in.}^4$$

$$k_E = \frac{160EI}{L^3} = \frac{(160)3(10)^3 1580}{(14.75)^3 144} = 1640 \text{ kips/ft}$$

$$y_E = \frac{R_m}{k_E} = \frac{68}{1640} = 0.0415 \text{ ft}$$

$$\text{Weight} = \frac{12.5(150)(14.75)}{(12)1000} = 2.3 \text{ kips}$$

$$\text{Mass } m = \frac{2.3}{32.2} = 0.0715 \text{ kip-sec}^2/\text{ft}$$

$$k_1 = \frac{185}{160} k_E = 1.155(1640) = 1890 \text{ kips/ft}$$

$$k_{ep} = \frac{384}{5(160)} k_E = 0.48(1640) = 788 \text{ kips/ft}$$

$$R_{1m} = \frac{8M_P}{L} = \frac{8(83.3)}{14.75} = 45.0 \text{ kips}$$

$$y_e = \frac{R_{1m}}{k_1} = \frac{45}{1890} = 0.0238 \text{ ft}$$

$$y_m = y_{ep} = y_e + \frac{R_m - R_{lm}}{k_{ep}} = 0.0238 + \frac{68 - 45.0}{788} = 0.053 \text{ ft}$$

g. Second Trial - Equivalent System Properties.

$$R_{me} = K_L R_m = 0.61(68) = 41.5 \text{ kips (eq 6.12)}$$

$$H_e = K_L H = 0.61(1.64) = 1.0 \text{ kip-sec (eq 6.8)}$$

$$m_e = K_M m = 0.47(0.0715) = 0.0336 \text{ kip-sec}^2/\text{ft (eq 6.2)}$$

$$T_n = 2\pi \sqrt{\frac{K_{LM} m}{k_E}} = 6.28 \sqrt{\frac{0.78(0.0715)}{1640}} = 0.0366 \text{ sec}$$

$$W_P = \frac{(H_e)^2}{2m_e} = \frac{(1.0)^2}{2(0.0336)} = 14.9 \text{ ft-kips (eq 6.10)}$$

h. Work Done vs Energy Absorption Capacity.

$$C_T = T/T_n = 0.061/0.0366 = 1.67$$

$$C_R = R_m/B = 68/53.8 = 1.26 \text{ (eqs 6.15, 6.16)}$$

$$t_m/T = 0.34 \text{ (fig. 5.29)}$$

$$t_m = (0.34)0.061 = 0.0207 \text{ sec}$$

Idealized load-time curve is satisfactory (fig. 7.71)

$$C_W = 0.118 \text{ (fig. 5.27)}$$

$$W_m = C_W W_P = 0.118(14.9) = 1.76 \text{ ft-kips (eq 6.17)}$$

$$E = R_{me}(y_m - 0.5y_e) = 41.5 [0.053 - 0.5(0.0238)] = 1.71 \text{ ft-kips (eq 6.18)}$$

$E \approx W$ , therefore the selected proportions are not satisfactory as a preliminary design.

i. Preliminary Design for Bond Stress.

At bottom of wall (at fixed end of idealized slab)

This section has smaller  $d$  than at midspan therefore  $p > 0.015$ .

From equation 4.16 for  $d = 10$  in. and  $M_P = 83.3$  kip-ft, required  $p \approx 0.0184$ .

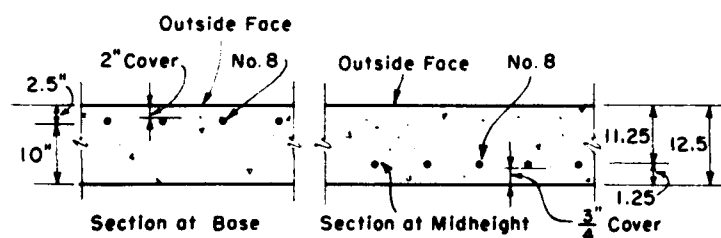
$$\text{Estimated } V_{\max} = 1/2 R_m$$

$$= 1/2 (68) = 34 \text{ kips}$$

$$\text{Allowable } u = 0.15f'_c$$

$$= 0.15(3000)$$

$$= 450 \text{ psi (par. 4-09)}$$



$$\Sigma o = \frac{V}{u_j d} = \frac{8(34,000)}{450(7)10} = 8.65 \text{ in.}$$

Try #8 at 4-1/4 in.,  $A_s = 2.23 \text{ in.}^2/\text{ft}$ ,  $\Sigma o = 8.9 \text{ in./ft}$

$$p = \frac{A_s}{bd} = \frac{2.23}{12(10)} = 0.0186$$

At top of wall (at pinned end of idealized slab)

$$\text{Estimated } V_{\max} = 1/3 R_m = 1/3 (68) = 23 \text{ kips}$$

$$\Sigma o = \frac{8(23,000)}{450(7)11.25} = 5.2 \text{ in.}$$

Bond stress is not critical

Try #8 at 4-1/4 in.,  $A_s = 2.23 \text{ in.}^2$ ,  $\Sigma o = 8.9 \text{ in.}$

$$p = \frac{A_s}{bd} = \frac{2.23}{11.25(12)} = 0.0165$$

j. Determination of Maximum Deflection and Dynamic Reactions by Numerical Integration.

$$M_{Pm} = p f_{dy} b d^2 \left( 1 - \frac{p f_{dy}}{1.7 f'_c} \right) = 0.0165(52)(1)(11.25)^2 \left[ 1 - \frac{(0.0165)52}{1.7(3.9)} \right]$$

$$= 94.5 \text{ kip-ft (eq 4.16)}$$

$$M_{Ps} = 0.0186(52)1(10)^2 \left[ 1 - \frac{0.0186(52)}{1.7(3.9)} \right] = 82.6 \text{ kip-ft}$$

$$I_g = b h^3 / 12 = (12.5)^3 = 1950 \text{ in.}^4$$

$$I_t = b d^3 \left[ \frac{k^3}{3} + n p (1 - k)^2 \right] = 12(11.25)^3 \left[ \frac{0.42^3}{3} + 0.148(1 - 0.42)^2 \right]$$

$$= 1290 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 0.5(1950 + 1290) = 1620 \text{ in.}^4$$

$$\text{Weight} = \frac{12.5(150)14.75}{12(1000)} = 2.3 \text{ kips}$$

$$\text{Mass } m = \frac{2.3}{32.2} = 0.0715 \text{ kip-sec}^2/\text{ft}$$

Elastic range:

$$R_{lm} = \frac{8M_{Ps}}{L} = \frac{8(82.6)}{14.75} = 44.8 \text{ kips}$$

$$k_1 = \frac{185EI}{L^3} = \frac{(185)3(10)^3 1620}{(14.75)^3 144} = 1940 \text{ kips/ft}$$

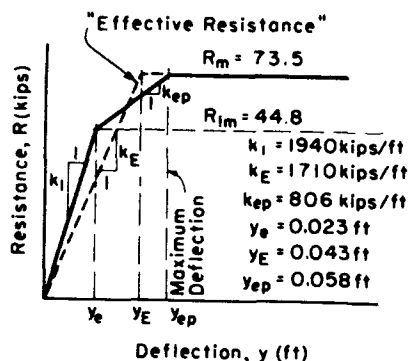
$$y_e = \frac{R_{lm}}{k_1} = \frac{44.8}{1940} = 0.023 \text{ ft}$$

Elasto-plastic range:

$$R_m = \frac{4(M_{Ps} + 2M_{Pm})}{L} = \frac{4[82.6 + 2(94.5)]}{14.75} = 73.5 \text{ kips}$$

$$k_{ep} = \frac{384EI}{5L^3} = \frac{384}{5(185)} k_1 = 806 \text{ kips/ft}$$

$$y_{ep} = y_e + \frac{R_m - R_{lm}}{k_{ep}} = 0.023 + \frac{73.5 - 44.8}{806} = 0.058 \text{ ft}$$



By providing equal areas under the "effective resistance" line and the computed elasto-plastic resistance line,  $y_E = 0.043 \text{ ft}$  (fig. 7.75)

$$k_E = \frac{R_m}{y_E} = \frac{73.5}{0.043} = 1710 \text{ kips/ft}$$

$$T_n = 2\pi \sqrt{\frac{K_{LM}^m}{k_E}} = 6.28 \sqrt{\frac{0.78(0.0715)}{1710}} = 0.0358 \text{ sec}$$

Figure 7.75. Resistance function for 12-12-in. slab spanning 14.75 ft, fixed at one support, pinned at other support

The basic equation for the numerical integration in table 7.28 is  $y_{n+1} = \ddot{y}_n(\Delta t)^2 +$

$2y_n - y_{n-1}$  (table 5.3) where

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(\Delta t)^2}{K_{LM}(m)}$$

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(0.003)^2}{0.78(0.0715)} = 1.62(10)^{-4}(P_n - R_n) \text{ ft, elastic range}$$

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(0.003)^2}{0.78(0.0715)} = 1.62(10)^{-4}(P_n - R_n) \text{ ft, elasto-plastic range}$$

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(0.003)^2}{0.66(0.0715)} = 1.907(10)^{-4}(P_n - R_n) \text{ ft, plastic range}$$

The time interval  $\Delta t = 0.003 \text{ sec}$  is less than  $T_n/10 = 0.00358 \text{ sec}$  (par. 5-08).

The dynamic reaction equations are listed in paragraph 7-48b. The  $P_n$  values for the second column are obtained from figure 7.71, multiplying by  $144(14.75)/1000 = 2.12$ .

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**Table 7.28. Determination of Maximum Deflection and Dynamic Reactions  
for Front Wall Slab**

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> -R <sub>n</sub> (kips)	$\ddot{y}_n(\Delta t)^2$ (ft)	y <sub>n</sub> (ft)	V <sub>1n</sub> (kips)	V <sub>2n</sub> (kips)
0	53.8	0	26.9	0.00436	0	6.5	10.2
0.003	51.2	8.5	42.7	0.00692	0.00436	8.3	13.4
0.006	48.5	30.3	18.2	0.00295	0.01564	13.7	22.3
0.009	45.9	50.4	-4.5	-0.00073	0.02987	24.7	24.7
0.012	43.2	61.2	-18.0	-0.00292	0.04337	28.7	28.7
0.015	40.6	69.7	-29.1	-0.00471	0.05395	31.7	31.7
0.018	38.0	73.5	-35.5	-0.00677	0.05982*	32.5	32.5
0.021	35.3	73.5	-38.2	-0.00728	0.05892	32.1	32.1
0.024	31.8	59.4			0.05074	19.2	31.5
*(y <sub>n</sub> ) <sub>max</sub> = 0.060 ft.							

In table 7.28 the maximum computed deflection is  $(y_n)_{\max} = 0.060$  ft. This is slightly more than the specified maximum displacement,  $y_{ep} = 0.054$  ft. This is an acceptable difference for blast resistant design.

**k. Shear Stress and Bond Strength.**

For bottom of wall (fixed end of idealized beam)

$$V_{\max} = 32.5 \text{ kips (table 7.28)}$$

For no shear reinforcement

$$\begin{aligned} v_y &= 0.04f'_c + 5000p = 0.04(3000) + 5000(0.0186) = 120 + 93 \\ &= 213 \text{ psi (eq 4.24)} \end{aligned}$$

$$v = \frac{8V}{7bd} = \frac{8(32,500)}{7(12)(10)} = 310 \text{ psi}$$

Shear reinforcement required for  $310 - 213 = 97$  psi

Contribution of shear reinforcement to allowable shear stress =  $rf_y$

$$r = \frac{97}{40,000} = 0.00242$$

Try 1 #4,  $A_s = 0.20 \text{ in.}^2$

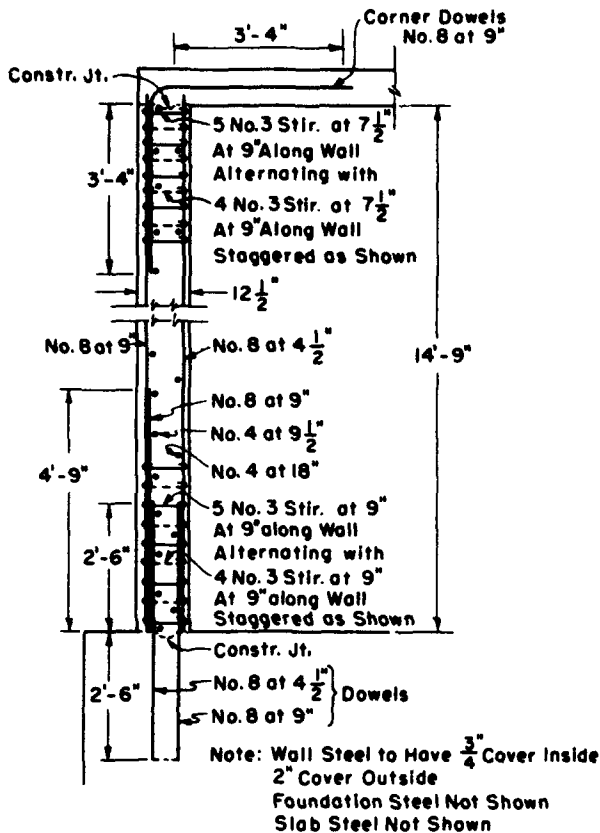
$$r = \frac{A_s}{bs} = \frac{0.20}{8-1/2 s} = 0.00242; \therefore s = 9.75 \text{ in., use } s = 9 \text{ in.}$$

For top of wall (pinned end of idealized beam)

$$V_{\max} = 32.5 \text{ kips (table 7.28)}$$

$$v_y = 0.04f'_c + 5000p = 0.04(3000) + 5000(0.0148) = 120 + 74 = 194 \text{ psi}$$

$$v = \frac{8V}{7bd} = \frac{8(32,500)}{7(12)(11.25)} = 275 \text{ psi}$$



Shear reinforcement required for

$$275 - 194 = 81 \text{ psi}$$

$$r = \frac{81}{40,000} = 0.00202$$

$$\text{Try 1 \#4, } A_s = 0.20 \text{ in.}^2$$

$$r = \frac{A_s}{bs} = \frac{0.20}{8-1/2 \text{ s}} = 0.00202;$$

$$\therefore s = 11.65 \text{ in., use } s = 11 \text{ in.}$$

Bond

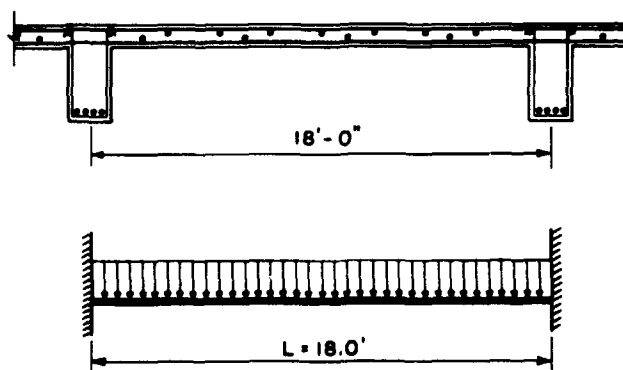
$$u = \frac{8V}{7\Sigma o d} = \frac{8(32,500)}{7(8.9)10} = 412 \text{ psi}$$

$$\begin{aligned} \text{Allowable } u &= 0.15f'_c \\ &= 0.15(3000) \\ &= 450 \text{ psi} > 412 \text{ psi; OK} \end{aligned}$$

1. Summary.

12-1/2-in. slab

7-49 DESIGN OF ROOF SLAB. The design procedure followed in designing this roof slab is similar to the design procedure for design of the wall slab in paragraph 7-48. Consideration is given to the static load stresses in this case because they reduce the maximum resistance of the slab. Since the procedures of this manual are limited to single-span elements, the slab is designed by considering the behavior of a one-foot-wide element fixed at both ends and spanning 18 ft between the frame centerlines. The slab is permitted to deflect through the elasto-plastic





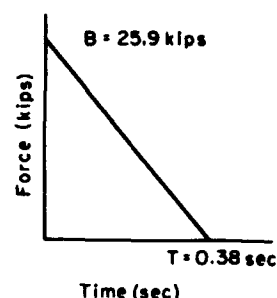
range but not into the plastic range (fig. 7.76, page 208).

a. Design Loading. Since the maximum deflection of this slab is limited to the elasto-plastic region, this maximum deflection occurs in a very short time (table 7.29, page 208). The rapid response means that the design load for all slab elements may be based on the incident overpressure regardless of the direction of motion of the blast wave. Only on the first few feet from the front edge, for the blast wave moving perpendicular to the long axis of the building, is the loading reduced by the vortex action before the slab reaches its maximum deflection (par. 3-08d).

The design load as idealized from the computed loading shown by figure 7.70 is defined by:

$$B = 10 \text{ psi} = \frac{10(144)18}{1000} = 25.9 \text{ kips}$$

$$T = 0.38 \text{ sec}$$



b. Dynamic Design Factors. (Refer to table 6.1.)

Elastic range:

$$K_L = 0.53,$$

$$K_M = 0.41,$$

$$K_{LM} = 0.77$$

$$R_{lm} = \frac{12M_P}{L},$$

$$k_1 = \frac{384EI}{L^3}$$

$$V = 0.36R_n + 0.14P_n$$

Elasto-plastic range:

$$K_L = 0.64,$$

$$K_M = 0.50,$$

$$K_{LM} = 0.78$$

$$R_m = \frac{16M_P}{L},$$

$$k_{ep} = \frac{384EI}{5L^3}$$

$$V = 0.39R_n + 0.11P_n$$

Average values:

$$K_L = 0.5(0.53 + 0.64) = 0.58$$

$$K_M = 0.5(0.41 + 0.50) = 0.455$$

$$K_{LM} = 0.5(0.77 + 0.78) = 0.77$$

$$R_{mf} = \frac{22M_P}{L} \text{ (fictitious maximum resistance)}$$

$$k_E = \frac{264EI}{L^4} \text{ "elastic" effective spring constant}$$

c. First Trial - Actual Properties.

Assume D.L.F. = 2.0 (experience)

$$R_{mf} = D.L.F.(B) = 2.0(25.9) = 51.8 \text{ kips (par. 6-11)}$$

Assume  $p = 0.015$

$$M_P = pf_{dy}bd^2 \left( 1 - \frac{pf_{dy}}{1.7f'_{dc}} \right) \text{ (eq 4.16)}$$

$$= 0.015(52)(1)d^2 \left[ 1 - \frac{0.015(52)}{1.7(3.9)} \right] = 0.688d^2 \text{ kip-ft (d in inches)}$$

$$R_{mf} = \frac{22M_P}{L} = \frac{(22)0.688d^2}{18} = 51.8, \therefore d = 7.85 \text{ in.}$$

Try  $h = 9-1/2 \text{ in.}$ ,  $d = 8-1/4 \text{ in.}$ ,  $p = 0.015$ ,  $np = 0.15$

$$M_P = 0.688d^2 = 0.688(8.25)^2 = 46.9 \text{ kip-ft}$$

$$R_{mf} = \frac{22M_P}{L} = \frac{22(46.9)}{18} = 57.3 \text{ kips}$$

$$I_g = bh^3/12 = (9.5)^3 = 857 \text{ in.}^4$$

$$I_t = bd^3 \left[ \frac{k^3}{3} + np(1 - k)^2 \right] = 12(d)^3 \left[ \frac{(0.42)^3}{3} + 0.15(1 - 0.42)^2 \right]$$

$$= 0.905d^3 = 0.905(8.25)^3 = 508 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 0.5(857 + 508) = 682 \text{ in.}^4$$

$$k_E = \frac{264EI}{L^3} = \frac{(264)3(10^3)682}{18^3(144)} = 644 \text{ kips/ft}$$

$$\text{Weight} = \left[ \frac{9.5(150)}{12} + 6.0 \right] \frac{18}{1000} = 2.25 \text{ kips}$$

$$\text{Mass } m = \frac{2.25}{32.2} = 0.07 \text{ kip-sec}^2/\text{ft}$$

d. First Trial - Equivalent Properties.

$$T_n = 2\pi\sqrt{K_{IM}m/k_E} = 6.28\sqrt{0.77(0.07)/644} = 0.0576 \text{ sec}$$

e. First Trial - Available Resistance vs Required Resistance.

$$C_T = T/T_n = 0.38/0.0576 = 6.6$$

D.L.F. = 1.92 (fig. 5.20)

$$t_m/T = 0.08 \text{ (fig. 5.20)}$$

$$t_m = 0.08(0.38) = 0.0304 \text{ sec}$$

$$\text{Required } R_{mf} = D.L.F.(B) = 1.92(25.9) = 49.8 \text{ kips} < 57.3 \text{ kips}$$

The required  $R_{mf} < \text{available } R_{mf}$ , therefore the selected proportions are satisfactory as a preliminary design.

f. Preliminary Design for Bond Stress.

$$\text{At } t_m = 0.0304, P = \frac{(9.2)144(18)}{1000} = 23.8 \text{ kips (fig. 7.70)}$$

$$V = 0.39R_m + 0.11P = 0.39(57.3) + 0.11(23.8) = 22.4 + 2.62 = 25.02 \text{ kips}$$

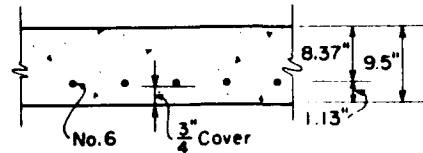
$$\text{Allowable } u = 0.15f'_c = 0.15(3000) = 450 \text{ psi (par. 4-09)}$$

$$\Sigma o = \frac{V}{u_j d} = \frac{8(25,020)}{450(7)(8.25)} = 7.7 \text{ in.}$$

$$\text{Try \#6 at } 3\text{-}1/2 \text{ in., } A_s = 1.51 \text{ in.}^2, \Sigma o = 8.1 \text{ in., } h = 9.5 \text{ in.,}$$

$$d = 8.37, p = \frac{A_s}{bd} = \frac{1.51}{12(8.37)} = 0.0151,$$

$$np = 10(0.0151) = 0.151$$



g. Determination of Maximum Deflection and Dynamic Reactions by Numerical Integration.

$$M_P = pf_{dy} bd^2 \left( 1 - \frac{pf_{dy}}{1.7f'_{dc}} \right) = 0.0153(52)(1)(8.37)^2 \left[ 1 - \frac{(0.0153)(52)}{1.7(3.9)} \right]$$

$$= 49.0 \text{ kip-ft (eq 4.16)}$$

$$I_g = bh^3/12 = (9.5)^3 = 857 \text{ in.}^4$$

$$k = \sqrt{n^2 p^2 + 2np} - np = 0.42$$

$$I_t = bd^3 \left[ \frac{k^3}{3} + np(1 - k)^2 \right] = 0.905d^3 = 0.905(8.37)^3 = 530 \text{ in.}^4$$

$$I_a = 0.5(I_g + I_t) = 0.5(857 + 530) = 693 \text{ in.}^4$$

$$\text{Weight} = \left[ \frac{9.5(150)}{12} + 6.0 \right] \frac{18}{1000} = 2.25 \text{ kips}$$

$$\text{Mass } m = \frac{2.25}{32.2} = 0.07 \text{ kip-sec}^2/\text{ft}$$

Elastic range:

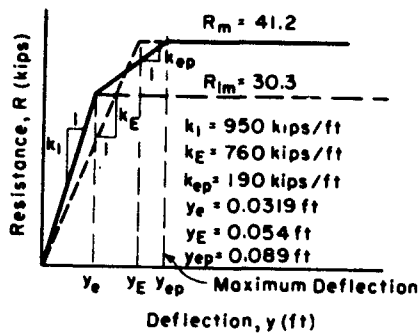
$$R_{lm} = \frac{12M_P}{L} - \text{weight} = \frac{12(49)}{18} - 2.25 = 32.6 - 2.25 = 30.3 \text{ kips}$$

$$k_1 = \frac{384EI}{L^3} = \frac{384(3)(10)^3 693}{(18)^3 (144)} = 950 \text{ kips/ft}$$

$$y_e = \frac{R_{1m}}{k_1} = \frac{30.3}{950} = 0.0319 \text{ ft}$$

Elasto-plastic range:

$$R_m = \frac{16M_p}{L} - \text{weight} = \frac{16(49)}{18} - 2.25 = 43.5 - 2.25 = 41.2 \text{ kips}$$



$$k_{ep} = \frac{384EI}{5L^3} = \frac{1}{5} k_1 = \frac{950}{5} = 190 \text{ kips/ft}$$

$$y_{ep} = y_e + \frac{R_m - R_{1m}}{k_{ep}} = 0.0319 + \frac{41.2 - 30.3}{190} = 0.0319 + 0.057 = 0.089 \text{ ft}$$

$$k_E = \frac{307EI}{L^3} = 760 \text{ kips/ft}$$

$$y_E = \frac{R_m}{k_E} = \frac{41.2}{760} = 0.054 \text{ ft}$$

Figure 7.76. Resistance function for 9-1/2-in. slab spanning 18 ft

$$T_n = 2\pi \sqrt{K_{LM}/k_E} = 6.28 \sqrt{0.77(0.07)/760} = 0.053 \text{ sec}$$

The basic equation for the numerical integration in table 7.29 is

$$y_{n+1} = \ddot{y}_n (\Delta t)^2 + 2y_n - y_{n-1} \text{ (table 5.3) where}$$

Table 7.29. Determination of Maximum Deflection and Dynamic Reactions for Roof Slab

t (sec)	P <sub>n</sub> (kips)	R <sub>n</sub> (kips)	P <sub>n</sub> - R <sub>n</sub> (kips)	$\ddot{y}_n (\Delta t)^2$ (ft)	y <sub>n</sub> (ft)	V <sub>n</sub> (kips)
0	25.9	0	12.9	0.0060	0	3.6
0.005	25.6	5.7	19.9	0.0092	0.0060	5.6
0.010	25.2	20.1	5.1	0.0024	0.0212	10.8
0.015	24.9	31.6	-6.7	-0.0031	0.0388	15.1
0.020	24.5	34.4	-9.9	-0.0045	0.0533	16.1
0.025	24.2	36.3	-12.1	-0.0055	0.0633	16.8
0.030	23.9	37.1	-13.2	-0.0060	0.0678	17.1
0.035	23.5	35.7	-12.2	-0.0057	0.0663	16.1
0.040	23.2	28.8			0.0591	13.6
0.045	22.8					11.4
0.050	22.5					11.2

$$\dot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(\Delta t)^2}{K_{IM}(m)}$$

$$\dot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(0.005)^2}{0.77(0.07)} = 4.638(10^{-4})(P_n - R_n) \text{ ft, elastic range}$$

$$\dot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(0.005)^2}{0.78(0.07)} = 4.578(10^{-4})(P_n - R_n) \text{ ft, elasto-plastic range}$$

The time interval  $\Delta t = 0.005$  sec is approximately  $T_n/10 = 0.0053$  (par. 5-08). The dynamic reaction equations are listed in paragraph 7-49b. The  $P_n$  values for the second column are obtained from figure 7.70, multiplying by  $144(18)/1000 = 2.59$ .

The maximum deflection  $(y_n)_{\max}$  computed in table 7.29 is 0.0678 ft which is less than the allowable  $y_m$  of 0.089 ft.

#### h. Shear Strength and Bond Stress.

$$V_{\max} = 17.0 \text{ kips (table 7.29)}$$

For no shear reinforcement

$$\text{Allowable } v_y = 0.04f'_c + 5000p \text{ (eq 4.24)}$$

$$v_y = 0.04(3000) + 5000(0.0151) = 120 + 76 = 196 \text{ psi}$$

$$v = \frac{8V}{7bd} = \frac{8(17,000)}{7(12)(8.37)} = 193 \text{ psi; OK}$$

No shear reinforcement required

$$u = \frac{8V}{7\Sigma o d} = \frac{8(17,000)}{7(8.1)(8.37)} = 287 \text{ psi}$$

$$\text{Allowable } u = 0.15f'_c = 0.15(3000) = 450 \text{ psi; OK}$$

#### i. Summary.

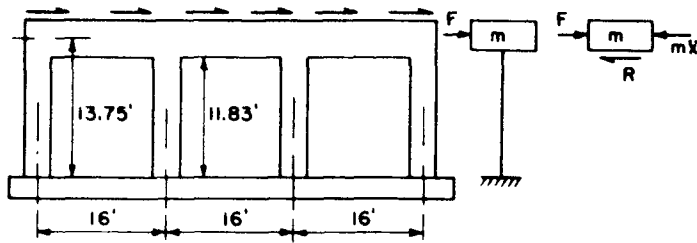
9-1/2-in. slab

$$p = 0.0151$$

$$\Sigma o = 8.1 \text{ in.}$$

No shear reinforcement required

7-50 PRELIMINARY DESIGN OF COLUMN. A single-story frame subject to lateral load behaves essentially as a single-degree-of-freedom system. In determining the requirement for the columns which are the springs of the system it is not necessary to use the equivalent system technique otherwise used in this manual for designing structural elements.



Combining the principles of paragraph 6-11 and the equations from paragraph 7-06 results in a procedure for preliminary design. In this preliminary design the girders are

assumed to be infinitely rigid to simplify the analysis.

The column height from the girder centerline to the top of the footing is 13.75 ft. This dimension is used to determine the spring constant. The maximum resistance computation is based on the clear column height, 11.83 ft.

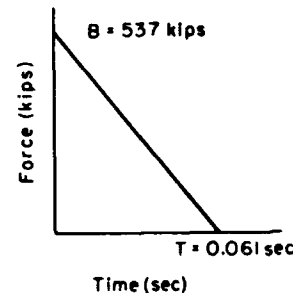
a. Design Loading. In the preliminary design it is an unnecessary refinement to use the dynamic reactions from the wall slabs. The design load is idealized from the net lateral overpressure curve. The height of wall considered to load the frame is equal to half the clear height of the wall plus the thickness of the roof slab

$$h' = \frac{14.75}{2} + 0.79 = 8.16 \text{ ft}$$

The design load as idealized from the computed loading shown by figure 7.73 is defined by:

$$B = 25.3 \text{ psi} = \frac{25.3(144)18(8.16)}{1000} = 537 \text{ kips}$$

$$T = 0.061 \text{ sec}$$



b. Mass Computation.

$$\text{Roof slab} \left[ \frac{9.5(150)}{12} + 6.0 \right] \frac{18(54)}{1000} = 122 \text{ kips}$$

$$\text{Girder (assumed)} \frac{18(32)50.5(150)}{12(12)1000} = 30 \text{ kips}$$

$$4 \text{ columns (assumed)} \frac{18(30)(11.83)150(4)}{12(12)(1000)} = 26.6 \text{ kips}$$

$$2 \text{ wall slabs} \frac{12.5(14.75)150(18)2}{12(1000)} = 83.0 \text{ kips}$$

Mass of single-degree-of-freedom system = total roof slab + total

$$\begin{aligned} \text{girder} + \frac{1}{3} \text{ columns} + \frac{1}{3} \text{ walls} &= \frac{122 + 30 + 0.33(26.6 + 83.0)}{32.2} \\ &= 5.85 \text{ kip-sec}^2/\text{ft} \end{aligned}$$

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c. First Trial - Actual Properties.

Assume D.L.F. = 1.2 (experience)

$$R_m = D.L.F.(B) = 1.2(537) = 645 \text{ kips (par. 6-11)}$$

$$\text{Required } M_D = R_m h / 2n = 645(11.83) / 2(4) = 955 \text{ kip-ft}$$

Estimated average roof pressure = 6.4 psi (fig. 7.74)

$$\text{Average blast load per column} = \frac{54(18)6.4(144)}{1000(4)} = 224 \text{ kips}$$

$$\text{Dead load per column} = \frac{1}{4} (122 + 30) = 38 \text{ kips}$$

$$\text{Average column design load} = P_D = 224 + 38 = 262 \text{ kips}$$

$$M_D = A_s f_{dy} d' + P_D \left( 0.5t - \frac{P_D}{1.7bf'_{dc}} \right) \text{ (eq 4.32)}$$

Let  $p = p' = 0.015$ ,  $d' = (t - 7.5) \text{ in.}$ ,  $d'' = 3.75 \text{ in.}$ ,  $b = 18$ , $A_s = pbt$ Substituting into previous  $M_D$  equation and solving for  $t$  gives

$$955(12) = 0.015(18)t(52)(t - 7.5) + 262 \left[ 0.5t - \frac{262}{1.7(18)3.9} \right]$$

$$14.0t^2 + 25.0t - 12,037 = 0, \text{ solving } t = 28.4 \text{ in.}$$

Try section 18 in. by 28 in.,  $d = 28 - 3.75 = 24.25 \text{ in.}$ Assume  $d''/d = 3.75/24.25 = 0.155$ ,

$$m = np + (n - 1)p' = 0.15 + 9(0.015) = 0.285$$

$$q = np + (n - 1)p' d''/d = 0.15 + 9(0.015)0.155 = 0.171$$

 $k = 0.37$  (table 11 RCDH of ACI)

$$I_g = \frac{bt^3}{12} = \frac{18(28)^3}{12} = 32,900 \text{ in.}^4$$

$$\begin{aligned} I_t &= bd^3 \left[ \frac{k^3}{3} + (n - 1)p' \{ k^2 - 2k(d''/d) + (d''/d)^2 \} + np(1 - k)^2 \right] \\ &= 18(24.25)^3 \left[ \frac{(0.37)^3}{3} + 9(0.015) \{ (0.37)^2 - 2(0.37)(0.155) + (0.155)^2 \} + 0.15(1 - 0.37)^2 \right] \\ &= 258,000 \left[ 0.0169 + 0.135(0.137 - 0.115 + 0.024) + 0.0595 \right] = 258,000(0.0826) = 21,300 \text{ in.}^4 \end{aligned}$$

$$I_a = 0.5(I_g + I_t) = 0.5(32,900 + 21,300) = 27,100 \text{ in.}^4$$

d. First Trial - Determination of D.L.F.

$$k = \frac{12EI_n}{h^3} = \frac{12(3)10^3(27,100)^4}{(13.75)^3 144} = 10,450 \text{ kips/ft (eq 7.10)}$$

$$x_e = R_m / k = 645 / 10,450 = 0.062 \text{ ft}$$

$$T_n = 2\pi\sqrt{m/k} = 6.28\sqrt{5.85/10,450} = 0.148 \text{ sec}$$

$$T/T_n = 0.061/0.148 = 0.41$$

$$D.L.F. = 1.06, t_m/T = 0.94 \text{ (fig. 5.20)}$$

$$t_m = 0.94(0.061) = 0.0575 \text{ sec}$$

The original idealized load-time curve should be revised to obtain a closer approximation to the total impulse up to time  $t_m$ . The impulse up to  $t = 0.060$  sec in figure 7.73 is

$$H = 0.898 \text{ psi-sec (obtained by graphical integration)}$$

$$T = 2H/B = 2(0.898)/25.3 = 0.071 \text{ sec}$$

$$T/T_n = 0.071/0.148 = 0.48$$

$$D.L.F. = 1.15, t_m/T = 0.85 \text{ (fig. 5.20)}$$

The revised idealized load is satisfactory because  $t_m = 0.85(0.071) = 0.060$  sec

$$\text{Required moment} = \frac{(B)D.L.F.(h)}{2n} = \frac{537(1.15)11.83}{2(4)} = 915 \text{ kip-ft}$$

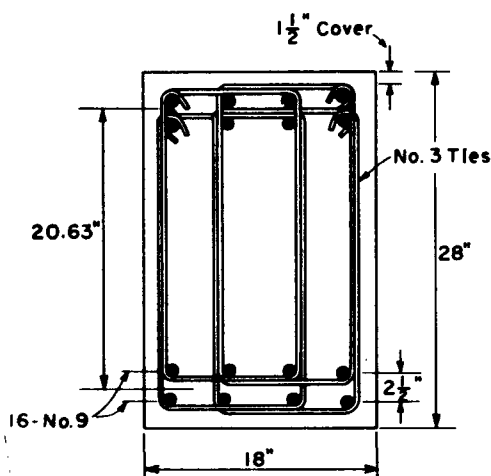
Required moment is less than available moment. Let us investigate possible reduction in size.

e. Second Trial - Actual Properties.

$$M_D = A_s f_{dy} d' + P_D \left( 0.5t - \frac{P_D}{1.7bf'_{dc}} \right) \text{ (eq 4.32)}$$

$$(915)12 = 0.015(18)t(52)(t - 7.5) + 262 \left[ 0.5t - \frac{262}{1.7(18)3.9} \right]$$

$$14.0t^2 + 25.0t - 11,547 = 0, \text{ solving } t = 27.8 \text{ in.}$$



The first trial proportions are satisfactory. An actual column section is selected to establish the column plastic bending moment for use in the girder design that follows in paragraph 7-51.

$$d'' = 3.687, d = 24.32, d' = 20.63$$

$$8 \#9, A_s = A'_s = 8.0 \text{ in.}^2$$

$$p = p' = 8.0/18(28) = 0.0159$$



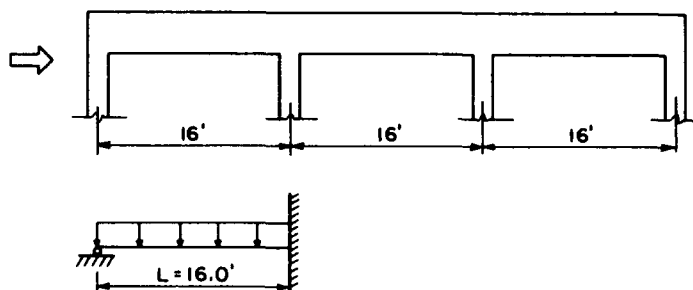
$$M_D = 8.0(52)(20.63) + 262 \left[ 14 - \frac{262}{1.7(18)3.9} \right] \quad (\text{eq 4.32})$$

$$= 8600 + 3070 = 11,670 \text{ kip-in.} = 970 \text{ kip-ft}$$

The previously computed value of  $I$  will be satisfactory for the computation in paragraph 7-51.

7-51 DESIGN OF ROOF GIRDER. The design of the girder in this example is performed in the same manner as the girder in paragraph 7-43 since both girders are designed for elastic action. Reference should be made to paragraph 7-11 for a detailed discussion of the design of roof girders.

The front girder span is critical. Previous consideration of the variation of local roof overpressure with location along the span from front to back of the building has shown that for the front girder the



incident overpressure may be used as the local roof overpressure because the maximum response of the girder generally occurs before the vortex action has an opportunity to cause the local roof loading to vary strongly from the incident overpressure. This can be seen from a comparison of the times for maximum displacement of the girders in tables 7.9, 7.21, and 7.26 with the local overpressure data in paragraph 7-23.

A fixed-pinned tee beam is considered. Reference is made to paragraph 6-20 and table 6.4 for design constants.

a. Load Determination. The girder loading is obtained in figure 7.77 from the numerical integration for the roof slab (par. 7-49d, table 7.29).

After  $t = 0.045$  sec the load on the girder from the slab is assumed to be equal to one-half the load on the slab. The time required for the shock front to traverse the girder is considered in establishing the load-time curve by plotting the dynamic slab reactions at the ends of the girder and averaging over the span. The time required for the shock wave to travel the length of the girder is

$$t_{\text{lag}} = \frac{16}{1403} = 0.0114 \text{ sec}$$

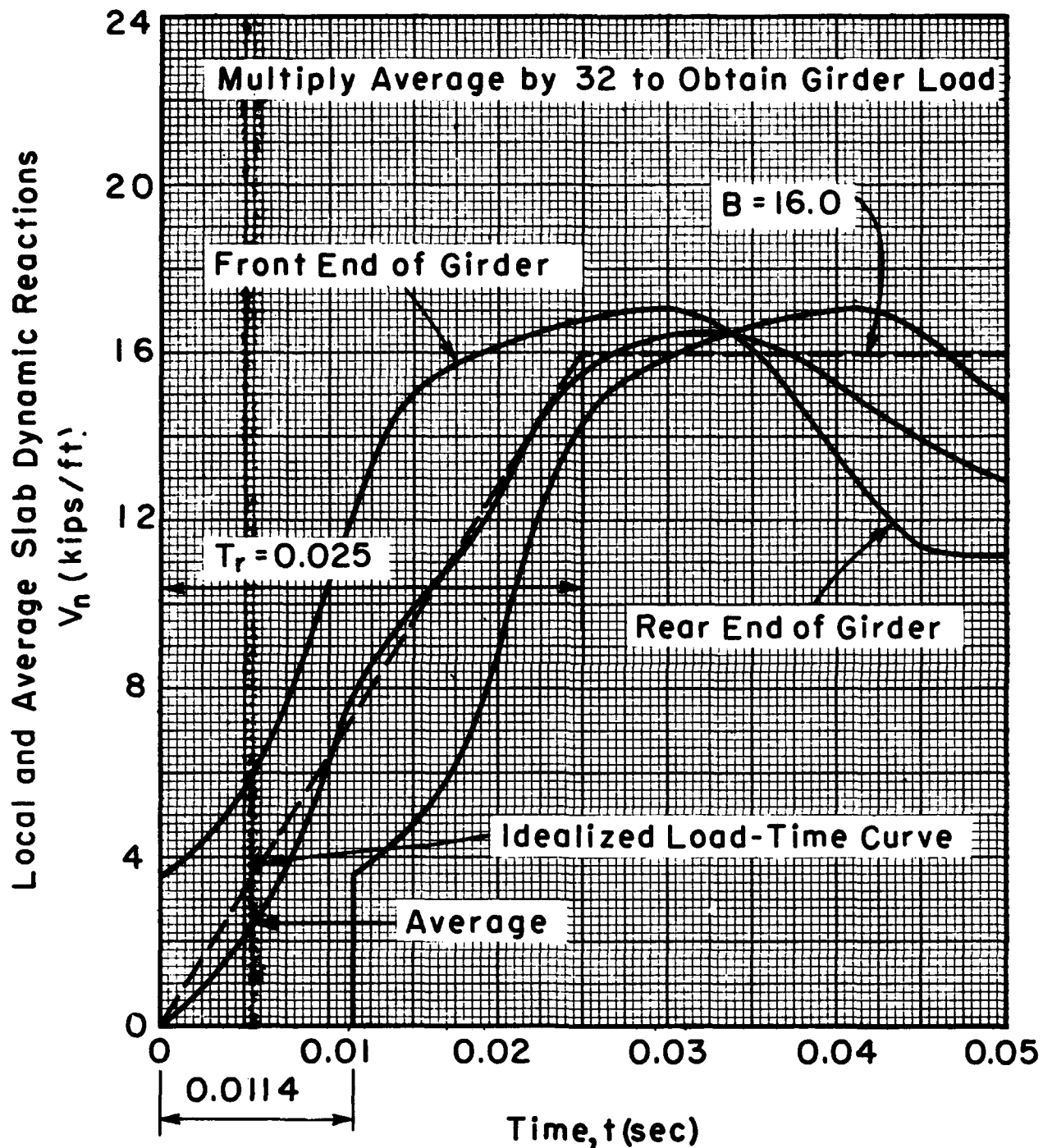
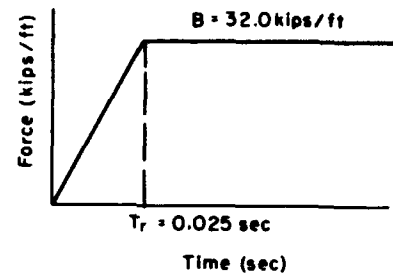


Figure 7.77. Local and average slab dynamic reactions for incident overpressure, blast wave parallel to girder

The idealized load-time curve imposed on the roof girder by a slab on one side is indicated by the dashed line in fig. 7.77. The design load on the girder is defined by



$$B = 2(16.0) = 32.0 \text{ kips/ft}$$

$$T_r = 0.025 \text{ sec}$$

b. Elastic Range Dynamic Design Factors. (Refer to table 6.4.)

$$K_L = 0.58,$$

$$K_M = 0.45,$$

$$K_{IM} = 0.78$$

$$k = f_3 EI / L^3 \text{ (fig. 6.29)}$$

$$R_m = f_1 M_{Ps} / L \text{ (fig. 6.27)}$$

$$M_{pos} = f_2 M_n \text{ (fig. 6.28)}, M_n = R_1 L / f_1 \text{ (fig. 6.27)}$$

$$V_1 = 0.26R + 0.12P$$

$$V_2 = 0.43R + 0.19P$$

c. Mass Computation.

$$\text{Slab and roofing} = \left[ \frac{9.5(150)}{12} + 6.0 \right] \frac{18(54)}{1000(3)} = 40.5 \text{ kips}$$

$$\text{Girder (estimate)} = \frac{18(36)150(16)}{12(12)1000} = 10.8 \text{ kips}$$

$$\text{Total mass} = \frac{40.5 + 10.8}{32.2} = 1.59 \text{ kip-sec}^2/\text{ft}$$

d. First Trial - Actual Properties.

Estimate tee beam action for first trial,  $f_1 = 9$ ,  $f_2 = 0.67$ ,  $f_3 = 240$

Assume D.L.F.(B) = 1.05 (experience)

$$R_m = \text{D.L.F.}(B) = 1.05(32.0)16 = 540 \text{ kips}$$

Moment in girder at interior support (fixed support) due to static loads

$$M = \frac{WL}{8} = \frac{(40.5 + 10.8)16}{8} = 103 \text{ kip-ft}$$

Moment in girder at midspan due to static loads

$$M = \frac{9WL}{128} = \frac{9(51.3)16}{1.28} = 58 \text{ kip-ft}$$

Support moment from column

$$\frac{2}{3} M_D = \frac{2}{3} (970) = 646 \text{ kip-ft (par. 7-11 and 7-50)}$$

Midspan moment from column = 0

Support moment required for vertical blast loads

$$M = \frac{R_m L}{9} = \frac{540(16)}{9} = 960 \text{ kip-ft}$$

Midspan moment due to vertical blast loads

$$M_{pos} = f_2 M_n = 0.67(960) = 640 \text{ kip-ft}$$

Total support moment = 960 + 646 + 103 = 1709 kip-ft

Total midspan moment = 640 + 0 + 58 = 698 kip-ft

Ratio of midspan tension reinforcing to interior support reinforcing steel = 698/1709 = 0.41

$$M_p = p f_{dy} b d^2 \left( 1 - \frac{p f_{dy}}{1.7 f'_{dc}} \right) = 0.015(52) b d^2 \left[ 1 - \frac{0.015(52)}{1.7(3.9)} \right]$$

$$M_p = 0.688 b d^2 = 1709(12) = 20,500 \text{ kip-in.}$$

Try b = 20 in.

$$d = \sqrt{20,500/13.76} = 38.6 \text{ in.}$$

Try d = 38.5, h = 42 in.

Rectangular section at the support

$$I_g = 20(42)^3/12 = 123,500 \text{ in.}^4$$

$$np = 10(12)/20(38.5) = 0.156$$

$$np' = 10(6)/20(38.5) = 0.078$$

$$m = np + (n - 1)p' = 0.156 + \frac{9}{10} (0.078) = 0.226$$

$$q = np + (n - 1)p' \frac{d'}{d} = 0.156 + \frac{9}{10} (0.078) \frac{2.25}{38.5} = 0.160$$

k = 0.39 (table 11, RCDH of ACI)

$$kd = 0.39(38.5) = 15.0 \text{ in.}$$

$$I_t = \frac{20(15)^3}{3} + 12(10)(23.5)^2 + 9(6)(12.75)^2$$

$$= 22,500 + 66,400 + 8,800 = 97,700 \text{ in.}^4$$

$$I_1 = (I_g + I_t)0.5 = 110,600 \text{ in.}^4$$

Tee section at midspanTension reinforcement at midspan =  $0.41(12) = 4.8$ , use 5 bars

$$\bar{y} = \frac{\frac{-96(9.5)^2}{2} + \frac{20(32.5)^2}{2}}{96(9.5) + 20(32.5)} = \frac{-4330 + 10,600}{911 + 650} = \frac{6270}{1561} = 4.0 \text{ in.}$$

$$I_g = \frac{96(9.5)^3}{3} + \frac{20(32.5)^3}{3} - 1561(4.0)^2 = 27,400 + 229,000 - 25,000 = 231,400 \text{ in.}^4$$

$$np = 10(5)/39.13(96) = 0.0133$$

$$np' = 10(3)/39.13(96) = 0.0080$$

$$m = np + (n - 1)p' = 0.0133 + \frac{9}{10} (0.008) = 0.0203$$

$$q = np + (n - 1)p' \frac{d'}{d} = 0.0133 + \frac{9}{10} (0.008) \frac{2.25}{39.75} = 0.0137$$

$$k = 0.145 \text{ (table 11, RCDH of ACI)}$$

$$kd = 0.145(39.75) = 5.76 \text{ in.}$$

$$I_t = \frac{96(5.76)^3}{3} + 9(4)(5.76 - 2.25)^2 + 10(8)(39.12 - 5.76)^2$$

$$I_t = 6150 + 443 + 89,000 = 95,600 \text{ in.}^4$$

$$I_2 = 0.5(I_g + I_t) = 163,500 \text{ in.}^4$$

$$I_1/I_2 = 110,600/163,500 = 0.68$$

$$f_1 = 9.0 \text{ (fig. 6.27)}$$

$$f_2 = 0.68 \text{ (fig. 6.28)}$$

$$f_3 = 246 \text{ (fig. 6.29)}$$

e. First Trial - Determination of D.L.F.

$$k_1 = \frac{f_1 EI_1}{L^3} = \frac{246(3)(10)^3(110,600)}{(16)^3 144} = 138,000 \text{ kips/ft}$$

$$T_n = 2\pi \sqrt{K_{IM}^m/k_1} = 6.28 \sqrt{0.78(1.59)/138,000} = 0.0189 \text{ sec}$$

$$T_r/T_n = 0.025/0.0189 = 1.325$$

$$\text{D.L.F.} = 1.2 \text{ (fig. 5.21)}$$

$$t_m/T_r \approx 1.25 \text{ (fig. 5.21); OK (see fig. 7.77)}$$

$$\text{Required } R = 1.2(32.0)(16) = 615 \text{ kips}$$

At the interior support section

$$M_p = \frac{0.0156(52)20(38.5)^2}{12} \left[ 1 - \frac{0.0156(52)}{1.7(3.9)} \right] = 1760 \text{ kip-ft}$$

The moment available at the support to resist the effects of vertical blast loads is

$$M = 1760 - 646 - 103 = 1011 \text{ kip-ft}$$

The available resistance is then

$$R_m = f_1 M/L = 9.0(1011)/16 = 569.0 \text{ kips}$$

This girder is inadequate,  $569 < 615$

Increase reinforcing at support to 13 bars and continue with same girder. The moments of inertia will not change appreciably (par. 7-43).

f. Preliminary Design for Bond Stress.

$$\text{Estimated } V_{\max} = 0.62R_m = 0.62(615) = 381 \text{ kips}$$

$$\text{Allowable } u = 0.15f'_c = 0.15(3000) = 450 \text{ psi (par. 4-09)}$$

$$\Sigma o = \frac{V}{u_j d} = \frac{8(381,000)}{450(7)(38.5)} = 25.0 \text{ in.}$$

$$A_s = 13 \text{ \#9 bars in two rows, } A_s = 13 \text{ in.}^2$$

$$\Sigma o = 46.0 \text{ in., } p = \frac{A_s}{bd} = \frac{13}{20(38.5)} = 0.0169$$

g. Determination of Maximum Deflection and Dynamic Reactions by Numerical Integration. Since the trial size is to be used for the numerical integration with only minor modification, reference is made to the previous computations for pertinent data.

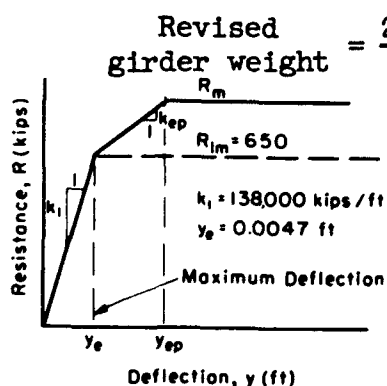


Figure 7.78. Resistance function for girder spanning 16 ft

$$\text{Revised girder weight} = \frac{20(32.5)16(150)}{12(12)1000} = 10.8 \text{ kips (same as estimate)}$$

$$\text{Total mass} = 1.59 \text{ kip-sec}^2/\text{ft}$$

$$M = 1760 \frac{13}{12} - 646 - 103 = 1156 \text{ kip-ft}$$

$$R_m = \frac{f_1 M}{L} = \frac{9.0(1156)}{16} = 650 \text{ kips}$$

$$k_1 = 138,000 \text{ kips/ft}$$

$$y_e = \frac{R_{1m}}{k_1} = \frac{650}{138,000} = 0.0047 \text{ ft}$$

Table 7.30. Determination of Maximum Deflection and Dynamic Reactions for Girder

t	P <sub>n</sub>	R <sub>n</sub>	P <sub>n</sub> -R <sub>n</sub>	$\ddot{y}_n(\Delta t)^2$	y <sub>n</sub>	V <sub>2n</sub>	V <sub>1n</sub>
(sec)	(kips)	(kips)	(kips)	(ft)	(ft)	(kips)	(kips)
0	0	0	3	0.000010	0		
0.002	19	1	18	0.000058	0.000010		
0.004	51	11	40	0.000129	0.000078		
0.006	90	38	52	0.000167	0.000275		
0.008	141	88	53	0.000171	0.000639		
0.010	202	162	40	0.000129	0.001174		
0.012	262	254	8	0.000026	0.001838		
0.014	301	349	-48	-0.000155	0.002526		
0.016	333	423	-90	-0.000290	0.003063		
0.018	365	456	-91	-0.000293	0.003308		
0.020	403	450	-47	-0.000151	0.003260		
0.022	454	422	32	0.000103	0.003061		
0.024	486	409	77	0.000248	0.002965		
0.026	506	430	76	0.000245	0.003117		
0.028	518	485	33	0.000106	0.003514		
0.030	525	554	-29	-0.000093	0.004017		
0.032	528	611	-83	-0.000267	0.004427		
0.034	525	631	-106	-0.000341	0.004570	371	227
0.036	518	603	-85	-0.000274	0.004372		
0.038	506				0.003900		

The basic equation for the numerical integration in table 7.30 is

$$y_{n+1} = \ddot{y}_n(\Delta t)^2 + 2y_n - y_{n-1} \quad (\text{table 5.3})$$

$$\ddot{y}_n(\Delta t)^2 = \frac{(P_n - R_n)(\Delta t)^2}{K_{IM}} = \frac{(P_n - R_n)(0.002)^2}{0.78(1.59)} = 3.22(10^{-6})(P_n - R_n) \text{ ft}$$

In table 7.30 the time interval  $\Delta t = 0.002$  sec is approximately  $T_n/10 = 0.00189$  sec (par. 5-08). The dynamic reaction equations are listed

in paragraph 7-51b. The  $P_n$  values for the second column are obtained from figure 7.77 multiplying by  $2(16) = 32$  to account for the 16-ft-span length of the girder and the two slabs loading the girder. The  $(y_n)_{\max}$  value =  $0.0044 \text{ ft} \approx y_e$ . The design is satisfactory for this consideration.

h. Shear and Bond Strength. For interior support end of girder (fixed end of idealized girder)

$$V_{\max} = 371 \text{ kips (table 7.30)}$$

For no shear reinforcement allowable  $v_y = 0.04f'_c + 5000p$  (eq 4.24)

$$v_y = 0.04(3000) + 5000(0.0156) = 120 + 78 = 198 \text{ psi}$$

$$v = \frac{8V}{7bd} = \frac{8(371,000)}{7(20)38.55} = 550 \text{ psi}$$

Shear reinforcement required for  $550 - 198 = 352 \text{ psi}$

Contribution of shear reinforcement to allowable shear stress =  $rf_y$

$$r = \frac{352}{40,000} = 0.0088$$

Try 4 #4 bars,  $A_s = 0.80 \text{ in.}^2$

$$r = \frac{A_s}{bs} = \frac{0.80}{20s} = 0.0088; \therefore s = 4.55 \text{ in., try } s = 4 \text{ in.}$$

For exterior support end of girder (pinned end of idealized girder)

$$V_{\max} = 227 \text{ kips (table 7.30)}$$

$$v_y = 0.04(3000) + 5000(0.00133) = 120 + 6.6 = 127 \text{ psi}$$

$$v = \frac{8V}{7bd} = \frac{8(227,000)}{7(20)39.75} = 326 \text{ psi}$$

Shear reinforcement required for  $326 - 127 = 199 \text{ psi}$

$$r = \frac{199}{40,000} = 0.00498$$

Try 4 #4 bars,  $A_s = 0.80 \text{ in.}^2$

$$r = \frac{A_s}{bs} = \frac{0.80}{20(s)} = 0.00498; \therefore s = 8.0 \text{ in. try } s = 8 \text{ in.}$$

$$u = \frac{8V}{7\Sigma bd} = \frac{8(371,000)}{7(46.0)38.55} = 238 \text{ psi}$$

Allowable  $u = 0.15f'_c = 0.15(3000) = 450 \text{ psi; OK}$

i. Summary.

20-in. by 42-in. tee beam

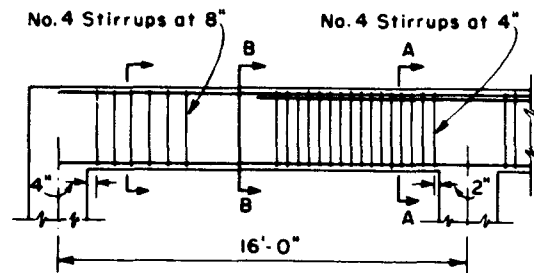
Maximum tension reinforcement = 13 #9 bars

$$p = 0.0156$$

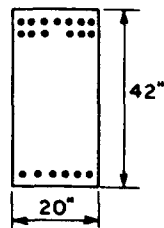
$$\Sigma o = 46 \text{ in.}$$



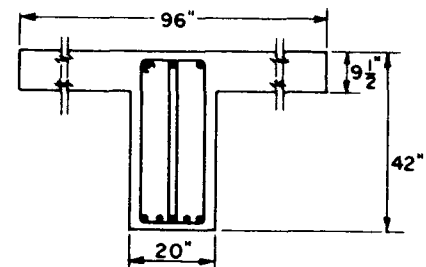
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Note: 2" Protection  
for All Steel



Effective Section at A-A



Effective Section at B-B

Shear reinforcement at interior end of girder, 4 #4 at 4 in.

Shear reinforcement at exterior end of girder, 4 #4 at 8 in.

7-52 FINAL DESIGN OF COLUMN. This column design for elastic behavior was begun in paragraph 7-50. The calculations which follow illustrate the steps which are needed to determine the adequacy of the preliminary design. The primary computation is a numerical integration in which is determined the time variation of the lateral displacement of the top of the columns.

In the preliminary design idealized loads and resistances are used in order to simplify the computations. In the final design, however, these idealizations are no longer used. This results in consideration of the following factors which have been neglected: (1) the variation of plastic hinge moment with direct stress, (2) the variation of column resistance with lateral deflection, (3) the effect of girder flexibility on the stiffness of the frame, and (4) the difference between the two design loads; one determined from the product of the front face overpressure and the wall area and the other determined from the wall slab dynamic reactions.

a. Mass Computation.

Roof slab = 122 kips (par. 7-50b)

Girder stem =  $\frac{20(35.5)50.5(150)}{12(12)1000} = 37.3$  kips

Columns =  $\frac{18(28)11.75(150)4}{12(12)1000} = 24.6$  kips

Walls = 83.0 kips (par. 7-50b)

Mass of single-degree-of-freedom system = total roof slab +

$$\text{total girder} + \frac{1}{3} (\text{columns} + \text{walls}) = \frac{122 + 37.3 + 0.33(24.6 + 83.0)}{32.2}$$

$$= 6.05 \text{ kip-sec}^2/\text{ft}$$

b. Column Properties. (See par. 7-50e.)

$$b = 18 \text{ in.}, t = 28 \text{ in.}, d = 24.32 \text{ in.}, d' = 20.63 \text{ in.}$$

$$d'' = 3.687 \text{ in.}, A_s = A'_s = 8.0 \text{ in.}^2, p = p' = \frac{8}{18(24.32)} = 0.0183$$

$$M_D = A_s f_{dy} d' + P_D \left( 0.5t - \frac{P_D}{1.7bf'_{dc}} \right) \text{ (eq 4.32)}$$

$$= \frac{8.0(52)20.63}{12} + \frac{P_D}{12} \left[ 14 - \frac{P_D}{1.7(18)3.9} \right]$$

$$M_D = 716 + 1.16P_D - 0.0007P_D^2$$

c. Effect of Girder Flexibility. Consideration of the relative flexibility of the girders and columns generally results in a value of the frame elastic spring constant  $k$  which is less than the value obtained for the assumption of infinitely stiff girders (par. 7-08) in the preliminary design. To obtain this revised value a simple sidesway analysis of the frame is made. From the sidesway analysis,  $k$  is the magnitude of the lateral load required to cause unit displacement.

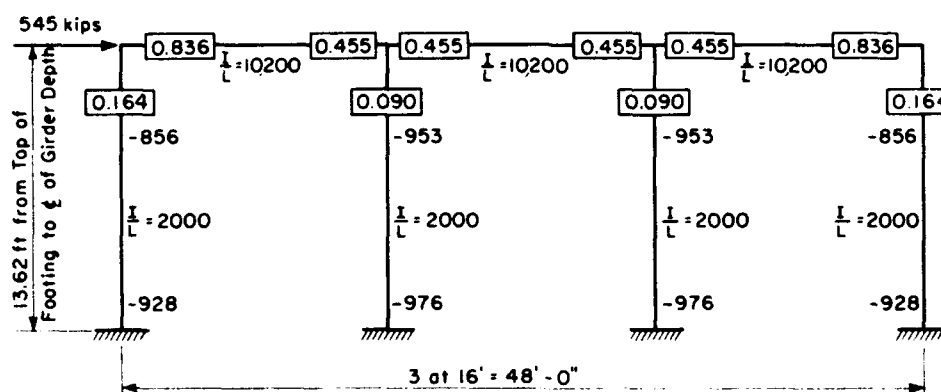


Figure 7.79. Sidesway analysis by moment distribution

The elastic sidesway analysis (the results of which are shown in figure 7.79) is performed for initial column moments of 1000 kip-ft at top and bottom of each column. This is equivalent to a lateral displacement of the top of the column.

$$x = \frac{(F.E.M.)h^2}{6EI} = \frac{1000(13.62)^2 144}{6(3)10^3(27,250)} = 0.0545 \text{ ft}$$

From figure 7.79

$$R = \frac{\Sigma M}{h} = \frac{2(856 + 928 + 953 + 976)}{13.62} = 545 \text{ kips}$$

$$k = \frac{R}{x} = \frac{545}{0.0545} = 10,000 \text{ kips/ft}$$

d. Loading. The  $F_n$  column of the numerical integration analysis (table 7.31) is obtained from figure 7.80. The first part of figure 7.80

Table 7.31. Determination of Column Adequacy

t (sec)	$P_n$ (kips)	$(P_{av})_n$ (kips)	$(M_D)_n$ (kip-ft)	$(R_m)_n$ (kips)	$F_n$ (kips)	$R_n$ (kips)	$F_n - R_n$ (kips)	$\ddot{x}_n(\Delta t)^2$ (ft)	$x_n$ (ft)
0	159	40	761	517	200	0	100.0	0.00165	0
0.01	467	117	843	573	475	16.5	458.5	0.00757	0.00165
0.02	775	194	915	622	475	108.7	366.3	0.00604	0.01087
0.03	1083	271	979	666	275	261.3	13.7	0.00023	0.02613
0.04	1358	340	1029	699	230	416.2	-186.2	-0.00307	0.04162
0.05	1308	327	1020	694	185	540.4	-355.4	-0.00586	0.05404
0.06	1259	315	1012	689	144	606.0	-462.0	-0.00762	0.06060*
0.07									0.05954

\*  $(x_n)_{\max} = 0.061 \text{ ft}$

is based on the  $V_{ln}$  dynamic reaction column of the wall slab analysis (table 7.28). The dynamic reactions for a one-foot width of wall are multiplied by 18, the width of one frame bay. The portion of the curve after  $t = 0.025$  sec is based on the net lateral overpressure curve (fig. 7.73). Here the values of  $F_n$  are obtained from  $\bar{P}_{net}$  in psi by using

$$F_n = \frac{144(18)8.16}{1000} \bar{P}_{net} = 21.2\bar{P}_{net} \text{ kips}$$

The dimension 8.16 is equal to one-half the front wall clear span plus the roof slab thickness  $\frac{(14.75)}{2} + 0.79 = 8.16 \text{ ft}$ .

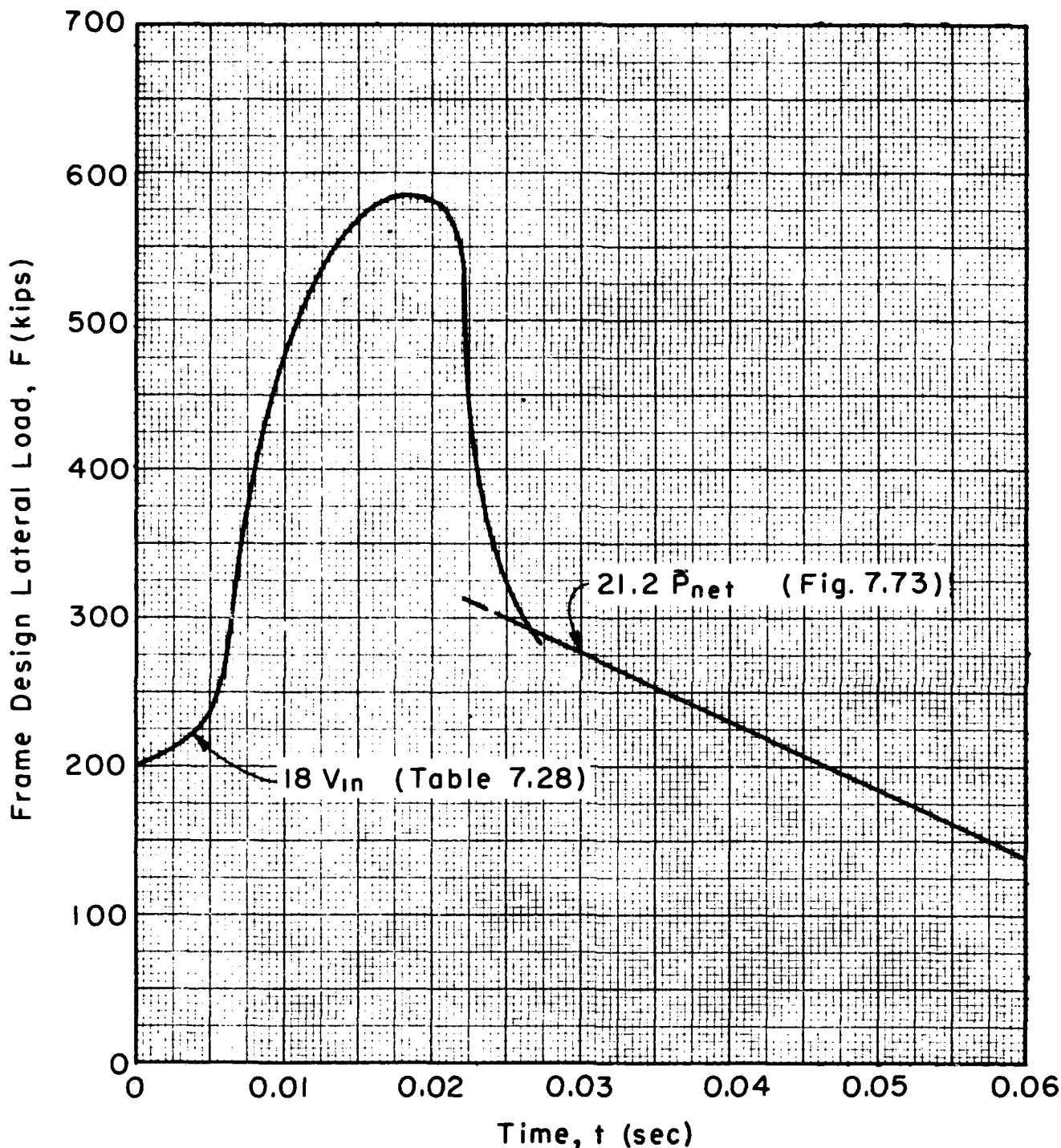


Figure 7.80. Frame design lateral load

e. Numerical Integration Computation to Determine Column Adequacy.

The total vertical load  $P_n$  in the second column is obtained by multiplying the average roof overpressure (fig. 7.74) by  $[54(18)144]/1000 = 140$  and adding the dead weight of the roof system (159.3 kips). The  $(P_{av})_n$  values are

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the average axial column loads and are obtained by dividing the total vertical load by the number of columns, 4. The  $(P_{av})_n$  column is used as  $P_D$  in the formula of paragraph 7-52b to obtain the value of  $(M_D)_n$ . The value of  $(M_D)_n$  is used in turn to obtain the maximum resistance at any time from the relation

$$(R_m)_n = \frac{2n}{h_c} (M_D)_n = \frac{2(4)(M_D)_n}{11.75} = 0.68(M_D)_n$$

$R_n$  is equal to  $kx_n = 10,000x_n$  in the elastic range.

The basic equation for the numerical integration analysis in table 7.31 is

$$x_{n+1} = \ddot{x}_n(\Delta t)^2 + 2x_n - x_{n-1} \quad (\text{table 5.3})$$

where

$$\ddot{x}_n(\Delta t)^2 = \frac{(F_n - R_n)}{m} (\Delta t)^2 = \frac{(F_n - R_n)}{6.05} (0.01)^2 = 0.165(10^{-4})(F_n - R_n) \text{ ft}$$

The time interval  $\Delta t = 0.01$  sec used in table 7.31 is based on the natural period,  $T_n = 2\pi\sqrt{m/k} = 6.28\sqrt{6.05/10,000} = 0.1545$  sec

$$\Delta t = 0.01 < T_n/10 = 0.01545 \text{ sec (par. 5-08)}$$

In table 7.31 the design is elastic because  $R_n < (R_m)_n$ , i.e.,  
606 < 689.

#### f. Shear and Bond Stress.

$$V_{\max} = \frac{606}{4} = 151.5 \text{ kips (table 7.31)}$$

For no shear reinforcement,

$$\text{Allowable } v_y = 0.04f'_c + 5000p \quad (\text{eq 4.24})$$

$$v_y = 0.04(3000) + 5000(0.0183) = 120 + 90 = 210 \text{ psi}$$

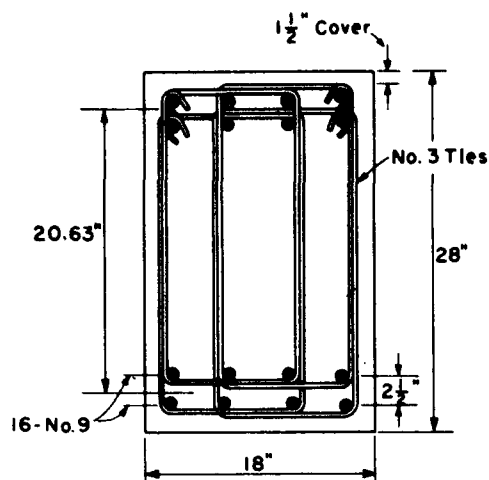
$$v = \frac{8V}{7bd} = \frac{8(151,500)}{7(18)(24.32)} = 395 \text{ psi}$$

Shear reinforcement required for  $395 - 210 = 185$  psi

$$3 \text{ \#3 column ties, } A_s = 6(0.11) = 0.66 \text{ in.}^2$$

$$r = \frac{185}{40,000} = 0.00462$$

$$r = \frac{A_s}{bs} = 0.00462 = \frac{0.66}{18(s)} ; \therefore s = 7.95 \text{ in., use } s = 8 \text{ in.}$$



$$\Sigma o = 32.0 \text{ in.}$$

$$u = \frac{8(151,500)}{7(32.0)(24.32)} = 222 \text{ psi}$$

$$\text{Allowable } u = 0.15f'_c = 0.15(3000) = 450 \text{ psi; OK}$$

g. Summary.

18 in. by 28 in. tied column

$$A_s = A'_s = 8.0 \text{ in.}^2$$

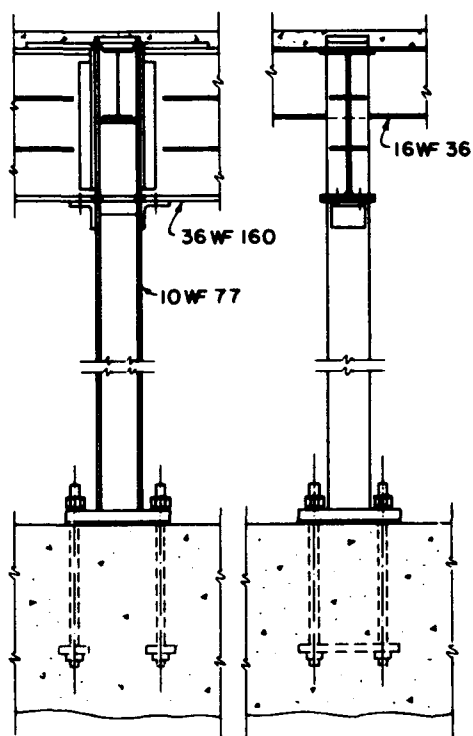
3 #3 ties at 8 in.

$$\Sigma o = 32.0 \text{ in.}$$

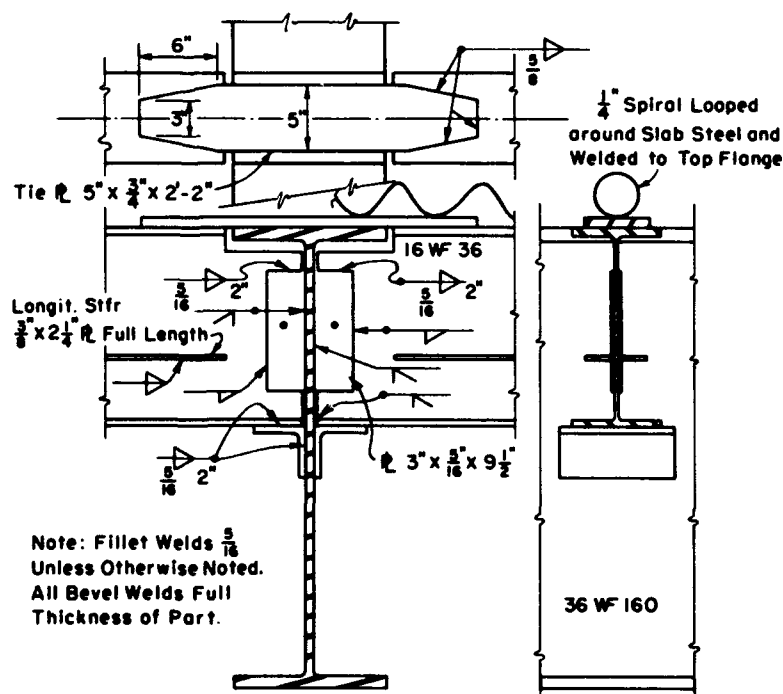
**DESIGN DETAILS**

**7-53 STRUCTURAL STEEL DETAILS.** To illustrate the type of design details that might be necessary, typical details for purlin to girder, girder to column, and column to foundation connections are presented.

Purlin to Girder Connection. The heavy end reactions and moments make necessary both seat angles and web plates. The web plates stiffen the purlin web near the connection where the longitudinal stiffeners must



Middle Column

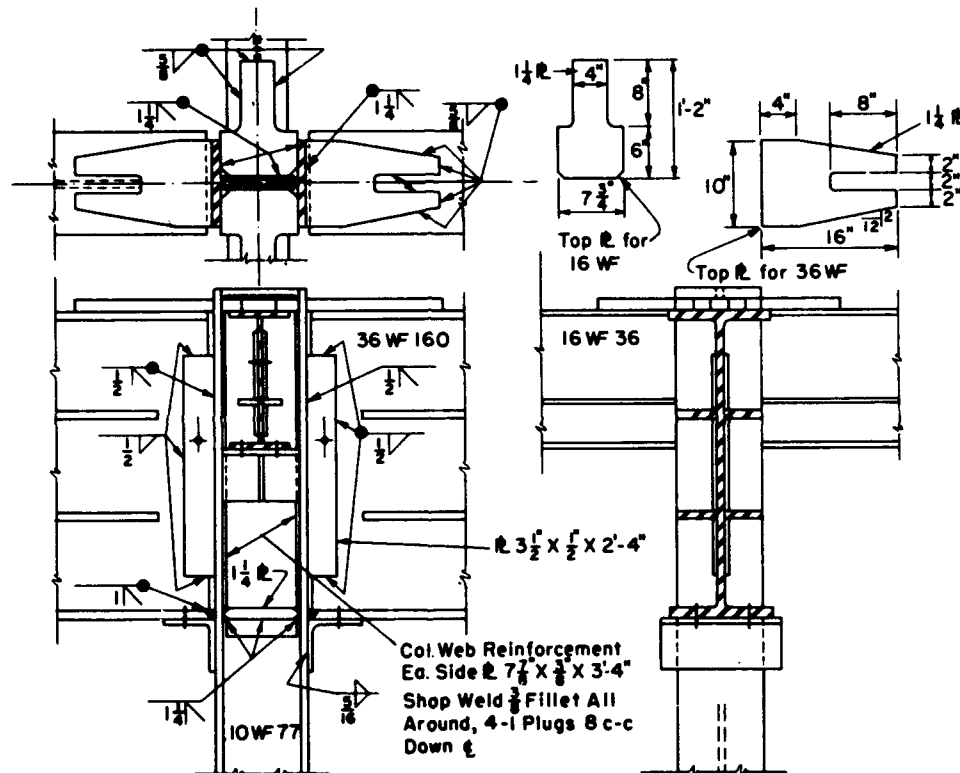


Note: Fillet Welds  $\frac{5}{16}$   
Unless Otherwise Noted.  
All Bevel Welds Full  
Thickness of Part.

Purlin to Girder Connection

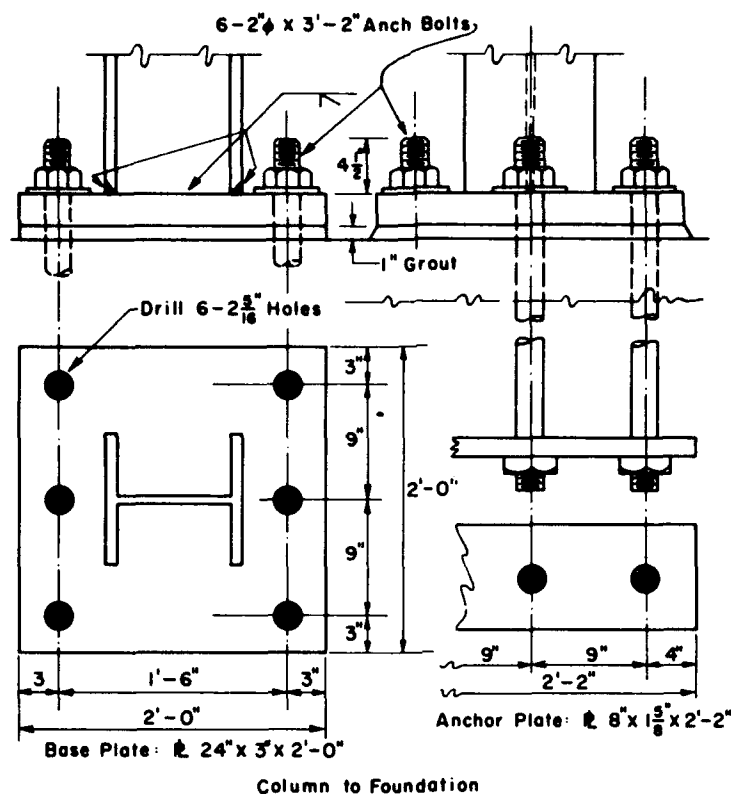
be cut. The stiffeners on the girders are not shown. The roof slab is held down by a steel spiral looped around the slab steel and welded to the purlin flange.

Girder to Column Connection. The shear force in the column web reaches a large value in the region between the girder flanges, necessitating column web reinforcing plates.



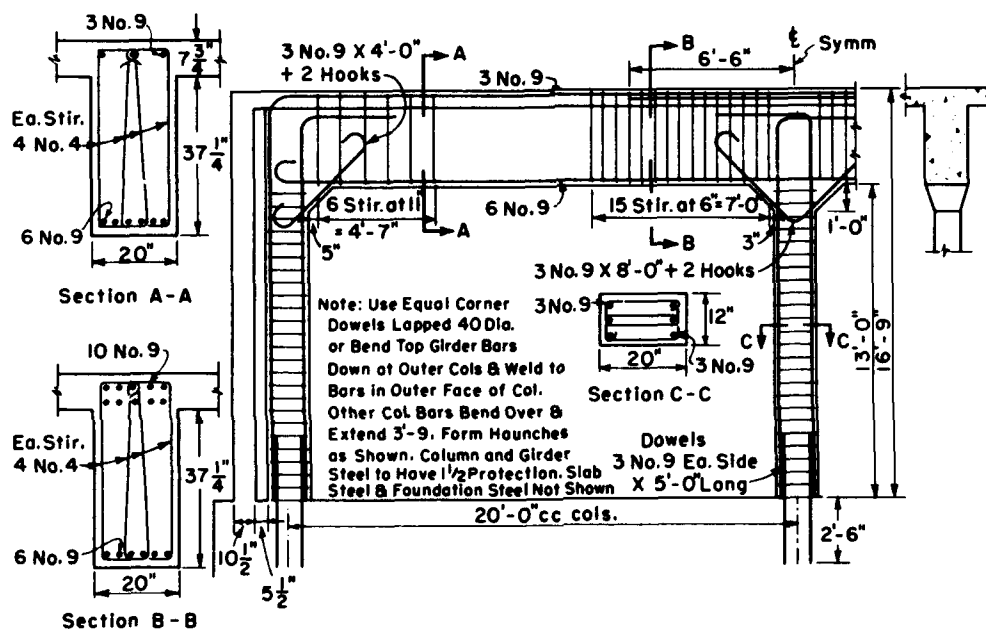
Girder To Column Conn.

Column to Foundation Connection. The base plate is shipped attached to the column to make possible a simple connection with reliable and easily inspected shop welds.



7-54 REINFORCED CONCRETE DETAILS. Typical details are presented to suggest possible treatments in critical locations.

Girder and Column Steel. Bars at the intersection of the girder and



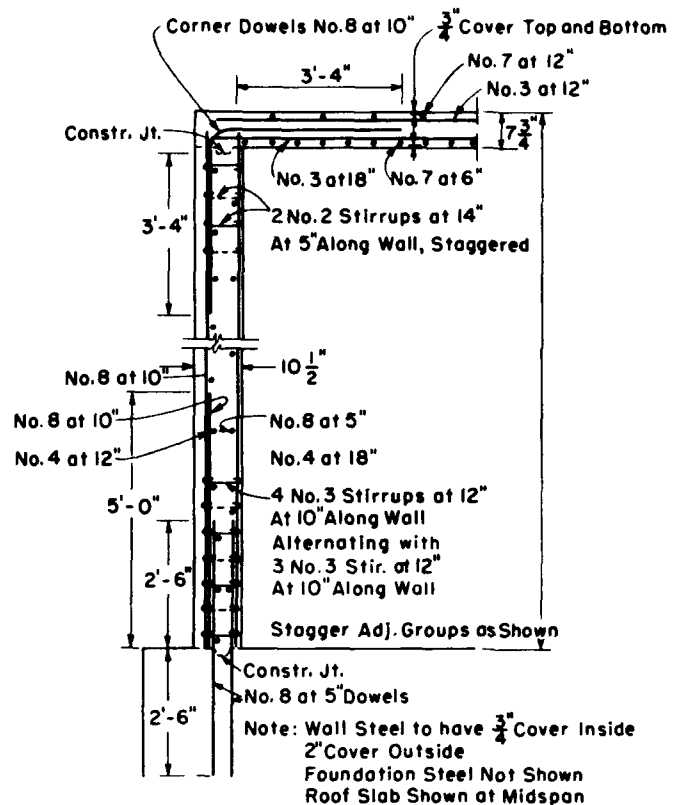
Girder and Column Steel



the outer column are shown butt-welded although dowels may be used. Dowels are shown elsewhere. The girder stirrups are bent to act as ties for bars in compression.

#### Wall and Roof Slab Section.

Corner dowels are placed to minimize any tendency to tear out of the concrete due to either inward or outward pressure. Shear reinforcement is arranged in a staggered pattern across the face of the wall.



Wall and Roof Slab Section

FOR THE CHIEF OF ENGINEERS

C. H. DUNN  
Colonel, Corps of Engineers  
Executive